KING SAUD UNIVERSITY DEPARTMENT OF MATHEMATICS
TIME: 3H, FULL MARKS: 40, SI /24/12/2020 MATH 204

Question 1. [4,4] a) Solve the differential equation

$$(2x^2 + y)dx + (x^2y - x)dy = 0.$$

b) Find the largest interval I for which the following initial value problem has a unique solution

$$\begin{cases} (x^2 - x - 2)y'' + \sqrt{16 - x^2}.y' + 4y = e^x \\ y(0) = 0, \ y'(0) = 1. \end{cases}$$

Question 2. a) [4,4,5]. Find only the form of the particular solution of the differential equation

$$y'' - 2y' - 3y = 4 - 2\cos x + x^2e^{3x}.$$

b) Solve the differential equation

$$y'' + y = \tan x, \quad 0 < x < \frac{\pi}{2}.$$

c) Determine the general solution of the DE:

$$x^2y'' + 3xy' + y = 4\ln x, \quad x > 0,$$

by using the method of reduction of order, given that $y_1 = \frac{1}{x}$ is a solution of the homogeneous equation.

Question 3 [5,4]. a) Solve the system of differential equations

$$\begin{cases} x' - x - y = t \\ y' - 3x + y = 1 - t \end{cases}$$

b) Determine a linear differential equation with constant coefficients having solutions

$$y_1 = 1, \ y_2 = x, \ y_3 = x^2, \ y_4 = e^{2x}, \ y_5 = e^{-x}.$$

Question 4. a) [5,5]. Consider the $2\pi-$periodic function f defined by:

$$f(x) = \pi - |x|, \quad -\pi \le x \le \pi$$

Sketch the graph of f on the interval $[-2\pi, 2\pi]$, find its Fourier series, and deduce the value of the numerical series $\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$.

b) Let $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} \pi, & |x| \le \pi \\ 0, & |x| > \pi \end{cases}$$

Sketch the graph of f, find its Fourier integral, and deduce the value of the integral

$$\int_0^\infty \frac{\sin(2\pi\lambda)}{\lambda} d\lambda.$$