<u>**Exercises 4 – chapter2**</u>

<u>Shortest path algorithms</u>

Ex. 2.8: Find the shortest path from vertex $\boldsymbol{u}$ to all vertices in the network given as following:

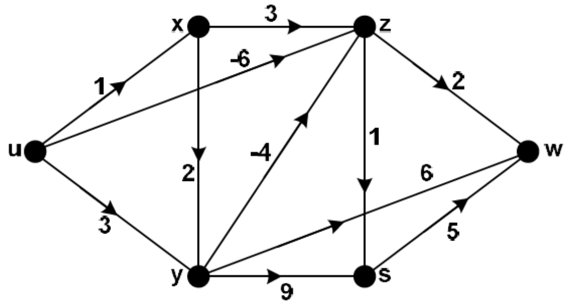


Figure 2.9: A weighted directed cycle-free graph.

<u>**Solution**</u>: **using Bellman algorithm:**

Starting vertex	Updating weights	S	Selected edges
u	$\pi(u) = 0$	{u}	--
x	$\pi(x) = \pi(u) + 1 = 1$	{u, x}	ux
y	$\pi(y) = \min\{\pi(x) + 2, \pi(u) + 3\} = \{3,3\} = 3$	{u,x,y}	xy or uy
z	$\pi(z) = \min\{\pi(x) + 3, \pi(u) - 6, \pi(y) - 4\} = \{4, -6, -1\} = -6$	{u,x,y,z}	uz
s	$\pi(s) = \min\{\pi(z) + 1, \pi(y) + 9\} = \{-5, 12\} = -5$	{u,x,y,z,s}	zs
w	$\pi(w) = \min\{\pi(z) + 2, \pi(y) + 6, \pi(s) + 5\} = \{-4,9,0\} = -4$	{u,x,y,z,s,w}	zw

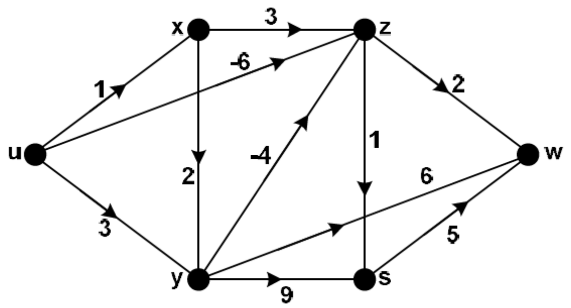
From above table , we have the following path

The path **uxy** and $\pi(y) = 3$ or **uy** and $\pi(y) = 3$

The path **uz** and $\pi(z) = -6$

The path **uzs** and $\pi(s) = -5$

The path **uzw** and $\pi(w) = -4$



Ex. 2.9: Find the shortest path from vertex u to all vertices in the network given in below.

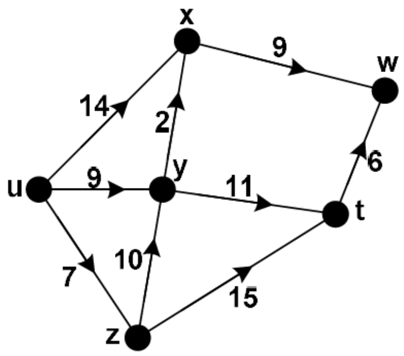
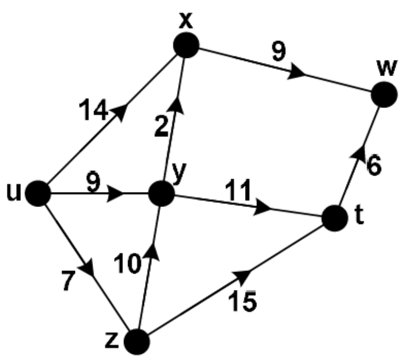


Figure 2.16: A weighted directed cycle-free graph.

Solution: using Bellman algorithm: H.W

Starting vertex	Updating weights	S	Selected edges
u	$\pi(u) = 0$	{u}	--
z	$\pi(x) = \pi(u) + 7 = 7$	{u, z}	uz
y	$\pi(y) = \min\{\pi(u) + 9, \pi(z) + 10\} = \{9,17\} = 9$	{u,z,y}	uy
x,t	$\pi(x) = \min\{\pi(u) + 14, \pi(y) + 2\} = \{14,11\} = 11$ $\pi(t) = \min\{\pi(y) + 11, \pi(z) + 15\} = \{20,22\} = 20$	{u,z,y,x,t}	yx yt
w	$\pi(w) = \min\{\pi(x) + 9, \pi(t) + 6\} = \{20,26\} = 20$	{u,z,y,x,t,w}	xw

From above table , we have the following path
The path uy and $\pi(y) = 9$
The path uyx and $\pi(x) = 11$
The path uyt and $\pi(t) = 20$
The path uyxw and $\pi(w) = 20$



Ex. 2.12: Find the shortest path from vertex **1** to vertex **6** for the following network.

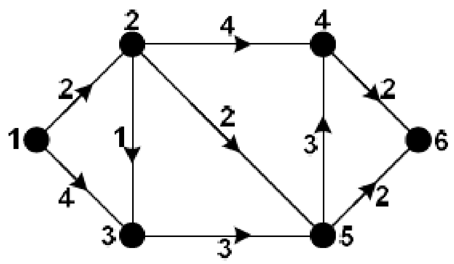


Figure 2.21: A digraph with cycle free.

Solution: **using Dijkstra's algorithm:**

	1	2	3	4	5	6
1	$\mathbf{0}^{permanent}$	∞	∞	∞	∞	∞
2	--	$\mathbf{2}^{permanent}_{12}$	$\mathbf{4}_{13}$	∞	∞	∞
3	--	--	$\mathbf{2+1} = \mathbf{3}^{permanent}_{123}$	$\mathbf{2+4} = \mathbf{6}_{124}$	$\mathbf{2+2} = \mathbf{4}_{125}$	∞
4			--	$\mathbf{6}_{124}$	~~$3+3=6$~~ $\mathbf{4}^{permanent}_{125}$	∞
5				~~$4+3=7$~~ $\mathbf{6}_{124}$	--	$\mathbf{4+2} = \mathbf{6}_{1256}$
6				$\mathbf{6}_{124}$		$\mathbf{6}_{1256}$

From above table , we have the path $1 \to 2 \to 5 \to 6$ and $\pi(6) = 6$

Ex. 2.14: Find the shortest path from vertex ***A*** to vertex **J** for the following network.

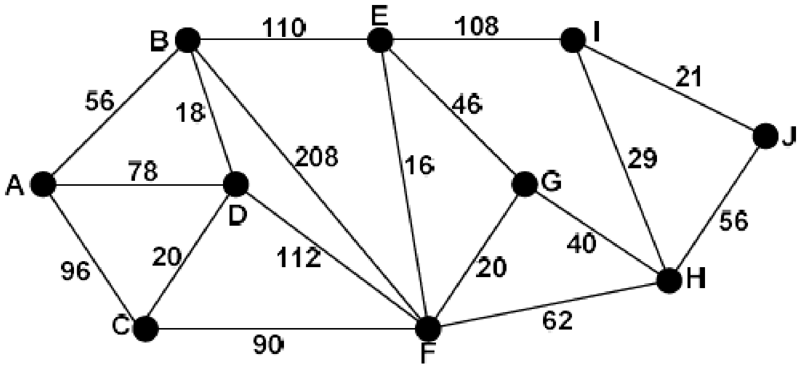


Figure 2.23: An undirected network.

Solution: **using Dijkstra's algorithm: HW**

	A	B	C	D	E	F	G	H	I	J
A	0^{per}	∞	∞	∞	∞	∞	∞	∞	∞	∞
B	--	56^{per}_{AB}	96_{AC}	78_{AD}		∞	∞	∞	∞	∞
C	--	--	96_{AC}	74^{per}_{ABD}	166_{ABE}	264_{ABF}	∞	∞	∞	∞
D	--	--	94^{per}_{ABDC}	--	166_{ABE}	186_{ABDF}	∞	∞	∞	∞
E	--	--	--	--	166^{per}_{ABE}	184_{ABDCF}	∞	∞	∞	∞
F	--	--	--	--	--	182^{per}_{ABEF}	214_{ABEG}	∞	274_{ABEI}	∞
G	--	--	--	--	--	--	202^{per}_{ABEFG}	244_{ABEFH}	274_{ABEI}	∞
H	--	--	--	--	--	--	--	242^{per}_{ABEFGH}	274_{ABEI}	∞
I	--	--	--	--	--	--	--	--	$271^{per}_{ABEFGHI}$	$289_{ABEFGHJ}$
J	--	--	--	--	--	--	--	--	--	$292_{ABEFGHIJ}$

From above table , we have the path $A \to B \to E \to F \to G \to H \to I \to G$ and $\pi(J) = 292$

Ex. 2.21: Apply the Bellman-Ford algorithm for the following network.

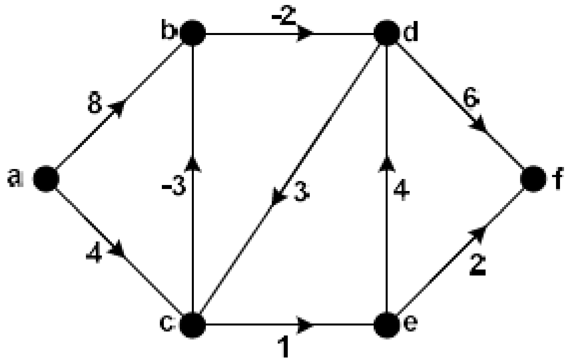


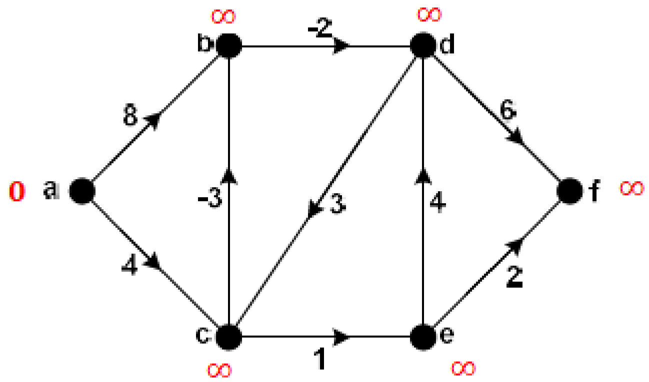
Figure 2.39: A directed network with directed cycles.

Solution: **Bellman-Ford algorithm :**

The above network contains **n=6** vertices then each edge will be relaxed **5** times (relaxation rules: $For\ k = 1,2, \dots , n-1$).

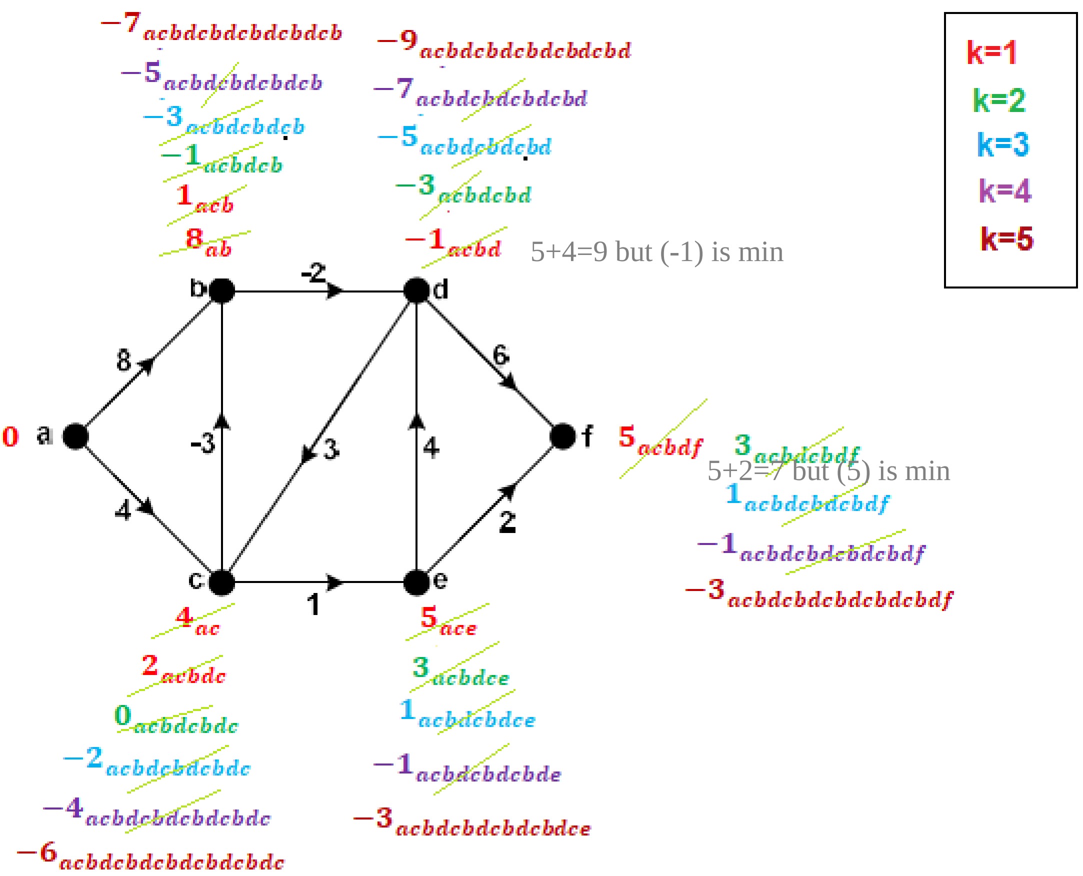
Let us first list the edges with the following order:

(a-b) , (a-c) , (c-b) ,(b-d) , (c-e) , (e-d) , (d-c) ,(d-f) and **(e-f).**

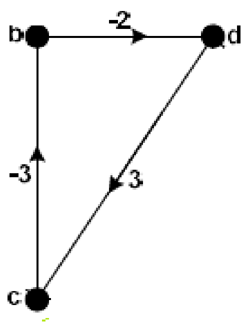


Initialization step

order: (a-b) , (a-c) , (c-b) ,(b-d) , (c-e) , (e-d) , (d-c) ,(d-f) and (e-f).



From the application of the algorithm, shows that the results obtained by the iterations are all different and hence shortest path is not existed. This is due to the cycle **c b d c** with length **−2**.



Ex. 2.20: Apply the Bellman-Ford algorithm to for the following network to find shortest path from 1 to 6 .

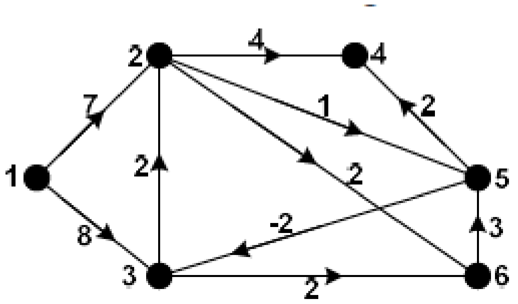
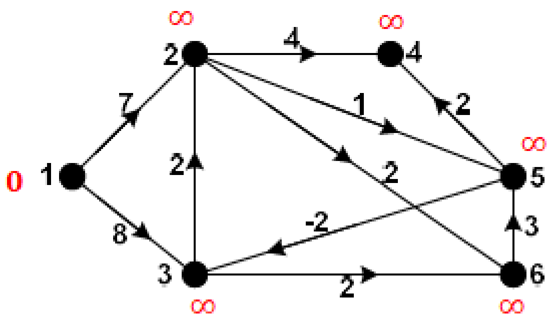


Figure 2.36: A directed network.

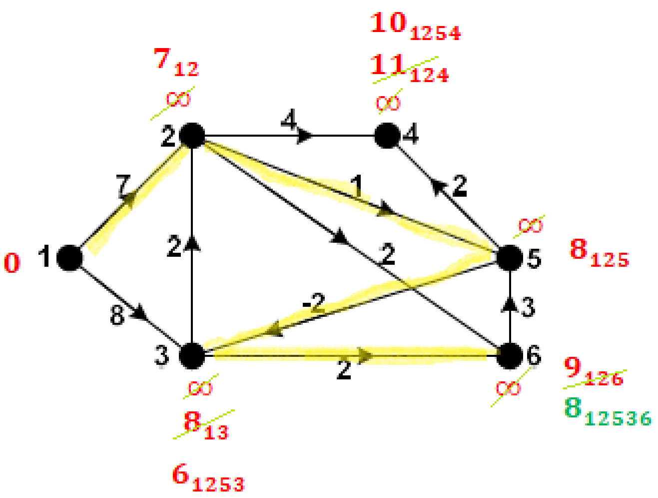
The above network contains **n=6** vertices then each edge will be relaxed **5** times (relaxation rules: $For\ k = 1,2, \dots , n-1$).

Let us first list the edges with the following order:

(1-2) , (1-3) , (3-2) ,(2-4) , (2-5) , (2-6) , (3-6) ,(6-5) , (5-3) and **(5-4).**



Initialization step



One can see that the relaxation in $k = 3$ is the same as the relaxation given in iteration $k = 2$ so there is no need to do the relaxation $k = 4$ as we will get the same result.
The shortest path is obtained through the path 12536 with cost 8.