

KING SAUD UNIVERSITY DEPARTMENT OF MATHEMATICS
TIME: 3H, FULL MARKS: 40, SI /29/04/1441 MATH 204

Question 1. [4,4] a) A boy with a thermometer in his pocket reading $40^{\circ}C$ falls in a swimming pool whose temperature is maintained at $30^{\circ}C$. If after 1 minute the thermometer reads $32^{\circ}C$, what will be the reading after 3 minutes.

b) Find the general solution of the differential equation

$$(4x \sin y + 6)dx + (x^2 \cos y)dy = 0, \quad x > 0.$$

Question 2. a) [4,5]. Solve the initial value problem

$$y' = \frac{(y - 2x + 1)^2}{y - 2x}, \quad y(0) = 4\sqrt{3}.$$

b) Find an interval I for which the following initial value problem has a unique solution

$$(4 - x^2)y'' + \frac{x}{\sqrt{x+1}}y' + y \ln\left(1 - \frac{x}{4}\right) = 0, \quad y(0) = 1, y'(0) = 0.$$

Question 3. a) [4,4]. Use undetermined coefficients method to solve the differential equation

$$y'' - y' - 2y = 4e^{3x} + 5 \sin x.$$

b) Solve the differential equation

$$y'' - 6y' + 9y = \frac{e^{3x}}{1+x}.$$

Question 4 [5]. Use power series method to find the power series solution about the ordinary point $x_0 = 0$ for the differential equation

$$(x - 1)y'' - xy' + y = 0.$$

Question 5. a) [5,5]. Let $f(x)$ be a 2π -periodic function defined by:

$$f(x) = \begin{cases} 1, & -\pi < x < -\frac{\pi}{2}, \quad \frac{\pi}{2} < x < \pi \\ 0, & |x| \leq \frac{\pi}{2}. \end{cases}$$

Sketch the graph of f , find its Fourier Series, and deduce that

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = \frac{\pi}{4}.$$

b) Consider the function

$$f(x) = \begin{cases} x, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$$

Sketch the graph of f , find its Fourier integral and deduce the value of the integral

$$\int_0^{\infty} \frac{2 \sin^2 \lambda - \lambda \sin(2\lambda)}{\lambda^2} d\lambda.$$