STAT 337

Design and Analysis of Experiments

Exercises

Editing by: Zahra Kaabi

Chapter 5

THE TWO-FACTOR FACTORIAL DESIGN

5.1. The following output was obtained from a computer program that performed a two-factor ANOVA on a factorial experiment.

Two-way ANOVA: y versus A, B

Source	DF	SS	MS	F	P
A	1	0.322	**(4)** ?	**(6)** ?	**(8)** ?
B	**(1)** ?	80.554	40.2771	4.59	**(9)** ?
Interaction	**(2)** ?	**(3)** ?	**(5)** ?	**(7)** ?	**(10)** ?
Error	12	105.327	8.7773		
Total	17	231.551			

The Critical values:

A: $F_{\alpha,(a-1),ab(n-1)}$

$= F_{0.05,1,12} = 4.75$

B: $F_{\alpha,(b-1),ab(n-1)}$

$= F_{0.05,2,12} = 3.86$

AB: $F_{\alpha,(a-1)(b-1),ab(n-1)}$

$= F_{0.05,2,12} = 3.86$

(a) Fill in the blanks in the ANOVA table. You can use bounds on the *P*-values.

- Given that $a-1=1 \Rightarrow a=2$

And $abn-1=17 \Rightarrow abn=18$

And $ab(n-1)=12 \Rightarrow abn-ab=12 \Rightarrow 18-2b=12 \Rightarrow b=3$

Then $(\mathbf{1}) = b-1=2$

- And $(\mathbf{2}) = (\boldsymbol{a}-\mathbf{1})(\boldsymbol{b}-\mathbf{1}) = \mathbf{2}$

- $(\mathbf{3})$ $SS_{AB} = SS_T - SS_E - SS_A - SS_B = 45.348$

- $(\mathbf{4}) MS_A = \frac{SS_A}{a-1}$ =0.322

- $(\mathbf{5}) MS_{AB} = \frac{SS_{AB}}{(\boldsymbol{a}-\mathbf{1})(\boldsymbol{b}-\mathbf{1})} = \frac{45.348}{2}$=22.674

- $(6) = F_0 = \frac{MS_A}{MS_E} = \frac{0.322}{0.8773} = 0.0367$
- $(7) = F_0 = \frac{MS_{AB}}{MS_E} = \frac{22.674}{0.8773} = 25.8452$
- $(8) = p - value = 0.8513$
- $(9) = p - value = 0.0331$
- $(10) = p - value = 0.1167$

(b) How many levels were used for factor B?

b= 3 levels.

(c) How many replicates of the experiment were performed?

$\Rightarrow abn = 18 \Rightarrow (2)(3)n = 18 \Rightarrow n = 3$ replicates.

(d) What conclusions would you draw about this experiment?

Only factor B is significant; factor A and the two-factor interaction are not significant.

5.2. The following output was obtained from a computer program that performed a two-factor ANOVA on a factorial experiment.

Two-way ANOVA: y versus A, B

Source	DF	SS	MS	F	P
A	1	(2)	0.0002	(6)	___
B	(1)	180.378	(3)	(7)	___
Interaction	3	8.479	(4)	(8)	0.932
Error	8	158.797	(5)		
Total	15	347.653			

The Critical values:

A: $F_{\alpha,(a-1),ab(n-1)} = F_{0.05,1,8} = 5.31766$

B: $F_{\alpha,(b-1),ab(n-1)} = F_{0.05,3,8} = 4.06618$

AB: $F_{\alpha,(a-1)(b-1),ab(n-1)} = F_{0.05,3,8} = 4.06618$

(a) Fill in the blanks in the ANOVA table. You can use bounds on the *P*-values.

- Given that $a-1=1$

And $(a-1)(b-1)=3 \Rightarrow (b-1)=3$

Then $(\mathbf{1}) = b-1 = 3$

- Given $MS_A = \frac{SS_A}{a-1} \Rightarrow 0.0002 = \frac{SS_A}{1}$

$\Rightarrow (\mathbf{2}) = SS_A = 0.0002$

- $(\mathbf{3}) MS_B = \frac{SS_B}{b-1} = \frac{180.378}{3} = 60.126$

- $(\mathbf{4}) MS_{AB} = \frac{SS_{AB}}{(\boldsymbol{a}-\mathbf{1})(\boldsymbol{b}-\mathbf{1})} = \frac{8.479}{3} = 2.286$

- $(\mathbf{5}) MS_E = \frac{SS_E}{ab(n-1)} = \frac{158.797}{8} = 19.85$

- $(\mathbf{6}) = F_0 = \frac{MS_A}{MS_E} = \frac{0.0002}{19.85} = 0.00001$

- $(\mathbf{7}) = F_0 = \frac{MS_B}{MS_E} = \frac{60.126}{19.85} = 3.029$

- $(\mathbf{8}) = F_0 = \frac{MS_{AB}}{MS_E} = \frac{2.286}{19.85} = 0.115$

- $(\mathbf{9}) = \boldsymbol{p-value} =$ 0.998

- $(\mathbf{10}) = \boldsymbol{p-value} =$ 0.093

(b) How many levels were used for factor *B*?
We have from (a) $b-1=3 \quad \Rightarrow b=4$ levels.

(c) How many replicates of the experiment were performed?
$abn-1=15 \Rightarrow abn=16 \Rightarrow (2)(4)n=16 \Rightarrow n=2$ replicates

(d) What conclusions would you draw about this experiment?
The two-factor interaction are not significant

$F_0 = 0.115 < F_{0.05,3,8} = 4.06618$

Factor *A* and is not significant

$F_0 = 0.00001 < F_{0.05,1,8} = 5.31766$

Factor *B* and is not significant

$F_0 = 3.029 < F_{0.05,3,8} = 4.06618$

5.3. The yield of a chemical process is being studied. The two most important variables are thought to be the pressure and the temperature. Three levels of each factor are selected, and a factorial experiment with two replicates is performed. The yield data are as follows:

	Pressure (psig)		
Temperature (°C)	**200**	**215**	**230**
150	90.4	90.7	90.2
	90.2	90.6	90.4
160	90.1	90.5	89.9
	90.3	90.6	90.1
170	90.5	90.8	90.4
	90.7	90.9	90.1

(**a**) Analyze the data and draw conclusions. Use $\alpha = 0.05$

a=3 , b=3 and n=2

		pressure		
	200	**215**	**230**	$Y_{i..}$
150	90.4 90.2 180.6	90.7 90.6 181.3	90.2 90.4 180.6	542.5
160	90.1 90.3 180.4	90.5 90.6 181.1	89.9 90.1 180	541.5
170	90.5 90.7 181.2	90.8 90.9 181.7	90.4 90.1 180.5	543.5
$Y_{.j.}$	542.2	544.1	541.1	$Y_{...}$=1627.4

temperature

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} Y_{ijk}{}^2 - \frac{Y_{...}{}^2}{abn}$$

$$= 90.4^2 + 90.2^2 + 90.7^2 + \cdots + 90.1^2 - \frac{1627.4^2}{18} = 1.298$$

$$SS_{Temperature} = \frac{1}{bn}\sum_{i=1}^{a} Y_{i..}{}^2 - \frac{Y_{...}{}^2}{abn}$$

$$= \frac{1}{6}(542.5^2 + 541.5^2 + 543.3^2) - \frac{1627.4^2}{18} = 0.768$$

$$SS_{pressure} = \frac{1}{an}\sum_{j=1}^{b} Y_{.j.}{}^2 - \frac{Y_{...}{}^2}{abn}$$

$$= \frac{1}{6}(542.2^2 + 544.1^2 + 541.1^2) - \frac{1627.4^2}{18} = 0.301$$

$$SS_{interaction} = \frac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b} Y_{ij.}{}^2 - \frac{Y_{...}{}^2}{abn} - SS_{Temperature} - SS_{pressure}$$

$$= \frac{1}{2}(180.6^2 + 181.3^2 + \cdots + 180.5^2) - \frac{1627.4^2}{18} - 0.768 - 0.301 = 0.069$$

$$SS_E = SS_T - SS_{interaction} - SS_{Temperature} - SS_{pressure} = 0.16$$

Zahra Kaabi

The Analysis of Variance Table for the Two-Factor Factorial, Fixed Effects Model

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A treatments	SS_A	$a-1$	$MS_A = \frac{SS_A}{a-1}$	$F_0 = \frac{MS_A}{MS_E}$
B treatments	SS_B	$b-1$	$MS_B = \frac{SS_B}{b-1}$	$F_0 = \frac{MS_B}{MS_E}$
Interaction	SS_{AB}	$(a-1)(b-1)$	$MS_{AB} = \frac{SS_{AB}}{(a-1)(b-1)}$	$F_0 = \frac{MS_{AB}}{MS_E}$
Error	SS_E	$ab(n-1)$	$MS_E = \frac{SS_E}{ab(n-1)}$	
Total	SS_T	$abn-1$		

The Analysis of Variance Table for the Two-Factor Factorial, Fixed Effects Model

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A treatments	0.301	2	0.1505	$F_0 = \frac{MS_A}{MS_E}$ =8.42
B treatments	0.768	2	0.384	$F_0 = \frac{MS_B}{MS_E}$ =21.33
Interaction	0.069	4	0.017	$F_0 = \frac{MS_{AB}}{MS_E}$ =0.94
Error	0.16	9	0.018	
Total	1.298	17		

Test the interaction between the two factors (Pressure and Temperature) :

(1) $H_0: (\tau\beta)_{ij} = 0 \; for \; all \; i, j$ VS H_1: at least one of $(\tau\beta)_{ij} \neq 0$

(2) The test statistic $F_0 = 0.94$

(3) The Critical value $= F_{\alpha,(a-1)(b-1),ab(n-1)} = F_{0.05,4,9} = 3.63$

(4) since $F_0 = 0.94 < 3.63 = F_{0.05,4,9}$ we cannot reject

$H_0: (\tau\beta)_{ij} = 0 \; for \; all \; i, j$

We conclude that there is no significant interaction between **Temperature** and **Pressure**.

NOTE:
If we reject H_0 (p-value $\leq \alpha$) → there is an interaction (stop)
If we cannot reject H_0 (p-value > α) → there is no an interaction (continue)

Since there is no significant interaction between the two factors, We will test the effectiveness of Temperature and pressure

Test the effect of Tempreture factor:

(1) $H_0: \tau_1 = \tau_2 = \tau_3 = 0$ VS H_1: at least one of $\tau_i \neq 0$

(2) The test statistic $F_0 = 8.42$

(3) The Critical value $= F_{\alpha, a-1, ab(n-1)} = F_{0.05,2,9} = 4.25$

(4) since $F_0 = 8.42 > 4.25 = F_{0.05,2,9}$ we reject $H_0: \tau_1 = \tau_2 = \tau_3$

That means there is a significant effect of **temperature** on the study .

Test the effect of Pressure factor:

(1) $H_0: \beta_1 = \beta_2 = \beta_3 = 0$ VS H_1: at least one of $\beta_j \neq 0$

(2) The test statistic $F_0 = 21.33$

(3) The Critical value $= F_{\alpha, b-1, ab(n-1)} = F_{0.05,2,9} = 4.25$

(4) since $F_0 = 21.33 > 4.25 = F_{0.05,2,9}$ we reject $H_0: \beta_1 = \beta_2 = \beta_3$

That means there is a significant effect of **pressure** on the study

- **Using Minitab to perform two factors Factorial Design**

You can watch this useful video

https://www.youtube.com/watch?v=Fu9cWxDRRuA

- Click Stat → DOE → **Factorial** → Create **Factorial**.
- A new window named "Create **Factorial Design**" pops up.
- Enter "2" as the "Number of **factors**."
- Select the **Full factorial** design.
- Click **Designs**.
- Enter "3" as the "**Number of level** " of factor A and "3" "**Number of level** " of factor B **and** . Enter "2" as the "Number of **replicates**.
- Click **OK**
- Click **factors.**
- In the row for **Factor A**, under **Name**, enter *Temprature*. . Under **level value**, enter *150, 160 and 170*
- In the row for **Factor B**, under **Name**, enter *Pressure*. . Under **level value**, enter *200,215 and 230*
- Click **OK**
- Click **Options**.
- Uncheck **randomize run**
- Click **OK**
- Click **OK**

You will get this table which is ready to enter the observations

StdOrder	RunOrder	PtType	Blocks	Temperature	pressure	obs
1	1	1	1	150	200	90.4
2	2	1	1	150	215	90.7
3	3	1	1	150	230	90.2
4	4	1	1	160	200	90.1
5	5	1	1	160	215	90.5
6	6	1	1	160	230	89.9
7	7	1	1	170	200	90.5
8	8	1	1	170	215	90.8
9	9	1	1	170	230	90.4
10	10	1	1	150	200	90.2

11	11	1	1	150	215	90.6
12	12	1	1	150	230	90.4
13	13	1	1	160	200	90.3
14	14	1	1	160	215	90.6
15	15	1	1	160	230	90.1
16	16	1	1	170	200	90.7
17	17	1	1	170	215	90.9
18	18	1	1	170	230	90.1

- Choose **Stat** > **DOE** > **Factorial** > **Analyze Factorial Design**
- In **Responses**, enter *obs*
- Click **OK**

The results:

General Factorial Regression: obs versus Temperature; pressure

```
Factor Information

Factor       Levels  Values
Temperature       3  150; 160; 170
pressure          3  200; 215; 230

Analysis of Variance

Source                      DF   Adj SS   Adj MS  F-Value  P-Value
Model                        8  1.13778  0.14222     8.00    0.003
  Linear                     4  1.06889  0.26722    15.03    0.001
    Temperature              2  0.30111  0.15056     8.47    0.009
    pressure                 2  0.76778  0.38389    21.59    0.000
  2-Way Interactions         4  0.06889  0.01722     0.97    0.470
    Temperature*pressure     4  0.06889  0.01722     0.97    0.470
Error                        9  0.16000  0.01778
Total                       17  1.29778


Model Summary

       S    R-sq  R-sq(adj)  R-sq(pred)
0.133333  87.67%     76.71%      50.68%


Coefficients

Term                     Coef  SE Coef  T-Value  P-Value   VIF
Constant              90.4111   0.0314  2876.86    0.000
Temperature
  150                  0.0056   0.0444     0.13    0.903  1.33
  160                 -0.1611   0.0444    -3.63    0.006  1.33
pressure
  200                 -0.0444   0.0444    -1.00    0.343  1.33
  215                  0.2722   0.0444     6.12    0.000  1.33
Temperature*pressure
  150 200             -0.0722   0.0629    -1.15    0.280  1.78
  150 215             -0.0389   0.0629    -0.62    0.551  1.78
  160 200             -0.0056   0.0629    -0.09    0.932  1.78
```

```
  160 215              0.0278   0.0629     0.44     0.669  1.78


Regression Equation

obs = 90.4111 + 0.0056 Temperature_150 - 0.1611 Temperature_160 + 0.1556 Temperature_170
      - 0.0444 pressure_200 + 0.2722 pressure_215 - 0.2278 pressure_230
      - 0.0722 Temperature*pressure_150 200 - 0.0389 Temperature*pressure_150 215
      + 0.1111 Temperature*pressure_150 230 - 0.0056 Temperature*pressure_160 200
      + 0.0278 Temperature*pressure_160 215 - 0.0222 Temperature*pressure_160 230
      + 0.0778 Temperature*pressure_170 200 + 0.0111 Temperature*pressure_170 215
      - 0.0889 Temperature*pressure_170 230
```

HW 5.4

5.4. An engineer suspects that the surface finish of a metal part is influenced by the feed rate and the depth of cut. He selects three feed rates and four depths of cut. He then conducts a factorial experiment and obtains the following data:

Feed Rate (in/min)	Depth of Cut (in)			
	0.15	0.18	0.20	0.25
	74	79	82	99
0.20	64	68	88	104
	60	73	92	96
	92	98	99	104
0.25	86	104	108	110
	88	88	95	99
	99	104	108	114
0.30	98	99	110	111
	102	95	99	107

a=3 , b=4 and n=3

(a) Analyze the data and draw conclusions. Use $\alpha = 0.05$.

The results:

General Factorial Regression: obs versus Feed Rate; Depth of cut

```
Factor Information

Factor        Levels  Values
```

Feed Rate	3	0.20; 0.25; 0.30
Depth of cut	4	0.15; 0.18; 0.20; 0.25

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	11	5842.7	531.15	18.49	0.000
Linear	5	5285.6	1057.12	36.81	0.000
Feed Rate	**2**	**3160.5**	1580.25	**55.02**	0.000
Depth of cut	**3**	**2125.1**	708.37	**24.66**	0.000
2-Way Interactions	**6**	**557.1**	92.84	**3.23**	0.018
Feed Rate*Depth of cut	6	557.1	92.84	3.23	0.018
Error	**24**	**689.3**	28.72		
Total	**35**	**6532.0**			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
5.35931	89.45%	84.61%	76.26%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	94.333	0.893	105.61	0.000	
Feed Rate					
0.20	-12.75	1.26	-10.09	0.000	1.33
0.25	3.25	1.26	2.57	0.017	1.33
Depth of cut					
0.15	-9.56	1.55	-6.18	0.000	1.50
0.18	-4.56	1.55	-2.94	0.007	1.50
0.20	3.56	1.55	2.30	0.031	1.50
Feed Rate*Depth of cut					
0.20 0.15	-6.03	2.19	-2.76	0.011	2.00
0.20 0.18	-3.69	2.19	-1.69	0.104	2.00
0.20 0.20	2.19	2.19	1.00	0.326	2.00
0.25 0.15	0.64	2.19	0.29	0.773	2.00
0.25 0.18	3.64	2.19	1.66	0.109	2.00
0.25 0.20	-0.47	2.19	-0.22	0.831	2.00

Regression Equation

```
obs = 94.333 - 12.75 Feed Rate_0.20 + 3.25 Feed Rate_0.25 + 9.50 Feed Rate_0.30
      - 9.56 Depth of cut_0.15 - 4.56 Depth of cut_0.18 + 3.56 Depth of cut_0.20
      + 10.56 Depth of cut_0.25 - 6.03 Feed Rate*Depth of cut_0.20 0.15
      - 3.69 Feed Rate*Depth of cut_0.20 0.18 + 2.19 Feed Rate*Depth of cut_0.20 0.20
      + 7.53 Feed Rate*Depth of cut_0.20 0.25 + 0.64 Feed Rate*Depth of cut_0.25 0.15
      + 3.64 Feed Rate*Depth of cut_0.25 0.18 - 0.47 Feed Rate*Depth of cut_0.25 0.20
      - 3.81 Feed Rate*Depth of cut_0.25 0.25 + 5.39 Feed Rate*Depth of cut_0.30 0.15
      + 0.06 Feed Rate*Depth of cut_0.30 0.18 - 1.72 Feed Rate*Depth of cut_0.30 0.20
      - 3.72 Feed Rate*Depth of cut_0.30 0.25
```

Welcome to Minitab, press F1 for help.

THE GENERAL FACTORIAL DESIGN

5.19. The quality control department of a fabric finishing plant is studying the effect of several factors on the dyeing of cotton–synthetic cloth used to manufacture men's shirts. Three operators, three cycle times, and two temperatures were selected, and three small specimens of cloth were dyed under each set of conditions. The finished cloth was compared to a standard, and a numerical score was assigned. The results are as follows. Analyze the data and draw conclusions. Comment on the model's adequacy.

	Temperature					
	300°C			350°C		
	Operator			Operator		
Cycle Time	**1**	**2**	**3**	**1**	**2**	**3**
	23	27	31	24	38	34
40	24	28	32	23	36	36
	25	26	29	28	35	39
	36	34	33	37	34	34
50	35	38	34	39	38	36
	36	39	35	35	36	31
	28	35	26	26	36	28
60	24	35	27	29	37	26
	27	34	25	25	34	24

Cycle time	Temperature						
	300			350			
	Operators			Operators			
	1	2	3	1	2	3	$Y_{i...}$
40	23 24 25 72	27 28 26 81	31 32 29 92	24 23 28 75	38 36 35 109	34 36 39 109	538
50	36 35 36 107	34 38 39 111	33 34 35 102	37 39 35 111	34 38 36 108	34 36 31 101	640
60	28 24 27 79	35 35 34 104	26 27 25 78	26 29 25 80	36 37 34 107	28 26 24 78	526
$Y_{.jk.}$	258	296	272	266	324	288	$Y_{....}$=1704
$Y_{.j..}$	826			878			

A	B	
	300	350
40	245	293
50	320	320
60	261	265
$Y_{.j..}$	826	878

$Y_{ij..}$

A	C		
	1	2	3
40	147	190	201
50	218	219	203
60	159	211	156
$Y_{..K.}$	524	620	560

$Y_{i.k.}$

a=3 , b=2, c=3 and n=3

A: Cycle time

B: Temperature

C: Operators

$$SS_A = \frac{1}{bcn}\sum_{i=1}^{a} Y_{i...}{}^2 - \frac{Y_{....}{}^2}{abcn}$$

$$= \frac{1}{2(3)(3)}(538^2 + 640^2 + 562^2) - \frac{1704^2}{3(2)(3)(3)} = \mathbf{436}$$

$$SS_B = \frac{1}{acn}\sum_{j=1}^{b} Y_{.j..}{}^2 - \frac{Y_{....}{}^2}{abcn}$$

$$= \frac{1}{3(3)(3)}(862^2 + 878^2) - \frac{1704^2}{3(2)(3)(3)} = \mathbf{50.07}$$

$$SS_C = \frac{1}{abn}\sum_{k=1}^{c} Y_{..k.}{}^2 - \frac{Y_{....}{}^2}{abcn}$$

$$= \frac{1}{3(2)(3)}(524^2 + 620^2 + 560^2) - \frac{1704^2}{3(2)(3)(3)} = \mathbf{261.33}$$

$$SS_{AB} = \frac{1}{cn}\sum_{i=1}^{a}\sum_{j=1}^{b} Y_{ij..}{}^2 - \frac{Y_{....}{}^2}{abcn} - SS_A - SS_B$$

$$= \frac{1}{(3)(3)}(245^2 + 293^2 + 320^2 + 320^2 + 261^2 + 265^2) - \frac{1704^2}{3(2)(3)(3)} - 436 - 50.07 = 78.82$$

$$SS_{AC} = \frac{1}{bn}\sum_{i=1}^{a}\sum_{k=1}^{c} Y_{i.k.}{}^2 - \frac{Y_{....}{}^2}{abcn} - SS_A - SS_C$$

$$= \frac{1}{(2)(3)}(147^2 + 190^2 + 201^2 + \cdots + 159^2 + 211^2 + 158^2) - \frac{1704^2}{3(2)(3)(3)} - 436 - 261.33$$

$$= \mathbf{355.67}$$

$$SS_{BC} = \frac{1}{an}\sum_{j=1}^{b}\sum_{k=1}^{c} Y_{.jk.}{}^2 - \frac{Y_{....}{}^2}{abcn} - SS_B - SS_C$$

$$= \frac{1}{(3)(3)}(258^2 + 296^2 + 272^2 + 266^2 + 324^2 + 288^2) - \frac{1704^2}{3(2)(3)(3)} - 50.07 - 261.33$$

$$= \mathbf{11.27}$$

$$SS_{ABC} = \frac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} Y_{ijk.}{}^{2} - \frac{Y_{....}{}^{2}}{abcn} - SS_A - SS_B - SS_C - SS_{AB} - SS_{AC} - SS_{BC}$$

$$= \frac{1}{3}(72^2 + 81^2 + 92^2 + \cdots + 107^2 + 78^2) - \frac{1704^2}{3(2)(3)(3)} - 436 - 50.07 - 261.33 - 78.82 - 355.67 - 11.27$$

$$= \mathbf{46.17}$$

$$SS_{subtotal(ABC)} = \frac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} Y_{ijk.}{}^{2} - \frac{Y_{....}{}^{2}}{abcn}$$

$$= \frac{1}{3}(72^2 + 81^2 + 92^2 + \cdots + 107^2 + 78^2) - \frac{1704^2}{3(2)(3)(3)} = 1239.33$$

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n} Y_{ijkl}{}^{2} - \frac{Y_{....}{}^{2}}{abcn}$$

$$= (23^2 + 27^2 + 31^2 + \cdots + 25^2 + 34^2 + 24^2) - \frac{1704^2}{3(2)(3)(3)} = 1357.33$$

$$SS_E = SS_T - SS_{subtotal(ABC)}$$

$$= 1357.33 - 1239.33 = 118$$

The Analysis of Variance Table for the Three-Factor Fixed Effects Model

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	Expected Mean Square	F_0
A	SS_A	$a-1$	MS_A	$\sigma^2 + \frac{bcn\sum \tau_i^2}{a-1}$	$F_0 = \frac{MS_A}{MS_E}$
B	SS_B	$b-1$	MS_B	$\sigma^2 + \frac{acn\sum \beta_j^2}{b-1}$	$F_0 = \frac{MS_B}{MS_E}$
C	SS_C	$c-1$	MS_C	$\sigma^2 + \frac{abn\sum \gamma_k^2}{c-1}$	$F_0 = \frac{MS_C}{MS_E}$
AB	SS_{AB}	$(a-1)(b-1)$	MS_{AB}	$\sigma^2 + \frac{cn\sum\sum (\tau\beta)_{ij}^2}{(a-1)(b-1)}$	$F_0 = \frac{MS_{AB}}{MS_E}$
AC	SS_{AC}	$(a-1)(c-1)$	MS_{AC}	$\sigma^2 + \frac{bn\sum\sum (\tau\gamma)_{ik}^2}{(a-1)(c-1)}$	$F_0 = \frac{MS_{AC}}{MS_E}$
BC	SS_{BC}	$(b-1)(c-1)$	MS_{BC}	$\sigma^2 + \frac{an\sum\sum (\beta\gamma)_{jk}^2}{(b-1)(c-1)}$	$F_0 = \frac{MS_{BC}}{MS_E}$
ABC	SS_{ABC}	$(a-1)(b-1)(c-1)$	MS_{ABC}	$\sigma^2 + \frac{n\sum\sum\sum (\tau\beta\gamma)_{ijk}^2}{(a-1)(b-1)(c-1)}$	$F_0 = \frac{MS_{ABC}}{MS_E}$
Error	SS_E	$abc(n-1)$	MS_E	σ^2	
Total	SS_T	$abcn-1$			

The Analysis of Variance Table for the Three-Factor Fixed Effects Model

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	Critical values	F_0
A	436	2	218	$F_{\alpha,(a-1),abc(n-1)} = F_{0.05,2,36} = 3.259$	66.46
B	50.07	1	50.07	$F_{\alpha,(b-1),abc(n-1)} = F_{0.05,1,36} = 4.11$	15.27
C	261.33	2	130.66	$F_{\alpha,(c-1),abc(n-1)} = F_{0.05,2,36} = 3.259$	39.84
AB	78.82	2	39.4:	$F_{\alpha,(a-1)(b-1),abc(n-1)} = F_{0.05,2,36} = 3.259$	12.01
AC	355.67	4	88.92	$F_{\alpha,(a-1)(c-1),abc(n-1)} = F_{0.05,4,36} = 2.63$	27.12
BC	11.27	2	5.63	$F_{\alpha,(b-1)(c-1),abc(n-1)} = F_{0.05,2,36} = 3.259$	1.72
ABC	46.17	4	11.54	$F_{\alpha,(a-1)(b-1)(c-1),abc(n-1)} = F_{0.05,4,36} = 2.63$	3.52
Error	118	36	3.28		
Total	1357.33	53			

Zahra Kaabi

Test the interaction between the three factors (A, B and C) :

(1) $H_0: (\tau\beta\gamma)_{ijk} = 0 \; for \; all \; i, j, k$ VS H_1: at least one of $(\tau\beta\gamma)_{ijk} \neq 0$

(2) The test statistic F_0 =**3.52**

(3) The Critical value $= F_{\alpha,(a-1)(b-1)(c-1),abc(n-1)} = F_{0.05,4,36} = 2.63$

(4) since $F_0 = 3.52 > 2.63 = F_{0.05,4,36}$ we reject

$$H_0: (\tau\beta\gamma)_{ijk} = 0 \; for \; all \; i, j, k$$

We conclude that there **is significant interaction** between the three factors**.**

> **NOTE:**
> If we reject H_0 (p-value $\leq \alpha$) → there is an interaction (stop)
> If we cannot reject H_0 (p-value > α) → there is no an interaction (continue)

- **Using Minitab to perform general Factorial Design**

- Click Stat → DOE → **Factorial** → Create **Factorial**.
- A new window named "Create **Factorial Design**" pops up.
- Enter "3" as the "Number of **factors**."
- Select the **general Full factorial design.**
- Click **Designs**.
- . Enter "3" as the "Number of **replicates**." And "**3**" as the "Number of **level"**
- Click **OK**
- Click **factors**.
- In the row for **Factor A**, under **Name**, enter ***cycle time***. . Under **level value**, enter *40, 50and 60*.
- In the row for **Factor B**, under **Name**, enter ***Temperature***. . Under **level value**, enter *300 and 350*
- In the row for **Factor C**, under **Name**, enter ***Operator***. Under **level value**, enter *1,2 and 3*
- Click **OK**
- Click **Options**.
- Uncheck **randomize run**
- Click **OK**
- Click **OK**

- You will get this table which is ready to enter the observations

StdOrder	RunOrder	PtType	Blocks	A	B	C	obs
1	1	1	1	40	300	1	23
2	2	1	1	40	300	2	27
3	3	1	1	40	300	3	31
4	4	1	1	40	350	1	24
5	5	1	1	40	350	2	38
6	6	1	1	40	350	3	34
7	7	1	1	50	300	1	36
8	8	1	1	50	300	2	34
9	9	1	1	50	300	3	33
10	10	1	1	50	350	1	37
11	11	1	1	50	350	2	34
12	12	1	1	50	350	3	34
13	13	1	1	60	300	1	28
14	14	1	1	60	300	2	35
15	15	1	1	60	300	3	26
16	16	1	1	60	350	1	26
17	17	1	1	60	350	2	36
18	18	1	1	60	350	3	28
19	19	1	1	40	300	1	24
20	20	1	1	40	300	2	28
21	21	1	1	40	300	3	32
22	22	1	1	40	350	1	23
23	23	1	1	40	350	2	36
24	24	1	1	40	350	3	36
25	25	1	1	50	300	1	35
26	26	1	1	50	300	2	38
27	27	1	1	50	300	3	34
28	28	1	1	50	350	1	39
29	29	1	1	50	350	2	38
30	30	1	1	50	350	3	36
31	31	1	1	60	300	1	24
32	32	1	1	60	300	2	35
33	33	1	1	60	300	3	27
34	34	1	1	60	350	1	29
35	35	1	1	60	350	2	37
36	36	1	1	60	350	3	26
37	37	1	1	40	300	1	25
38	38	1	1	40	300	2	26
39	39	1	1	40	300	3	29

40	40	1	1	40	350	1	28
41	41	1	1	40	350	2	35
42	42	1	1	40	350	3	39
43	43	1	1	50	300	1	36
44	44	1	1	50	300	2	39
45	45	1	1	50	300	3	35
46	46	1	1	50	350	1	35
47	47	1	1	50	350	2	36
48	48	1	1	50	350	3	31
49	49	1	1	60	300	1	27
50	50	1	1	60	300	2	34
51	51	1	1	60	300	3	25
52	52	1	1	60	350	1	25
53	53	1	1	60	350	2	34
54	54	1	1	60	350	3	24

- Choose **Stat** > **DOE** > **Factorial** > **Analyze Factorial Design** Uncheck **randomize run**
- In **Responses**, enter *obs*
- Click **OK**

The results:

Multilevel Factorial Design

```
Factors:        3     Replicates:     2
Base runs:     18     Total runs:    36
Base blocks:    1     Total blocks:   1

Number of levels: 3; 2; 3
```

Results for: Worksheet 2

Multilevel Factorial Design

```
Factors:        3     Replicates:     3
Base runs:     18     Total runs:    54
Base blocks:    1     Total blocks:   1

Number of levels: 3; 2; 3
```

General Factorial Regression: obs versus A; B; C

```
Factor Information
```

Factor	Levels	Values
A	3	40; 50; 60
B	2	300; 350
C	3	1; 2; 3

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	17	1239.33	72.902	22.24	0.000
Linear	5	747.41	149.481	45.60	0.000
A	**2**	**436.00**	218.000	**66.51**	0.000
B	**1**	**50.07**	50.074	**15.28**	0.000
C	**2**	**261.33**	130.667	**39.86**	0.000
2-Way Interactions	8	445.74	55.718	17.00	0.000
A*B	**2**	**78.81**	39.407	**12.02**	0.000
A*C	**4**	**355.67**	88.917	**27.13**	0.000
B*C	**2**	**11.26**	5.630	**1.72**	0.194
3-Way Interactions	4	46.19	11.546	3.52	0.016
A*B*C	**4**	**46.19**	11.546	**3.52**	0.016
Error	**36**	**118.00**	**3.278**		
Total	**53**	**1357.33**			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
1.81046	91.31%	87.20%	80.44%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	31.556	0.246	128.08	0.000	
A					
40	-1.667	0.348	-4.78	0.000	1.33
50	4.000	0.348	11.48	0.000	1.33
B					
300	-0.963	0.246	-3.91	0.000	1.00
C					
1	-2.444	0.348	-7.02	0.000	1.33
2	2.889	0.348	8.29	0.000	1.33
A*B					
40 300	-1.704	0.348	-4.89	0.000	1.33
50 300	0.963	0.348	2.76	0.009	1.33
A*C					
40 1	-2.944	0.493	-5.98	0.000	1.78
40 2	-1.111	0.493	-2.25	0.030	1.78
50 1	3.222	0.493	6.54	0.000	1.78
50 2	-1.944	0.493	-3.95	0.000	1.78
B*C					
300 1	0.519	0.348	1.49	0.145	1.33
300 2	-0.593	0.348	-1.70	0.098	1.33
A*B*C					
40 300 1	1.648	0.493	3.34	0.002	1.78
40 300 2	-1.407	0.493	-2.86	0.007	1.78
50 300 1	-1.185	0.493	-2.41	0.021	1.78
50 300 2	1.093	0.493	2.22	0.033	1.78

Regression Equation

obs = 31.556 - 1.667 A_40 + 4.000 A_50 - 2.333 A_60 - 0.963 B_300 + 0.963 B_350 - 2.444 C_1

```
+ 2.889 C_2 - 0.444 C_3 - 1.704 A*B_40 300 + 1.704 A*B_40 350 + 0.963 A*B_50 300
- 0.963 A*B_50 350 + 0.741 A*B_60 300 - 0.741 A*B_60 350 - 2.944 A*C_40 1
- 1.111 A*C_40 2 + 4.056 A*C_40 3 + 3.222 A*C_50 1 - 1.944 A*C_50 2 - 1.278 A*C_50 3
- 0.278 A*C_60 1 + 3.056 A*C_60 2 - 2.778 A*C_60 3 + 0.519 B*C_300 1 - 0.593 B*C_300 2
+ 0.074 B*C_300 3 - 0.519 B*C_350 1 + 0.593 B*C_350 2 - 0.074 B*C_350 3
+ 1.648 A*B*C_40 300 1 - 1.407 A*B*C_40 300 2 - 0.241 A*B*C_40 300 3 - 1.648 A*B*C_40
350 1 + 1.407 A*B*C_40 350 2 + 0.241 A*B*C_40 350 3 - 1.185 A*B*C_50 300 1
+ 1.093 A*B*C_50 300 2 + 0.093 A*B*C_50 300 3 + 1.185 A*B*C_50 350 1 - 1.093 A*B*C_50
350 2 - 0.093 A*B*C_50 350 3 - 0.463 A*B*C_60 300 1 + 0.315 A*B*C_60 300 2
+ 0.148 A*B*C_60 300 3 + 0.463 A*B*C_60 350 1 - 0.315 A*B*C_60 350 2 - 0.148 A*B*C_60
350 3
```

Fits and Diagnostics for Unusual Observations

Obs	obs	Fit	Resid	Std Resid	
8	34.00	37.00	-3.00	-2.03	R
40	28.00	25.00	3.00	2.03	R

R Large residual