Chapter 3 (Maximum Flow Problems)

Ex. 3.3: Apply Ford-Fulkerson algorithm to find the maximum flow for the following network.

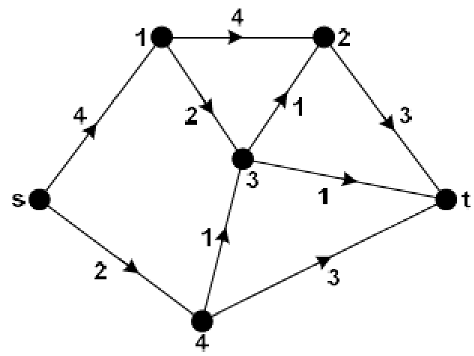

Figure 3.9: A directed Network.

Let us first construct the flow network by setting flow on all edges of the network with 0 and hence the total flow will be **F = 0** as follows (flow/capacity).

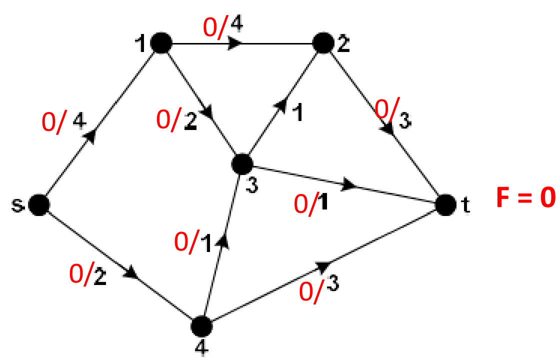

Figure 3.9: A directed Network.

Now we choose the augmenting paths. Choosing the augmenting path is randomly as there is no method shows how to choose such paths.

Let us start with the following augmenting path:

$$\boldsymbol{s-4-t} \text{ such that } \mathbf{min\{2-0, 3-0\} = 2}$$

so the maximum flow that can be sent through this augmenting path is **2** and hence the residual network becomes,

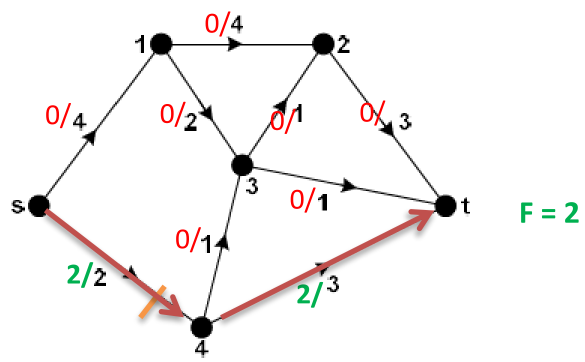

Figure 3.9: A directed Network.

Let us choose the augmenting path: $\boldsymbol{s - 1 - 3 - t}$ such that the maximum flow that can be sent through this path is $\mathbf{min}\{4 - 0, 2 - 0, 1 - 0\} = \mathbf{1}$ and hence the updating residual network becomes

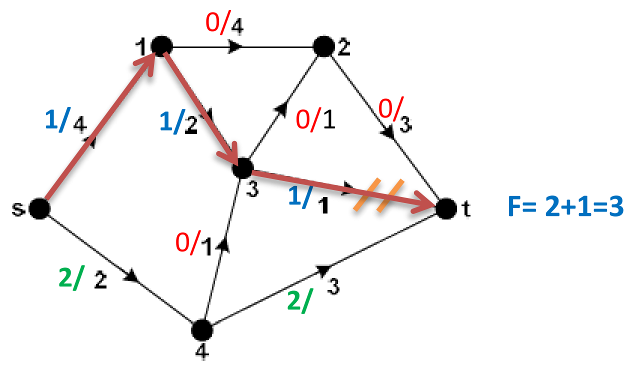

Figure 3.9: A directed Network.

Let us choose the augmenting path: $\mathbf{s - 1 - 2 - t}$ such that the maximum flow that can be sent through this path is $\mathbf{min}\{4 - 1, 4 - 0, 3 - 0\} = \mathbf{3}$ and hence the updating residual network becomes,

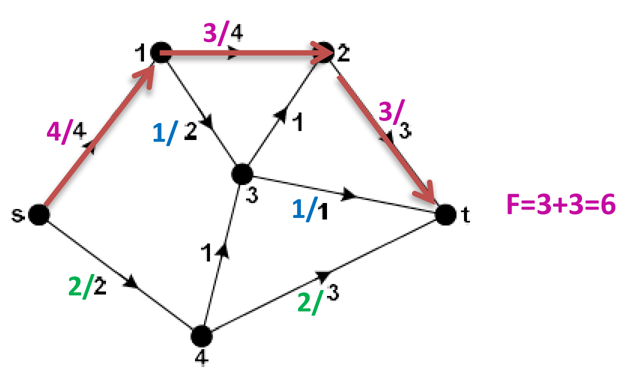

Figure 3.9: A directed Network.

There are no other augmenting paths and hence the maximum flow is **6**.

Ex. 3.7: Apply Edmonds-Karp algorithm on the network given in Ex. 3.3.

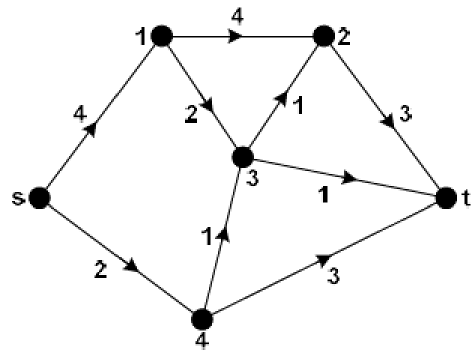

Figure 3.9: A directed Network.

We apply step 1 by searching for the augmenting path using **BFS** (breadth first search) as follows.

Starting vertex	Adjacent vertices (not visited yet)	Visited vertex	FIFO-queue	Edges
s	**1 ,4**	**s**	**1 ,4**	**s-1 , s-4**
1	**2, 3**	**1**	**4 , 2 , 3**	**1-2 , 1-3**
4	**t**	**4**	**2, 3**	**4-3 , 4-t**

The obtained augmenting path is $s \rightarrow 4 \rightarrow t$ with **flow** $= f_1 = \min\{2, 3\} = 2$.

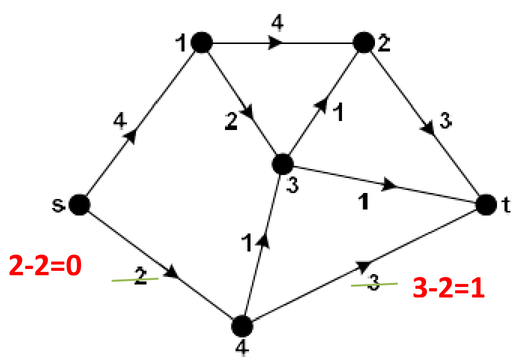

Figure 3.9: A directed Network.

We apply step 1 one more time for new augmenting path as follows.

Starting vertex	Adjacent vertices (not visited yet)	Visited vertex	FIFO-queue	Edges
s	**1**	**s**	**1**	**s-1**
1	**2, 3**	**1**	**2, 3**	**1-2 , 1-3**
2	**t**	**2**	**3 , t**	**2-t**

The obtained augmenting path is $\mathbf{s} \rightarrow \mathbf{1} \rightarrow \mathbf{2} \rightarrow \mathbf{t}$ with $\mathbf{flow} = \mathbf{f_2} = \mathbf{min}\{4, 4, 3\} = \mathbf{3}$. So the revised network takes the following form.

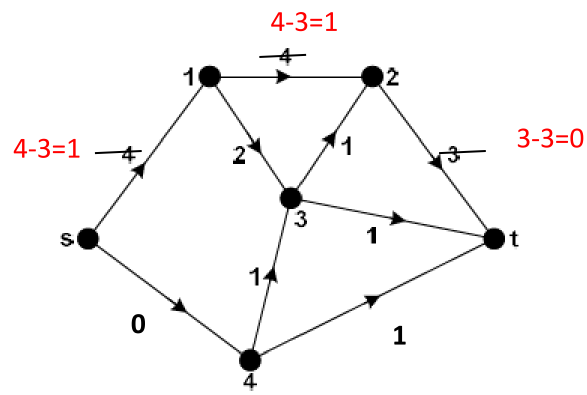

Figure 3.9: A directed Network.

We apply step 1 one more time for new augmenting path as follows.

Starting vertex	Adjacent vertices (not visited yet)	Visited vertex	FIFO-queue	Edges
s	**1**	**s**	**1**	**s-1**
1	**2, 3**	**1**	**2,3**	**1-2 ,1-3**
3	**t**	**3**	**t**	**3-t**

The obtained augmenting path is $\mathbf{s} \rightarrow \mathbf{1} \rightarrow \mathbf{3} \rightarrow \mathbf{t}$ with $\mathbf{flow} = \mathbf{f_3} = \mathbf{min}\{1, 2, 1\} = \mathbf{1}$. So the revised network takes the following form.

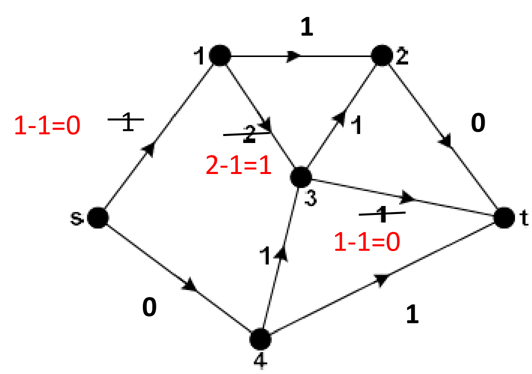

Figure 3.9: A directed Network.

As one can see that there are no more augmenting paths and hence the maximum flow is $\mathbf{f_1} + \mathbf{f_2} + \mathbf{f_3} = \mathbf{2 + 3 + 1 = 6}$.