

KING SAUD UNIVERSITY

Math Department

November 2022

Time: 180mn

Final exam Math106

Question 1(3+3+3)

- a) Use the substitution $u = x^2 + 4$ to compute $\int x^3 \sqrt{x^2 + 4} dx$.
- b) Find the number(s) c that satisfies the conclusion of the mean value theorem for the function $f(x) = -x^2 + 4x$ on $[0, 3]$.
- c) Evaluate the integral $\int \frac{1}{\sqrt{e^{4x} - 16}} dx$

Question 2(2+3+3)

- a) Evaluate the integral $\int \frac{\sinh x}{\sqrt{9 + (\cosh x)^2}} dx$
- b) Find the indefinite integral $\int \frac{\ln x}{x^5} dx$
- c) Compute $\int (\sec x)^6 dx$

Question 3(3+3+2)

- a) Find $\lim_{x \rightarrow 0} (1 + 8x^2)^{\frac{1}{x^2}}$
- b) Evaluate the integral $\int \frac{8x + 32}{(x - 2)(x^2 - 4)} dx$
- c) Compute $\int \frac{1}{\sqrt{x^2 - 4x + 2}} dx$

Question 4(3+3+3)

a) Sketch the region bounded by $y = \sqrt{x+2}$, $y = -x$, and the x-axis and find its area.

b) Find the volume of the solid obtained by revolving the region bounded by

$$y = (x - 1)^2 \text{ and } y = 3 - x \text{ about the x-axis.}$$

c) Compute the area of the surface obtained by revolving the curve

$$y = \sqrt{9 - x^2} \quad -2 \leq x \leq 2 \text{ about the x-axis.}$$

Question 5(3+3)

a) Find the arc length of the parametric curve $x = \frac{t^4}{4}$, $y = \frac{t^6}{6}$ $0 \leq t \leq 1$

b) Sketch the region inside $r = 2$ and outside $r = 4 - 4\cos\theta$ and find its area.