Exercises chapter 7

7.9 Exercises

7.1 The number of deaths from leukemia and other cancers among survivors of the Hiroshima atom bomb are shown in Table 7.12, classified by the radiation dose received. The data refer to deaths during the period 1950–1959 among survivors who were aged 25 to 64 years in 1950 (from data set 13 of Cox and Snell 1981, attributed to Otake 1979).

(a) Obtain a suitable model to describe the dose–response relationship between radiation and the proportional cancer mortality rates for leukemia.

(b) Examine how well the model describes the data.

(c) Interpret the results.

Table 7.12 *Deaths from leukemia and other cancers classified by radiation dose received from the Hiroshima atomic bomb.*

	Radiation dose (rads)					
Deaths	0	1–9	10–49	50–99	100–199	200+
Leukemia	13	5	5	3	4	18
Other cancers	378	200	151	47	31	33
Total cancers	391	205	156	50	35	51

7.2 **Odds ratios**. Consider a 2×2 contingency table from a prospective study in which people who were or were not exposed to some pollutant are followed up and, after several years, categorized according to the presence or absence of a disease. Table 7.13 shows the probabilities for each cell. The odds of disease for either exposure group is $O_i = \pi_i/(1 - \pi_i)$, for

$i = 1, 2$, and so the odds ratio

$$\phi = \frac{O_1}{O_2} = \frac{\pi_1(1-\pi_2)}{\pi_2(1-\pi_1)}$$

is a measure of the relative likelihood of disease for the exposed and not exposed groups.

Table 7.13 *2×2 table for a prospective study of exposure and disease outcome.*

	Diseased	Not diseased
Exposed	π_1	$1-\pi_1$
Not exposed	π_2	$1-\pi_2$

(a) For the simple logistic model $\pi_i = e^{\beta_i}/(1+e^{\beta_i})$, show that if there is no difference between the exposed and not exposed groups (i.e., $\beta_1 = \beta_2$), then $\phi = 1$.

(b) Consider J 2×2 tables like Table 7.13, one for each level x_j of a factor, such as age group, with $j = 1, \ldots, J$. For the logistic model

$$\pi_{ij} = \frac{\exp(\alpha_i + \beta_i x_j)}{1+\exp(\alpha_i + \beta_i x_j)}, \qquad i = 1, 2, \quad j = 1, \ldots, J.$$

Show that $\log \phi$ is constant over all tables if $\beta_1 = \beta_2$ (McKinlay 1978).

7.3 Tables 7.14 and 7.15 show the survival 50 years after graduation of men and women who graduated each year from 1938 to 1947 from various Faculties of the University of Adelaide (data compiled by J.A. Keats). The columns labelled S contain the number of graduates who survived and the columns labelled T contain the total number of graduates. There were insufficient women graduates from the Faculties of Medicine and Engineering to warrant analysis.

(a) Are the proportions of graduates who survived for 50 years after graduation the same all years of graduation?

(b) Are the proportions of male graduates who survived for 50 years after graduation the same for all Faculties?

(c) Are the proportions of female graduates who survived for 50 years after graduation the same for Arts and Science?

(d) Is the difference between men and women in the proportion of graduates who survived for 50 years after graduation the same for Arts and Science?

7.4 Let $l(\mathbf{b}_{\min})$ denote the maximum value of the log-likelihood function for the minimal model with linear predictor $\mathbf{x}^T\boldsymbol{\beta} = \beta_1$, and let $l(\mathbf{b})$ be the corresponding value for a more general model $\mathbf{x}^T\boldsymbol{\beta} = \beta_1 + \beta_2 x_1 + \ldots + \beta_p x_{p-1}$.

Table 7.14 *Fifty years survival for men after graduation from the University of Adelaide.*

Year of graduation	Faculty Medicine S	T	Arts S	T	Science S	T	Engineering S	T
1938	18	22	16	30	9	14	10	16
1939	16	23	13	22	9	12	7	11
1940	7	17	11	25	12	19	12	15
1941	12	25	12	14	12	15	8	9
1942	24	50	8	12	20	28	5	7
1943	16	21	11	20	16	21	1	2
1944	22	32	4	10	25	31	16	22
1945	12	14	4	12	32	38	19	25
1946	22	34			4	5		
1947	28	37	13	23	25	31	25	35
Total	177	275	92	168	164	214	100	139

Table 7.15 *Fifty years survival for women after graduation from the University of Adelaide.*

Year of graduation	Faculty Arts S	T	Science S	T
1938	14	19	1	1
1939	11	16	4	4
1940	15	18	6	7
1941	15	21	3	3
1942	8	9	4	4
1943	13	13	8	9
1944	18	22	5	5
1945	18	22	16	17
1946	1	1	1	1
1947	13	16	10	10
Total	126	157	58	61

(a) Show that the likelihood ratio chi-squared statistic is

$$C = 2\left[l(\mathbf{b}) - l(\mathbf{b}_{\min})\right] = D_0 - D_1,$$

where D_0 is the deviance for the minimal model and D_1 is the deviance for the more general model.

(b) Deduce that if $\beta_2 = \ldots = \beta_p = 0$, then C has the central chi-squared distribution with $(p-1)$ degrees of freedom.