

Math 106

Integral Calculus

Exercises Chapter 6 (Logarithm and Exponential Functions)

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6.2 Natural Logarithm Function

Natural Logarithm Function

Example

Find $f'(x)$ where

① $f(x) = \ln(3x^2 - 2x + 1)$

② $f(x) = \ln |4 - x|$

③ $f(x) = x \ln x$

④ $f(x) = \frac{1}{\ln x} + \ln \frac{1}{x}$

Natural Logarithm Function

Example

Find $f'(x)$ where

① $f(x) = \ln |\sqrt[3]{4x-5} (3x+8)^2|$

② $f(x) = \ln \frac{\sqrt{x^2+1}}{(9x-4)^2}$

③ $f(x) = \ln |\sin x|$

Natural Logarithm Function

Example

Find y'

$$3y - x^2 + \ln xy = 2$$

Natural Logarithm Function

Example

Use logarithmic differentiation to find y' .

① $y = (4x + 2)^3(6x + 1)^2$

② $y = \sqrt{4x + 7}(x - 5)^2$

③ $y = \sqrt{(3x^2 + 2)\sqrt{6x - 7}}$

6.3 Exponential Function

Exponential Function

Example

Find $f'(x)$ where

① $f(x) = e^{-5x}$

② $f(x) = e^{3x^2}$

③ $f(x) = \frac{1}{e^x + 1}$

④ $f(x) = e^{\frac{1}{x}} + \frac{1}{e^x}$

Exponential Function

Example

Find y'

① $e^{xy} - x^3 + 3y^2 = 11$

② $e^x \cot y = xe^{2y}$

6.4 Integration(natural logarithm and exponential functions)

Integration (natural logarithm and exponential functions)

Example

Evaluate the following integrals

① $\int_{-2}^1 \frac{1}{2x+7} dx$

② $\int_1^2 \frac{4x}{x^2-9} dx$

③ $\int_1^2 x^2 e^{3x^3} dx$

Integration (natural logarithm and exponential functions)

Example

Evaluate the following integrals

① $\int \frac{x-2}{x^2-4x+9} dx$

② $\int \frac{\ln x}{x} dx$

③ $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

Integration (natural logarithm and exponential functions)

Example

Evaluate the following integrals

$$\textcircled{1} \int \frac{3 \sin x}{1 + 2 \cos x} dx$$

$$\textcircled{2} \int e^{\cos x} \sin x dx$$

$$\textcircled{3} \int \frac{\cos x \sin x}{\cos^2 x - 1} dx$$

Integration (natural logarithm and exponential functions)

Example

Evaluate the following integrals

1 $\int \frac{\cos^2 x}{\sin x} dx$

6.5 General Exponential and Logarithmic Functions

General Exponential and Logarithmic Functions

Definition

For every $a > 0$, and $x \in \mathbb{R}$,

$$a^x = e^{x \ln a}$$

Theorem

$$\int a^x dx = \frac{1}{\ln a} a^x + c$$

General Exponential and Logarithmic Functions

Definition

The general logarithmic function with base a where $a > 0$ and $a \neq 1$ is denoted by $\log_a x$ and it is the inverse function of the general exponential function a^x

$$y = \log_a x \quad \text{iff} \quad x = a^y$$

$$\log_a x = \frac{\ln x}{\ln a}$$

General Exponential and Logarithmic Functions

Example

Find $f'(x)$

① $f(x) = 7^x$

② $f(x) = \log(x^4 + 3x^2 + 1)$

③ $f(x) = \log \ln x$

④ $f(x) = x^e + e^x + e^e$

⑤ $f(x) = x^{x^2}$

General Exponential and Logarithmic Functions

Example

Evaluate the following integrals

① $\int_{-2}^1 7^x dx$

② $\int \frac{2^x}{2^x + 1} dx$

③ $\int \frac{1}{x \log x} dx$

④ $\int \frac{5^{\tan x}}{\cos^2 x} dx$

Exam Problem

Exam problem

$$\int \frac{1}{x \ln x \ln(\ln x)} dx$$

6.7+ 6.8 Inverse Trigonometric Functions and Hyperbolic Functions

Integration of Inverse Trigonometric Functions

Theorem

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + c,$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

$$\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \sec^{-1} \frac{x}{a} + c$$

Integration of inverse hyperbolic functions

Theorem

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C, \quad a > 0$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C, \quad 0 < a < x$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{a} \tanh^{-1}\left(\frac{x}{a}\right) + C, \quad |x| < a$$

$$\int \frac{dx}{x\sqrt{a^2 - x^2}} = -\frac{1}{a} \operatorname{sech}^{-1}\left(\frac{|x|}{a}\right) + C, \quad 0 < |x| < a$$

Hyperbolic Functions

Definition

The hyperbolic sine function ($\sinh$), and hyperbolic cosine function ($\cosh$) are defined by

$$\sinh x = \frac{e^x - e^{-x}}{2}, \quad \forall x \in \mathbb{R}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}, \quad \forall x \in \mathbb{R}$$

Theorem

$$\cosh^2 x - \sinh^2 x = 1$$

Derivatives of hyperbolic functions

Theorem

$$\frac{d}{dx}(\sinh x) = \cosh x$$

$$\frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$

$$\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$$

$$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$$

Inverse Trigonometric and Hyperbolic Functions

Example

Find $f'(x)$

① $f(x) = \sin^{-1} \sqrt{x}$

② $f(x) = \tan^{-1}(3x - 5)$

③ $f(x) = \ln(\tan^{-1} x^2)$

④ $f(x) = 3^{\sin^{-1}(x^3)}$

Inverse Trigonometric and Hyperbolic Functions

Example

Evaluate the following integrals

① $\int_0^4 \frac{1}{x^2 + 16} dx$

② $\int_0^1 \frac{e^x}{1 + e^{2x}} dx$

③ $\int \frac{\cos x}{\sqrt{9 - \sin^2 x}} dx$

Inverse Trigonometric and Hyperbolic Functions

Example

Evaluate the following integrals

① $\int \frac{1}{\sqrt{x}(1+x)} dx$

② $\int \frac{e^x}{\sqrt{16 - e^{2x}}} dx$

③ $\int \frac{1}{\sqrt{e^{2x} - 25}} dx$

Inverse Trigonometric and Hyperbolic Functions

Example

Evaluate the following integrals

① $\int \frac{1}{x\sqrt{1+x}} dx$

② $\int \frac{1}{\sqrt{81+16x^2}} dx$

③ $\int \frac{1}{\sqrt{16x^2-9}} dx$

Inverse Trigonometric and Hyperbolic Functions

Example

Evaluate the following integrals

① $\int \frac{1}{49 - 4x^2} dx$

② $\int \frac{1}{x\sqrt{9 - x^4}} dx$

③ $\int \frac{1}{\sqrt{5 - e^{2x}}} dx$

Exam Problem

Exam problem

$$\int \frac{1}{\sec x \sqrt{9 \sin^2 x - \sin^4 x}} dx$$
$$0 < x < \frac{\pi}{2}$$

Hyperbolic Functions

Example

Evaluate the following integrals

① $\int x^2 \cosh x^3 dx$

② $\int \frac{1}{\operatorname{sech} 7x} dx$

③ $\int \frac{\sinh \sqrt{x}}{\sqrt{x}} dx$

Hyperbolic Functions

Example

Evaluate the following integrals

① $\int \sinh x \operatorname{sech}^2 x \, dx$

② $\int \cosh x \operatorname{csch}^2 x \, dx$

Hyperbolic Functions

Example

Find $f'(x)$

① $f(x) = \sinh^{-1} 5x$

② $f(x) = \cosh^{-1} \sqrt{x}$

③ $f(x) = \tanh^{-1}(-4x)$

④ $f(x) = \operatorname{sech}^{-1}(x^2)$