Exercises Math 254

CH	Sections	Problem Set
1	1.2, 1.3	7. Find the absolute and relative errors in approximating π by 3.1416. What are the corresponding errors in the approximation $100\pi \approx 314.16$? 8. Calculate the error, relative error, and number of significant digits in the following approximations, with $p \approx x$: *(a)* $x = 25.234, \quad p = 25.255.$ *(b)* $x = e, \quad p = 19/7.$ *(c)* $x = \sqrt{2}, \quad p = 1.414.$
2	2.1, 2.2, 2.3, 2.4, 2.5.	*Book:* Page 76: 1 (a,c), 2(b), 3⟶ 15.
3	3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7.	*Book:* Page 178: 2⟶16 36. Consider the following system of equations: $$\begin{aligned} 4x_1 + 2x_2 + x_3 &= 1 \\ x_1 + 7x_2 + x_3 &= 4 \\ x_1 + x_2 + 20x_3 &= 7. \end{aligned}$$ *(a)* Show that the Jacobi method converges using $\|T_J\|_\infty < 1$. *(b)* Compute the second approximation $\mathbf{x}^{(2)}$, starting with $\mathbf{x}^{(0)} = [0, 0, 0]^T$. *(c)* Compute an error estimate $\|\mathbf{x} - \mathbf{x}^{(2)}\|_\infty$ for your approximation. 38. Consider the following system of equations: $$\begin{aligned} 4x_1 + 2x_2 + x_3 &= 11 \\ -x_1 + 2x_2 &= 3 \\ 2x_1 + x_2 + 4x_3 &= 16. \end{aligned}$$ *(a)* Show that the Gauss-Seidel method converges using $\|T_G\|_\infty < 1$. *(b)* Compute the second approximation $\mathbf{x}^{(2)}$, starting with $\mathbf{x}^{(0)} = [1, 1, 1]^T$. *(c)* Compute an error estimate $\|\mathbf{x} - \mathbf{x}^{(2)}\|_\infty$ for your approximation.

		~~44.~~ Consider the following system: $$\begin{aligned} 4x_1 - 2x_2 - x_3 &= 1 \\ -x_1 + 4x_2 - x_4 &= 2 \\ -x_1 + 4x_3 - x_4 &= 0 \\ -x_2 - x_3 + 4x_4 &= 1. \end{aligned}$$ Using $\mathbf{x}^{(0)} = \mathbf{0}$, how many iterations are required to approximate the solution to within five decimal places using: *(a)* Jacobi method, *(b)* Gauss-Seidel method
4	4.1, 4.2, 4.3.	*Book:* Page 236: 1⟶19.
5	5.1, 5.2, 5.3, 5.4, 5.5.	*Book:* Page 293: 1⟶21 ~~11.~~ Use the most accurate formula to determine approximations that will complete the following table: x | f(x) | $f'(x)$ 2.1 | -1.709847 | 2.2 | -1.373823 | 2.3 | -1.11921 | 2.4 | -0.916014 | ~~35.~~ Evaluate $\int_0^1 e^{x^2} dx$ by Simpson's rule choosing h small enough to guarantee five decimal accuracy. How large can h be?
6	6.1, 6.2, 6.3.	*Book:* Page 316: 3⟶5