STAT 337

Design and Analysis of Experiments

Exercises

Editing by: Zahra Kaabi

Chapter 3

3-3: A computer ANOVA output is shown below. Fill in the blanks. You may give bounds on the P-value.

One-way ANOVA

Source	DF	SS	MS	F	P
Factor	3	36.15	(3) ?	(5)?	(6) ?
Error	(1) ?	(2) ?	(4) ?		
Total	19	196.04			

(1) We have N-1=19 ⇒**N=20** and a-1=3 ⇒**a=4**
Then (1)=N-a=20-4=16.

(2) $SS_E = SS_{Total} - SS_{Treatments} = 196.04 - 36.15 = 159.89$

(3) $MS_{Treatments} = \frac{SS_{Treatments}}{a-1} = \frac{36.15}{3} = 12.05$

(4) $MS_E = \frac{SS_E}{N-a} = \frac{159.89}{16} = 9.99$

(5) $F = \frac{MS_{Treatments}}{MS_E} = \frac{12.05}{9.99} = 1.21$

(6) p-value=0.34

How to calculate p-value:

p-value= $P(F > F_{statistic})$ where, $df_1 = a - 1 = 3$ **and** $df_2 = N - a = 16$

$$= P(F > 1.21)$$
$$= 1 - P(F < 1.21)$$
$$= 1 - 0.661906$$
$$= 0.338094 \approx 0.34$$

Minitab: $P(F < 1.21)$

calc→probability distribution→ F

→ *cumulative distribution*

→ *Numerator degree of freedom* = **3**

Denominator degree of freedom = **16**

input constant=**1.21**

→OK

The answer = 0.661906

Zahra Kaabi

3-10: A product developer is investigating the tensile strength of a new synthetic fiber that will be used to make cloth for men's shirts. Strength is usually affected by the percentage of cotton used in the blend of materials for the fiber. The engineer conducts a completely randomized experiment with five levels of cotton content and replicates the experiment five times. The data are shown in the following table.

Cotton Weight Percent	Observations				
15	7	7	15	11	9
20	12	17	12	18	18
25	14	19	19	18	18
30	19	25	22	19	23
35	7	10	11	15	11

(a) Is there evidence to support the claim that cotton con- tent affects the mean tensile strength? Use □ □ α =0.05.

(b) Use the Fisher LSD method to make comparisons between the pairs of means. What conclusions can you draw?

C1	C2
15	7
15	7
15	15
15	11
15	9
20	12
20	17
20	12
20	18
20	18
25	14
25	19
25	19

a=5 (15,20,25,30,35)

n= 5 (number of observations in each group)

N= 25 (number of all observations)

25	18
25	18
30	19
30	25
30	22
30	19
30	23
35	7
35	10
35	11
35	15
35	11

Perform an ANOVA

- Choose Stat > ANOVA > One-Way.
- Choose Response data are in one column for all factor levels.
- In Response, enter .C2 .
- In factors, enter C1..
- Click **Comparisons.**
- Under Comparison procedures assuming equal variances, check **Fisher**
- Click OK.

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$ VS

H_1: at least one of the means tensile strength for the five levels of cotton content different to others means

Since p-value $= 0.000 < 0.05 = \alpha$

$\Rightarrow$ We reject $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$, that means at least one of the means different to others means.

One-way ANOVA: C2 versus C1

```
Source  DF      SS      MS      F      P
C1       4  475.76  118.94  14.76  0.000
Error   20  161.20    8.06
Total   24  636.96

S = 2.839   R-Sq = 74.69%   R-Sq(adj) = 69.63%


                               Individual 95% CIs For Mean Based on
                               Pooled StDev
Level  N    Mean  StDev  ------+---------+---------+---------+---
15     5   9.800  3.347  (-----*----)
20     5  15.400  3.130              (----*----)
25     5  17.600  2.074                  (----*----)
30     5  21.600  2.608                          (----*----)
35     5  10.800  2.864    (-----*----)
                         ------+---------+---------+---------+---
                            10.0      15.0      20.0      25.0

Pooled StDev = 2.839
```

Grouping Information Using Fisher Method

C1	N	Mean	Grouping
30	5	21.600	A
25	5	17.600	B
20	5	15.400	B
35	5	10.800	C
15	5	9.800	C

Means that do not share a letter are significantly different.

Fisher 95% Individual Confidence Intervals
All Pairwise Comparisons among Levels of C1

Simultaneous confidence level = 73.57%

C1 = 15 subtracted from:

```
C1   Lower  Center   Upper  --------+---------+---------+---------+-
20   1.855   5.600   9.345                    (----*----)
25   4.055   7.800  11.545                       (----*---)
30   8.055  11.800  15.545                            (----*---)
35  -2.745   1.000   4.745               (---*----)
                            --------+---------+---------+---------+-
                                 -8.0       0.0       8.0      16.0
```

C1 = 20 subtracted from:

```
C1   Lower  Center   Upper  --------+---------+---------+---------+-
25  -1.545   2.200   5.945                (----*---)
30   2.455   6.200   9.945                     (----*---)
35  -8.345  -4.600  -0.855         (---*----)
                            --------+---------+---------+---------+-
                                 -8.0       0.0       8.0      16.0
```

C1 = 25 subtracted from:

```
C1    Lower  Center   Upper  --------+---------+---------+---------+-
30    0.255   4.000   7.745                  (----*----)
35  -10.545  -6.800  -3.055      (----*---)
                             --------+---------+---------+---------+-
                                  -8.0       0.0       8.0      16.0
```

C1 = 30 subtracted from:

```
C1    Lower   Center   Upper  --------+---------+---------+---------+-
35  -14.545  -10.800  -7.055  (----*---)
                              --------+---------+---------+---------+-
                                   -8.0       0.0       8.0      16.0
```

Manually: for example

If we want to compare μ_1 and μ_2 (*Fisher Least Significant Difference (LSD) Method*)

$H_0: \mu_1 = \mu_2$ VS $H_1: \mu_1 \neq \mu_2$

We reject $H_0: \mu_1 = \mu_2$ *if* $|\bar{Y}_{1.} - \bar{Y}_{2.}| > LSD = t_{\frac{\alpha}{2}, N-a}\sqrt{Ms_E\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$

$LSD = t_{\frac{\alpha}{2}, N-a}\sqrt{Ms_E\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = 2.09\sqrt{8.06\left(\frac{2}{5}\right)}$=3.75

$|\bar{Y}_{1.} - \bar{Y}_{2.}| = |9.8 - 15.4| = 5.6$ >3.75=LSD

$\Rightarrow$ We reject $H_0: \mu_1 = \mu_2$, that means there is a difference between them.

Construct a 95 percent confidence interval on the difference between μ_1 and μ_2

$$\mu_i - \mu_j \in \bar{Y}_{i.} - \bar{Y}_{j.} \pm t_{\frac{\alpha}{2}, N-a}\sqrt{Ms_E\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

$\mu_1 - \mu_2 \in \bar{Y}_{1.} - \bar{Y}_{2.} \pm t_{\frac{\alpha}{2}, N-a}\sqrt{Ms_E\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$

$\bar{Y}_{1.} - \bar{Y}_{2.} = 9.8 - 15.4 = -5.6$

$t_{\frac{\alpha}{2}, N-a}\sqrt{Ms_E\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = 2.09\sqrt{8.06\left(\frac{2}{5}\right)}$=3.75

$$\mu_1 - \mu_2 \in -5.6 \pm \mathbf{3.75} \Rightarrow \mu_1 - \mu_2 \in (-9.35, -1.85)$$

If we want to compare μ_1 *and* μ_5 (*Fisher Least Significant Difference (LSD) Method*)

$H_0: \mu_1 = \mu_5$ VS $H_1: \mu_1 \neq \mu_5$

We reject $H_0: \mu_1 = \mu_5$ *if* $|\bar{Y}_{1.} - \bar{Y}_{5.}| > LSD = t_{\frac{\alpha}{2}, N-a}\sqrt{Ms_E\left(\frac{1}{n_1} + \frac{1}{n_5}\right)}$

$$LSD = t_{\frac{\alpha}{2}, N-a}\sqrt{Ms_E\left(\frac{1}{n_1} + \frac{1}{n_5}\right)} = 2.09\sqrt{8.06\left(\frac{2}{5}\right)} = 3.75$$

$|\bar{Y}_{1.} - \bar{Y}_{5.}| = |9.8 - 10.8| = 1 < 3.75 = LSD$

$\Rightarrow$ We cannot reject $H_0: \mu_1 = \mu_5$, that means there is no significant difference between them.

Construct a 95 percent confidence interval on the difference between μ_1 *and* μ_5

$$\mu_1 - \mu_5 \in \bar{Y}_{1.} - \bar{Y}_{5.} \pm t_{\frac{\alpha}{2}, N-a}\sqrt{Ms_E\left(\frac{1}{n_1} + \frac{1}{n_5}\right)}$$

$$\bar{Y}_{1.} - \bar{Y}_{5.} = 9.8 - 10.8 = -1$$

$$t_{\frac{\alpha}{2}, N-a}\sqrt{Ms_E\left(\frac{1}{n_1} + \frac{1}{n_5}\right)} = 2.09\sqrt{8.06\left(\frac{2}{5}\right)} = 3.75$$

$$\mu_1 - \mu_5 \in -1 \pm \mathbf{3.75} \Rightarrow \mu_1 - \mu_5 \in (-4.75, 2.75)$$

3.12: A pharmaceutical manufacturer wants to investigate the bioactivity of a new drug. A completely randomized single-factor experiment was conducted with three dosage levels, and the following results were obtained.

Dosage	Observations			
20 g	24	28	37	30
30 g	37	44	31	35
40 g	42	47	52	38

(**a**) Is there evidence to indicate that dosage level affects bioactivity? Use $\alpha = 0.05$.

(**b**) If it is appropriate to do so, make comparisons between the pairs of means. What conclusions can you draw?

Dosage	observations
20	24
20	28
20	37
20	30
30	37
30	44
30	31
30	35
40	42
40	47
40	52
40	38

a=3

n= 4

N= 12

Perform an ANOVA

- Choose Stat > ANOVA > One-Way.
- Choose Response data are in one column for all factor levels.
- In Response, enter. **observations**
- In Factors, enter **Dosage**
- Click **Comparisons**.
- Under Comparison procedures assuming equal variances, check **Tukey**.

  Click **OK**

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$H_0: \mu_1 = \mu_2 = \mu_3$ **VS** H_1: at least one of the means different to others means

Since p-value = $0.014 < 0.05 = \alpha$

$\Rightarrow$ We reject $H_0: \mu_1 = \mu_2 = \mu_3$, there appears to be a different in the dosages.

One-way ANOVA: observations versus dosage

```
Source  DF     SS     MS     F      P
dosage   2  450.7  225.3  7.04  0.014
Error    9  288.3   32.0
Total   11  738.9

S = 5.659   R-Sq = 60.99%   R-Sq(adj) = 52.32%


                         Individual 95% CIs For Mean Based on
                         Pooled StDev
Level  N    Mean  StDev  -------+---------+---------+---------+--
20     4  29.750  5.439  (---------*--------)
30     4  36.750  5.439            (---------*--------)
40     4  44.750  6.076                        (--------*--------)
                         -------+---------+---------+---------+--
                             28.0      35.0      42.0      49.0

Pooled StDev = 5.659
```

```
Grouping Information Using Tukey Method

dosage  N    Mean  Grouping
40      4  44.750  A
30      4  36.750  A B
20      4  29.750    B
```

Means that do not share a letter are significantly different.

```
Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons among Levels of dosage

Individual confidence level = 97.91%


dosage = 20 subtracted from:

dosage   Lower  Center   Upper  ------+---------+---------+---------+---
30      -4.177   7.000  18.177               (--------*--------)
40       3.823  15.000  26.177                     (---------*--------)
                                ------+---------+---------+---------+---
                                    -12         0        12        24


dosage = 30 subtracted from:

dosage   Lower  Center   Upper  ------+---------+---------+---------+---
40      -3.177   8.000  19.177               (---------*--------)
                                ------+---------+---------+---------+---
                                    -12         0        12        24
```

Manually: for example

If we want to compare μ_1 *and* μ_2 *(Tukey's Test)*

$H_0: \mu_1 = \mu_2$ VS $H_1: \mu_1 \neq \mu_2$

We reject $H_0: \mu_1 = \mu_2$ *if* $|\bar{Y}_{1.} - \bar{Y}_{2.}| > T_\alpha = q_{\alpha(a,f)}\sqrt{Ms_E\left(\frac{1}{n}\right)}$

$q_{\alpha(a,f)}$ *where* $a = 3$ *and* $N = 12 \Rightarrow f = N - a = 12 - 3 = 9$ (table VII pdf 718)

$T_{0.05} = q_{0.05}(3,9)\sqrt{Ms_E\left(\frac{1}{n}\right)} = 3.948\sqrt{32\left(\frac{1}{4}\right)}=11.16$

$|\bar{Y}_{1.} - \bar{Y}_{2.}| = |29.75 - 36.75| = 7 < 11.16 = T_\alpha$

$\Rightarrow$ We cannot reject $H_0: \mu_1 = \mu_2$, that means there is no significant difference between them.

Construct a 95 percent confidence interval on the difference between μ_1 *and* μ_2

$$\mu_i - \mu_j \in \bar{Y}_{i.} - \bar{Y}_{j.} \pm q_\alpha(a,f)\sqrt{\frac{Ms_E}{n}}$$

$\mu_1 - \mu_2 \in \bar{Y}_{1.} - \bar{Y}_{2.} \pm q_\alpha(a,f)\sqrt{\frac{Ms_E}{n}}$

$\bar{Y}_{1.} - \bar{Y}_{2.} = 29.75 - 36.75 = -7$

$q_{0.05}(3,9)\sqrt{\frac{Ms_E}{n}} = 3.948\sqrt{32\left(\frac{1}{4}\right)}=11.16$

$\mu_1 - \mu_2 \in -7 \pm 11.16 \Rightarrow \mu_1 - \mu_2 \in (-18.16, 4.16)$

If we want to compare μ_1 *and* μ_3 *(Tukey's Test)*

$H_0: \mu_1 = \mu_3$ VS $H_1: \mu_1 \neq \mu_3$

We reject $H_0: \mu_1 = \mu_3$ *if* $|\bar{Y}_{1.} - \bar{Y}_{3.}| > T_\alpha = q_{\alpha(a,f)}\sqrt{Ms_E\left(\frac{1}{n}\right)}$

$T_\alpha = q_{0.05(3,9)}\sqrt{Ms_E\left(\frac{1}{n}\right)} = 3.948\sqrt{8.06\left(\frac{2}{5}\right)}$=11.16

$|\bar{Y}_{1.} - \bar{Y}_{3.}| = |29.75 - 44.75| = 15$ >12.49=T_α

$\Rightarrow$ We reject $H_0: \mu_1 = \mu_3$, that means there is a difference between them.

Construct a 95 percent confidence interval on the difference between μ_1 *and* μ_3

$\mu_1 - \mu_3 \in \bar{Y}_{1.} - \bar{Y}_{3.} \pm q_\alpha(a,f)\sqrt{\frac{Ms_E}{n}}$

$\bar{Y}_{1.} - \bar{Y}_{3.} = 29.75 - 44.75 = -15$

$q_{0.05}(3,9)\sqrt{\frac{Ms_E}{n}} = 3.948\sqrt{32\left(\frac{1}{4}\right)}$=11.16

$\mu_1 - \mu_3 \in -15 \pm 11.16 \Rightarrow \mu_1 - \mu_3 \in (-26.16, -3.84)$

If we want to compare μ_1 *and* μ_2 (Scheffé's Method)

suppose that the contrast of interests $\Gamma_1 = \mu_1 - \mu_2$

$H_0: \Gamma_1 = 0$ VS $H_1: \Gamma_1 \neq 0$ $\equiv$ $H_0: \mu_1 = \mu_2$ VS $H_1: \mu_1 \neq \mu_2$

The numerical value of this contrast is $C_1 = \bar{Y}_{1.} - \bar{Y}_{2.} = 29.75 - 36.75 = -7$

The standard error is $S_{C_1} = \sqrt{Ms_E \sum_{i=1}^{3} \frac{c^2{}_{i1}}{n_i}} = \sqrt{32\left(\frac{1}{4} + \frac{1}{4}\right)} = 4$ $(n_i = 4)$

The Critical value against C_1 which should be compared is

$S_{\alpha,1} = S_{C_1}\sqrt{(a-1)F_{\alpha,a-1,N-a}} = 4\sqrt{2F_{0.05,2,9}} = 4\sqrt{2(4.26)} = 11.68$ (a = 3)

$F_{0.05,2,9}$ *(by using minitab* $F_{0.95,2,9} = $ 4.25649 *)*

Minitab: $F_{0.95,2,9}$

calc → probability distribution→ F

→ *iverse cumulative distribution*

→ *Numerator degree of freedom* = **2** *Denominator degree of freedom* = **9** input constant=**0.95**

→OK

If $|C_1| = |\bar{Y}_{1.} - \bar{Y}_{2.}| > S_{\alpha,1}$ the hypothesis that the contrast $\Gamma_1 = 0$ is rejected

Here, $|C_1| = 7$ and $S_{\alpha,1}$=11.68 $\Rightarrow |\boldsymbol{C_1}| = \mathbf{7} < 11.68 = \boldsymbol{S_{\alpha,1}}$

Then, we cannot reject the hypothesis that contrast $\Gamma_1 = 0$, that means there is no significant difference between μ_1 *and* μ_2.

3.13: A rental car company wants to investigate whether the type of car rented affects the length of the rental period. An experiment is run for one week at a particular location, and 10 rental contracts are selected at random for each car type. The results are shown in the following table.

Type of Car	Observations									
Subcompact	3	5	3	7	6	5	3	2	1	6
Compact	1	3	4	7	5	6	3	2	1	7
Midsize	4	1	3	5	7	1	2	4	2	7
Full size	3	5	7	5	10	3	4	7	2	7

. Is there evidence to support a claim that the type of car rented affects the length of the rental contract? Use $\alpha = 0.05$. If so, which types of cars are responsible for the difference?

Type of car	observations
subcompact	3
subcompact	5
subcompact	3
subcompact	7
subcompact	6
subcompact	5
subcompact	3
subcompact	2
subcompact	1
subcompact	6
compact	1
compact	3
compact	4
compact	7
compact	5
compact	6
compact	3
compact	2
compact	1
compact	7
midsize	4
midsize	1
midsize	3

a=4

n=10

N=40

midsize	5
midsize	7
midsize	1
midsize	2
midsize	4
midsize	2
midsize	7
fullsize	3
fullsize	5
fullsize	7
fullsize	5
fullsize	10
fullsize	3
fullsize	4
fullsize	7
fullsize	2
fullsize	7

Perform an ANOVA

- Choose Stat > ANOVA > One-Way.
- Choose Response data are in one column for all factor levels.
- In Response, enter. **Observations**
- In Factors, enter **Type of car**
- Click **OK**

One-way ANOVA: observations versus Type of car

```
Source        DF      SS    MS     F      P
Type of car    3   16.67  5.56  1.11  0.358
Error         36  180.30  5.01
Total         39  196.98

S = 2.238   R-Sq = 8.47%   R-Sq(adj) = 0.84%


                              Individual 95% CIs For Mean Based on
                              Pooled StDev
Level        N   Mean  StDev  --+---------+---------+---------+-------
compact     10  3.900  2.283     (----------*-----------)
fullsize    10  5.300  2.452                (-----------*-----------)
midsize     10  3.600  2.221  (-----------*-----------)
subcompact  10  4.100  1.969      (-----------*-----------)
                              --+---------+---------+---------+-------
                              2.4       3.6       4.8       6.0
```

Pooled StDev = 2.238

(a) We are testing here

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$ VS H_1: at least one of the means different to others means

Since p-value $= 0.358 > 0.05 = \alpha$

$\Rightarrow$ We cannot reject $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

which means the type of car rented does not affects the length of the rental contract .

HW 3.15

3.15: A regional opera company has tried three approaches to solicit donations from 24 potential sponsors. The 24 poten- tial sponsors were randomly divided into three groups of eight, and one approach was used for each group. The dollar amounts of the resulting contributions are shown in the fol- lowing table.

Approach	Contributions (in $)							
1	1000	1500	1200	1800	1600	1100	1000	1250
2	1500	1800	2000	1200	2000	1700	1800	1900
3	900	1000	1200	1500	1200	1550	1000	1100

approach	contributions
1	1000
1	1500
1	1200
1	1800
1	1600
1	1100
1	1000
1	1250
2	1500

2	1800
2	2000
2	1200
2	2000
2	1700
2	1800
2	1900
3	900
3	1000
3	1200
3	1500
3	1200
3	1550
3	1000
3	1100

One-way ANOVA: contributions versus approach

```
Source    DF       SS      MS     F      P
approach   2  1362708  681354  9.41  0.001
Error     21  1520625   72411
Total     23  2883333

S = 269.1   R-Sq = 47.26%   R-Sq(adj) = 42.24%


                               Individual 95% CIs For Mean Based on Pooled StDev
Level  N    Mean  StDev    --+---------+---------+---------+--------
1      8  1306.3  295.7         (-------*-------)
2      8  1737.5  272.2                           (-------*------)
3      8  1181.3  235.9     (-------*-------)
                           --+---------+---------+---------+--------
                           1000      1250      1500      1750

Pooled StDev = 269.1
```

One-way ANOVA

$H_0: \mu_1 = \mu_2 = \mu_3$ VS H_1: at least one of the means different to others means

Since p-value $= 0.001 < 0.05 = \alpha$

$\Rightarrow$ We reject $H_0: \mu_1 = \mu_2 = \mu_3$,

That means there is a difference between the three approaches.

Zahra Kaabi

```
Grouping Information Using Tukey Method

approach  N    Mean  Grouping
2         8  1737.5  A
1         8  1306.3    B
3         8  1181.3    B
```

Means that do not share a letter are significantly different.

```
Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons among Levels of approach

Individual confidence level = 98.00%


approach = 1 subtracted from:

approach   Lower  Center  Upper  --------+---------+---------+---------+-
2           92.6   431.3  769.9                     (------*-----)
3         -463.7  -125.0  213.7          (------*-----)
                                 --------+---------+---------+---------+-
                                      -500         0       500      1000


approach = 2 subtracted from:

approach   Lower  Center   Upper  --------+---------+---------+---------+-
3         -894.9  -556.3  -217.6  (------*------)
                                  --------+---------+---------+---------+-

                                       -500         0       500      1000
```

Manually: for example

If we want to compare μ_1 *and* μ_2 *(Tukey's Test)*

$H_0: \mu_1 = \mu_2$ VS $H_0: \mu_1 \neq \mu_2$

We reject $H_0: \mu_1 = \mu_2$ if $|\bar{Y}_{1.} - \bar{Y}_{2.}| > T_\alpha = q_{\alpha(a,f)}\sqrt{Ms_E\left(\frac{1}{n}\right)}$

$$T_{0.05} = q_{0.05}(3,21)\sqrt{Ms_E\left(\frac{1}{n}\right)} = 3.565\sqrt{72411\left(\frac{1}{8}\right)}=339.17$$

$$|\bar{Y}_{1.} - \bar{Y}_{2.}| = |1306.3 - 1737.5| = 431.2 > 339.17 = T_\alpha$$

$\Rightarrow$ We reject $H_0: \mu_1 = \mu_2$, that means there a difference between them.

Construct a 95 percent confidence interval on the difference between μ_1 *and* μ_2

$$\mu_i - \mu_j \in \bar{Y}_{i.} - \bar{Y}_{j.} \pm q_\alpha(a, f)\sqrt{\frac{Ms_E}{n}}$$

$$\mu_1 - \mu_2 \in \bar{Y}_{1.} - \bar{Y}_{2.} \pm q_\alpha(a, f)\sqrt{\frac{Ms_E}{n}}$$

$$\bar{Y}_{1.} - \bar{Y}_{2.} = 1306.3 - 1737.5 = -431.2$$

$$q_{0.05}(3, 21)\sqrt{\frac{Ms_E}{n}} = 3.565\sqrt{72411\left(\frac{1}{8}\right)} = 339.17$$

$$\mu_1 - \mu_2 \in -431.2 \pm 339.17 \Rightarrow \mu_1 - \mu_2 \in (-770.37, -92.03)$$

If we want to compare μ_1 *and* μ_3 *(Tukey's Test)*

$H_0: \mu_1 = \mu_3$ VS $H_0: \mu_1 \neq \mu_3$

We reject $H_0: \mu_1 = \mu_3$ if $|\bar{Y}_{1.} - \bar{Y}_{3.}| > T_\alpha = q_{\alpha(a,f)}\sqrt{Ms_E\left(\frac{1}{n}\right)}$

$$T_\alpha = q_{0.05}(3{,}21)\sqrt{\frac{Ms_E}{n}} = 3.565\sqrt{72411\left(\frac{1}{8}\right)} = 339.17 = T_\alpha$$

$$|\bar{Y}_{1.} - \bar{Y}_{3.}| = |1306.3 - 1181.3| = 125 > 339.17 = T_\alpha$$

$\Rightarrow$ We reject $H_0: \mu_1 = \mu_3$, that means there is a difference between them.

Construct a 95 percent confidence interval on the difference between μ_1 and μ_3

$$\mu_1 - \mu_3 \in \bar{Y}_{1.} - \bar{Y}_{3.} \pm q_\alpha(a,f)\sqrt{\frac{Ms_E}{n}}$$

$$\bar{Y}_{1.} - \bar{Y}_{3.} = 1306.3 - 1181.3 = 125$$

$$q_{0.05}(3,21)\sqrt{\frac{Ms_E}{n}} = 3.565\sqrt{72411\left(\frac{1}{8}\right)} = 339.17$$

$$\mu_1 - \mu_3 \in 125 \pm 339.17 \Rightarrow \mu_1 - \mu_3 \in (-214.17, 464.17)$$

```
Grouping Information Using Fisher Method

approach  N    Mean  Grouping
2         8  1737.5  A
1         8  1306.3    B
3         8  1181.3    B

Means that do not share a letter are significantly different.



Fisher 95% Individual Confidence Intervals
All Pairwise Comparisons among Levels of approach

Simultaneous confidence level = 88.16%


approach = 1 subtracted from:

approach   Lower  Center  Upper  -------+---------+---------+---------+--
2          151.4   431.3  711.1                     (-----*----)
3         -404.8  -125.0  154.8           (-----*----)
                                 -------+---------+---------+---------+--
                                      -500         0       500      1000


approach = 2 subtracted from:

approach   Lower  Center   Upper  -------+---------+---------+---------+--
3         -836.1  -556.3  -276.4  (-----*----)
                                  -------+---------+---------+---------+--
                                       -500         0       500      1000
```

Manually: for example

If we want to compare μ_1 *and* μ_2 (LSD method)

$H_0: \mu_1 = \mu_2$ VS $H_0: \mu_1 \neq \mu_2$

We reject $H_0: \mu_1 = \mu_2$ if $|\bar{Y}_{1.} - \bar{Y}_{2.}| > LSD = t_{\frac{\alpha}{2}, N-a}\sqrt{Ms_E\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$

$$LSD = t_{\frac{\alpha}{2}, N-a}\sqrt{Ms_E\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = t_{0.025,21}\sqrt{72411\left(\frac{2}{8}\right)}$$

$$= 2.08\sqrt{72411\left(\frac{2}{8}\right)} = 279.86$$

$|\bar{Y}_{1.} - \bar{Y}_{2.}| = |1306.3 - 1737.5| = 431.2 > 279.86 = \text{LSD}$

$\Rightarrow$ We reject $H_0: \mu_1 = \mu_2$, that means there is a difference between them.

Construct a 95 percent confidence interval on the difference between μ_1 *and* μ_2

$$\mu_i - \mu_j \in \bar{Y}_{i.} - \bar{Y}_{j.} \pm t_{\frac{\alpha}{2}, N-a}\sqrt{Ms_E\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

$$\mu_1 - \mu_2 \in \bar{Y}_{1.} - \bar{Y}_{2.} \pm t_{\frac{\alpha}{2}, N-a}\sqrt{Ms_E\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$\bar{Y}_{1.} - \bar{Y}_{2.} = 1306.3 - 1737.5 = -431.2$$

$$t_{\frac{\alpha}{2}, N-a}\sqrt{Ms_E\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = t_{0.025,21}\sqrt{72411\left(\frac{2}{8}\right)}$$

$$= 2.08\sqrt{72411\left(\frac{2}{8}\right)}=279.86$$

$$\boldsymbol{\mu_1} - \boldsymbol{\mu_2} \in -431.2 \pm 279.86 \Rightarrow \boldsymbol{\mu_1} - \boldsymbol{\mu_2} \in (-711.06, -151.34)$$

If we want to compare μ_1 *and* μ_3 (LSD method)

$H_0: \mu_1 = \mu_3$ VS $H_0: \mu_1 \neq \mu_3$

We reject $H_0: \mu_1 = \mu_3$ if $|\bar{Y}_{1.} - \bar{Y}_{3.}| > LSD = t_{\frac{\alpha}{2}, N-a}\sqrt{Ms_E\left(\frac{1}{n_1} + \frac{1}{n_3}\right)}$

$LSD = t_{\frac{\alpha}{2}, N-a}\sqrt{Ms_E\left(\frac{1}{n_1} + \frac{1}{n_3}\right)} = t_{0.025,21}\sqrt{72411\left(\frac{2}{8}\right)} =$

$2.08\sqrt{72411\left(\frac{2}{8}\right)}=279.86$

$|\bar{Y}_{1.} - \bar{Y}_{3.}| = |1306.3 - 1181.3| = 125 < 279.86 = LSD \Rightarrow$ We cannot reject $H_0: \mu_1 = \mu_3$, that means there is no significant difference between them.

Construct a 95 percent confidence interval on the difference between μ_1 *and* μ_3

$$\boldsymbol{\mu_1} - \boldsymbol{\mu_3} \in \bar{\boldsymbol{Y}}_{1.} - \bar{\boldsymbol{Y}}_{3.} \pm t_{\frac{\alpha}{2}, N-a}\sqrt{Ms_E\left(\frac{1}{n_1} + \frac{1}{n_3}\right)}$$

$\bar{\boldsymbol{Y}}_{1.} - \bar{\boldsymbol{Y}}_{3.} = 1306.3 - 1181.3 = 125$

$t_{\frac{\alpha}{2}, N-a}\sqrt{Ms_E\left(\frac{1}{n_1} + \frac{1}{n_5}\right)} = t_{0.025,21}\sqrt{72411\left(\frac{2}{8}\right)} = 2.08\sqrt{72411\left(\frac{2}{8}\right)}=279.86$

$\boldsymbol{\mu_1} - \boldsymbol{\mu_3} \in 125 \pm 279.86 \Rightarrow \boldsymbol{\mu_1} - \boldsymbol{\mu_3} \in (-154.86, 404.86)$.