254 MATH (Numerical Methods)

Textbook:

An Introduction to Numerical Methods and Analysis Using MATLAB

by Rizwan Butt.

Chapter	Sections	Exercises
1	1.2, 1.3	1) Find the maximum and minimum of the following functions a) $f(x) = \frac{x}{x^2+1}$ on $[0, 2]$. b) $f(x) = \frac{ln(x)}{x}$ on $[1, 3]$. c) $f(x) = x^2 - x - 6$, on $[-4, 4]$. d) $f(x) = \lvert x^2 - x - 6 \rvert$, on $[-4, 4]$. (Hint: $\lvert x \rvert$ is not differentiable at $x = 0$). 2) Find the absolute and relative errors in approximating π by 3.1416. What are the corresponding errors in the approximation $100\pi \approx 314.16$? 3) Calculate the error, relative error, and number of significant digits in the following approximations, with $p \approx x$: a) $x = 25.234,\ p = 25.255$. b) $x = e,\ p = 19/7$. c) $x = \sqrt{2},\ p = 1.414$.
2	2.1, 2.2, 2.3, 2.4, 2.5	1) Show that the following functions have a unique fixed point on $[-1, 1]$. a) $g(x) = \frac{x^3-1}{6}$ b) $g(x) = \frac{x^2}{6} - 1$ 2) Use the definition to show that a) $x_n = \frac{1}{n^2}$ converges linearly to zero.

		b) $x_n = 10^{-2^n}$ converges quadratically to zero. Textbook: Example: 2.4, 2.63. Exercises: page 76: 1(a,c), 2(b), 3 -15.
3	3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7	Textbook: Example: 3.20, 3.26, 3.41, 3.43, 3.51, 3.55 Exercises: page 178: 2 -16. 1) Consider the following system of equations: $$\begin{aligned} 4x_1 + 2x_2 + x_3 &= 1 \\ x_1 + 7x_2 + x_3 &= 4 \\ x_1 + x_2 + 20x_3 &= 7 \end{aligned}$$ a) Show that the Jacobi method converges using $\|T_J\|_\infty < 1$. b) Compute the second approximation $x^{(2)}$, starting with $x^{(0)} = [0,0,0]^T$. c) Compute an error estimate $\|x - x^{(2)}\|_\infty$ for your approximation. 2) Consider the following system of equations: $$\begin{aligned} 4x_1 + 2x_2 + x_3 &= 11 \\ -x_1 + 2x_2 + &= 3 \\ 2x_1 + x_2 + 4x_3 &= 16 \end{aligned}$$ a) Show that the Gauss-Seidel method converges using $\|T_G\|_\infty < 1$. b) Compute the second approximation $x^{(2)}$, starting with $x^{(0)} = [1,1,1]^T$. c) Compute an error estimate $\|x - x^{(2)}\|_\infty$ for your approximation. 3) Consider the following system of equations: $$\begin{aligned} 4x_1 - 2x_2 - x_3 &= 1 \\ -x_1 + 4x_2 + - x_4 &= 2 \\ -x_1 + 4x_3 - x_4 &= 0 \\ - x_2 - x_3 + 4x_4 &= 1 \end{aligned}$$ Using $x^{(0)} = 0$, how many iterations are required to approximate the solution to within five decimal places using: (a)Jacobi method, (b) Gauss-Seidel method.

<table>
<tr><td>4</td><td>4.1, 4.2, 4.3</td><td>Textbook: page 236: 1 -19.</td></tr>
<tr><td>5</td><td>5.1, 5.2, 5.3, 5.4, 5.5</td><td>Textbook: page 293: 1 -21.
1) Use the most accurate formula to determine approximations that will complete the following table:
<table><tr><td>x</td><td>f(x)</td><td>df/dx</td></tr><tr><td>2.1</td><td>- 1.709847</td><td></td></tr><tr><td>2.2</td><td>-1.373823</td><td></td></tr><tr><td>2.3</td><td>- 1.11921</td><td></td></tr><tr><td>2.4</td><td>- 0.916014</td><td></td></tr></table>2) Evaluate $\int_0^1 e^{x^2} dx$ by Simpson's rule choosing h small enough to guarantee five decimal accuracy. How large can h be?</td></tr>
<tr><td>6</td><td>6.1, 6.2, 6.3</td><td>Textbook: page 316: 3 -5.</td></tr>
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