

Math 106

Integral Calculus

Exercises Chapter 4 (Integrals)

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Exercises

- 1 4.1 + 4.2 Indefinite Integral and Substitution
- 2 4.3 + 4.4 Summation Notation, Reimann Sum and Definite Integrals
- 3 4.5 Definite Integrals
- 4 4.6 Fundamental Theorem of Calculus
- 5 4.7 Numerical Integration

4.1 + 4.2 Indefinite Integral and Substitution

Indefinite Integral and Substitution

Example

Find following integrals

① $\int (3x - 1)^2 dx$

② $\int (2x - 5)(3x + 1) dx$

③ $\int \frac{8x - 5}{\sqrt[3]{x}} dx$

Indefinite Integral and Substitution

Example

Find following integrals

① $\int \frac{x^3 - 1}{x - 1} dx$

② $\int \frac{7}{\csc x} dx$

③ $\int \frac{\sec t}{\cos t} dt$

Indefinite Integral and Substitution

Example

Find following integrals

① $\int \csc x \cot x \sec x \, dx$

② $\int \frac{d}{dx} \sqrt{x^2 + 1} \, dx$

③ $\int a^2 \, dx$

Indefinite Integral and Substitution

Example

Show that $\int x^2 dx \neq x \int x dx$

Indefinite Integral and Substitution

Example

Solve the differential equation

$$f'(x) = 12x^2 - 6x + 1$$

$$f(1) = 5$$

Indefinite Integral and Substitution

Example

Find following integrals

① $\int x(2x^2 + 3)^{10} dx$

② $\int x^2 \sqrt[3]{3x^3 + 7} dx$

③ $\int \frac{(1 + \sqrt{x})^3}{\sqrt{x}} dx$

Indefinite Integral and Substitution

Example

Find following integrals

① $\int \sqrt{x} \cos \sqrt{x^3} dx$

② $\int \sqrt{3x-2} dx$

③ $\int x\sqrt{9-x^2} dx$

Indefinite Integral and Substitution

Example

Find following integrals

① $\int (3 - x^3)^2 x \, dx$

② $\int \cos(4x - 3) \, dx$

③ $\int \frac{\sin 2x}{\sqrt{1 - \cos 2x}} \, dx$

Indefinite Integral and Substitution

Example

Find following integrals

① $\int \frac{\sin x}{\cos^4 x} dx$

Exam Problem

Exam problem

Compute the integral $\int \frac{(6x + 9\sqrt{x})dx}{(x^2 + 2x\sqrt{x})^4}$

Exam Problem

Exam problem

Use the substitution $u = 1 + x^2$ to compute $\int x^5 \sqrt{1 + x^2} dx$

Exam Problem

Exam problem

Compute the integral $\int \frac{1 + (\sec x)^2}{(\tan x + x)^5} dx$

Exam Problem

Exam problem

Find $\int \frac{\left(1 + x^{\frac{2}{3}}\right)^4}{x^{\frac{1}{3}}} dx$

4.3 + 4.4 Summation Notation, Reimann Sum and Definite Integrals

Summation Notation

Example

$$\textcircled{1} \sum_{k=0}^5 k(k-1)$$

$$\textcircled{2} \sum_{k=1}^{10} [1 + (-1)^k]$$

$$\textcircled{3} \sum_{k=1}^n k^2 + 3k + 5$$

Example

Find the area under $f(x) = x^3 + 1$ on $[0, 2]$

Example

Write the limit as definite integral

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n (3w_k^2 - 2w_k + 5) \Delta x$$

on $[-1, 2]$

Definite Integrals

Example

If

$$\int_1^4 \sqrt{x} \, dx = \frac{14}{3}$$

evaluate

$$\int_4^1 \sqrt{x} \, dx$$

Definite Integrals

Example

Use Reimann sum to find

$$\int_{-3}^2 (2x + 6) dx$$

4.5 Definite Integrals

Definite Integrals

Example

If

$$\int_1^4 x^2 dx = 21, \quad \int_1^4 x dx = \frac{15}{2}$$

Find

$$\int_1^4 (3x^2 + 5) dx$$

$$\int_1^4 (3x + 2)^2 dx$$

Definite Integrals

Example

Verify the inequality

$$\int_1^2 (3x^2 + 4) dx \geq \int_1^2 (2x^2 + 5) dx$$

Definite Integrals

Example

Verify the inequality

$$\int_0^{2\pi} (1 + \sin x) dx \geq 0$$

Definite Integrals

Example

Express as one integral

$$\int_5^1 f(x) dx + \int_{-3}^5 f(x) dx$$

Definite Integrals

Example

Express as one integral

$$\int_c^m f(x) dx - \int_d^m f(x) dx$$

4.6 Fundamental Theorem of Calculus

Fundamental Theorem of Calculus

Example

Evaluate

$$\textcircled{1} \int_1^4 (x^2 - 4x - 3) dx$$

$$\textcircled{2} \int_1^2 \frac{5}{x^6} dx$$

$$\textcircled{3} \int_{-8}^8 (\sqrt[3]{x^2} + 2) dx$$

$$\textcircled{4} \int_1^0 x^2 (\sqrt[3]{x} - \sqrt{x}) dx$$

Fundamental Theorem of Calculus

Example

Evaluate

① $\int_3^2 \frac{x^2 - 1}{x - 1} dx$

② $\int_1^1 (4x^2 - 5)^{100} dx$

③ $\int_{-3}^6 |x - 4| dx$

Fundamental Theorem of Calculus

Example

Evaluate

① $\int_1^4 \frac{1}{\sqrt{x}(\sqrt{x} + 1)^3} dx$

② $\int_0^{\pi/3} \frac{\sin x}{\cos^2 x} dx$

Fundamental Theorem of Calculus

Example

Evaluate

$$\textcircled{1} \quad \frac{d}{dx} \int_0^3 \sqrt{x^2 + 16} \, dx$$

$$\textcircled{2} \quad \frac{d}{dx} \int_0^x \frac{dt}{t+1}$$

Exam Problem

Exam problem

If $F(x) = \int_{\cos x}^{\sin x} \sqrt{1-t^2} dt$, find $F' \left(\frac{\pi}{4} \right)$

Exam Problem

Exam problem

Let $F(x) = \int_{\tan x}^{x^2} \frac{dt}{2+t^4}$, Find $F'(x)$

Exam Problem

Exam problem

If $F(x) = \cos(x^2) \int_0^{x^2} \cos^4(t) dt$. Find $F'(0)$

4.7 Numerical Integration

Trapezoidal Rule

Let $f : [a, b] \rightarrow \mathbb{R}$ with regular partition $P = \{x_0, \dots, x_n\}$ into n subintervals each of width $\Delta x = \frac{b-a}{n}$. Then the definite integral $\int_a^b f(x) dx$ can be approximated by

$$T_n = \frac{1}{2}(L_n + R_n) = \sum_{k=1}^n \frac{1}{2}[f(x_{k-1}) + f(x_k)]\Delta x$$

$$T_n = \frac{b-a}{2n}[f(x_0) + \textcolor{red}{2}f(x_1) + \textcolor{red}{2}f(x_2) + \dots \textcolor{red}{2}f(x_{n-1}) + f(x_n)]$$

Simpson's Rule

Let $f : [a, b] \rightarrow \mathbb{R}$ with regular partition $P = \{x_0, \dots, x_n\}$ into n subintervals **where n is even** each of width $\Delta x = \frac{b-a}{n}$. Then the definite integral $\int_a^b f(x) dx$ can be approximated by

$$S_n = \frac{b-a}{\mathbf{3}n} [f(x_0) + \mathbf{4}f(x_1) + \mathbf{2}f(x_2) + \dots \mathbf{4}f(x_{n-1}) + f(x_n)]$$

Trapezoidal Rule

Example

Approximate $\int_1^3 \sqrt{1+x^3} dx$ using trapezoidal rule with $n = 5$, and $n = 10$

Exam Problem

Exam problem

Use the trapezoid rule with $n = 4$ to approximate the integral $\int_0^1 \frac{dx}{1+x^2}$

Simpson's Rule

Example

Approximate $\int_0^4 \frac{dx}{1+x^3}$ using Simpson's rule with $n = 4$ and $n = 16$

Exam Problem

Exam problem

Use Simpson's rule with $n = 4$ to approximate $\int_0^4 \frac{dx}{\sqrt{2+x^3}}$