KING SAUD UNIVERSITY
COLLEGE OF SCIENCE
DEPARTMENT OF MATHEMATICS
M - 203 (DIFFERENTIAL AND INTEGRAL CALCULUS)
FINAL EXAMINATION (II SEMESTER 1447/1448, 2025/2026

Max. Marks: 40 Time: 3 Hours

Marking Scheme: [Q.1. (4+4+4); Q.2. (4+4+4); Q.3. (4+4+4+4)]

Note: No calculator is allowed.

Q#1) (a) Test the convergence or divergence of the series

$$\sum_{n=1}^{\infty}\left(\frac{1}{2n+1}-\frac{1}{2n+2}\right)$$

(b) Find the interval of convergence and the radius of convergence for the power series

$$\sum_{n=1}^{\infty}\frac{(-1)^n x^n}{n\sqrt{n}\,4^{n+2}}$$

(c) Find the Maclaurin series of e^x and use its first three non-zero terms to approximate the series

$$\int_0^1 \frac{e^{x^2}-1-x^2}{x^4}\,dx$$

Q#2) (a) Evaluate the integral by reversing the order of integration

$$\int_0^1 \int_{\sin^{-1} y}^{\frac{\pi}{2}} \cos^2(x)\,dxdy$$

(b) Use polar coordinates to evaluate the integral

$$\int_1^2 \int_0^x \frac{1}{\sqrt{x^2+y^2}}\,dydx$$

(c) Evaluate the iterated integral

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{2}(x^2+y^2)\,dzdydx$$

by changing to cylindrical coordinates.

Q#3) (a) Show that the line integral

$$\int_{(1,2,3)}^{(3,2,1)}(2xy-z^2)dx+(x^2+2z)dy+(2z+2y-2xz)dz$$

is independent of path and find its value.

(b) Use Green's theorem to find the area enclosed by the ellipse $9x^2+4y^2=36$.

(c) Let S be the surface that is boundary of the region bounded by the graphs of the equations $z=2-x^2-y^2$, $z=x^2+y^2$ and let $\mathbf{F}$ be the force give by $\mathbf{F}=y\mathbf{i}+x\mathbf{j}+z\mathbf{k}$ acting on the points of S. Find the flux of $\mathbf{F}$ through S.

(d) Use Stokes' theorem to evaluate the surface integral

$$\int\int_S((curl\ \mathbf{F})\cdot\mathbf{n}\,dS$$

where $\mathbf{F}(x,y,z)=y^2z\mathbf{i}+xz\mathbf{j}+x^2y^2\mathbf{k}$ and S is the part of the paraboloid $z=x^2+y^2$ that lies inside the cylinder $x^2+y^2=1$, oriented upward.