

STAT 337

**Design and Analysis of
Experiments**

Exercises

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Chapter 3

3-3: A computer ANOVA output is shown below. Fill in the blanks. You may give bounds on the P -value.

One-way ANOVA					
Source	DF	SS	MS	F	P
Factor	3	36.15	(3) ?	(5)?	(6) ?
Error	(1) ?	(2) ?	(4) ?		
Total	19	196.04			

(1) We have $N-1=19 \Rightarrow N=20$ and $a-1=3 \Rightarrow a=4$
Then $(1)=N-a=20-4=16$.

(2) $SS_E = SS_{\text{Total}} - SS_{\text{Treatments}} = 196.04 - 36.15 = 159.89$

(3) $MS_{\text{Treatments}} = \frac{SS_{\text{Treatments}}}{a-1} = \frac{36.15}{3} = 12.05$

(4) $MS_E = \frac{SS_E}{N-a} = \frac{159.89}{16} = 9.99$

(5) $F = \frac{MS_{\text{Treatments}}}{MS_E} = \frac{12.05}{9.99} = 1.21$

(6) $p\text{-value}=0.34$

How to calculate p-value:

$p\text{-value} = P(F > F_{\text{statistic}})$ where, $df_1 = a - 1 = 3$ and $df_2 = N - a = 16$
 $= P(F > 1.21)$
 $= 1 - P(F < 1.21)$
 $= 1 - 0.661906$
 $= 0.338094 \approx 0.34$

Minitab: $P(F < 1.21)$

calc→probability distribution→ F

→ cumulative distribution

→ Numerator degree of freedom = 3

Denominator degree of freedom = 16

input constant=1.21

→OK

The answer = 0.661906

3-10: A product developer is investigating the tensile strength of a new synthetic fiber that will be used to make cloth for men's shirts. Strength is usually affected by the percentage of cotton used in the blend of materials for the fiber. The engineer conducts a completely randomized experiment with five levels of cotton content and replicates the experiment five times. The data are shown in the following table.

Cotton Weight Percent	Observations				
15	7	7	15	11	9
20	12	17	12	18	18
25	14	19	19	18	18
30	19	25	22	19	23
35	7	10	11	15	11

- (a) Is there evidence to support the claim that cotton content affects the mean tensile strength? Use $\alpha = 0.05$.
- (b) Use the Fisher LSD method to make comparisons between the pairs of means. What conclusions can you draw?

C1	C2
15	7
15	7
15	15
15	11
15	9
20	12
20	17
20	12
20	18
20	18
25	14
25	19
25	19

$a=5$ (15,20,25,30,35)
 $n= 5$ (number of observations in each group)
 $N= 25$ (number of all observations)

25	18
25	18
30	19
30	25
30	22
30	19
30	23
35	7
35	10
35	11
35	15
35	11

Perform an ANOVA

- Choose Stat > ANOVA > One-Way.
- Choose Response data are in one column for all factor levels.
- In Response, enter .C2 .
- In factors, enter C1..
- Click **Comparisons**.
- Under Comparison procedures assuming equal variances, check **Fisher**
- Click OK.

One-way ANOVA: C2 versus C1

Source	DF	SS	MS	F	P
C1	4	475.76	118.94	14.76	0.000
Error	20	161.20	8.06		
Total	24	636.96			

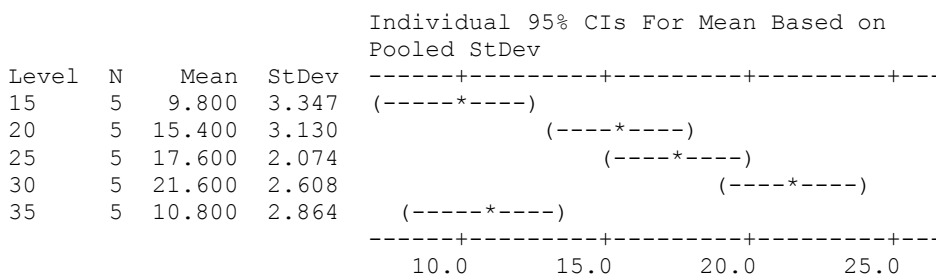
S = 2.839 R-Sq = 74.69% R-Sq(adj) = 69.63%

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$ VS

H_1 : at least one of the means tensile strength for the five levels of cotton content different to others means

Since p-value = 0.000 < 0.05 = α

⇒ We reject $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$, that means at least one of the means different to others means.

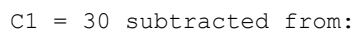
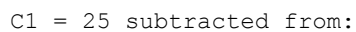
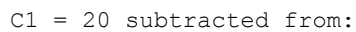


Pooled StDev = 2.839

C1	N	Mean	Grouping
30	5	21.600	A
25	5	17.600	B
20	5	15.400	B
35	5	10.800	C
15	5	9.800	C

Fisher 95% Individual Confidence Intervals
All Pairwise Comparisons among Levels of C1

C1 = 15 subtracted from:



Manually: for example

If we want to compare μ_1 and μ_2 (Fisher Least Significant Difference (LSD) Method)

$$H_0: \mu_1 = \mu_2 \text{ VS } H_1: \mu_1 \neq \mu_2$$

We reject $H_0: \mu_1 = \mu_2$ if $|\bar{Y}_1 - \bar{Y}_2| > LSD = t_{\frac{\alpha}{2}, N-a} \sqrt{MS_E \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$

$$LSD = t_{\frac{\alpha}{2}, N-a} \sqrt{MS_E \left(\frac{1}{n_1} + \frac{1}{n_2} \right)} = 2.09 \sqrt{8.06 \left(\frac{2}{5} \right)} = 3.75$$

$$|\bar{Y}_1 - \bar{Y}_2| = |9.8 - 15.4| = 5.6 > 3.75 = LSD$$

\Rightarrow We reject $H_0: \mu_1 = \mu_2$, that means there is a difference between them.

Construct a 95 percent confidence interval on the difference between μ_1 and μ_2

$$\mu_i - \mu_j \in \bar{Y}_i - \bar{Y}_j \pm t_{\frac{\alpha}{2}, N-a} \sqrt{MS_E \left(\frac{1}{n_i} + \frac{1}{n_j} \right)}$$

$$\mu_1 - \mu_2 \in \bar{Y}_1 - \bar{Y}_2 \pm t_{\frac{\alpha}{2}, N-a} \sqrt{MS_E \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$$

$$\bar{Y}_1 - \bar{Y}_2 = 9.8 - 15.4 = -5.6$$

$$t_{\frac{\alpha}{2}, N-a} \sqrt{MS_E \left(\frac{1}{n_1} + \frac{1}{n_2} \right)} = 2.09 \sqrt{8.06 \left(\frac{2}{5} \right)} = 3.75$$

$$\mu_1 - \mu_2 \in -5.6 \pm 3.75 \Rightarrow \mu_1 - \mu_2 \in (-9.35, -1.85)$$

If we want to compare μ_1 and μ_5 (Fisher Least Significant Difference (LSD) Method)

$$H_0: \mu_1 = \mu_5 \text{ VS } H_1: \mu_1 \neq \mu_5$$

We reject $H_0: \mu_1 = \mu_5$ if $|\bar{Y}_1 - \bar{Y}_5| > LSD = t_{\frac{\alpha}{2}, N-a} \sqrt{MS_E \left(\frac{1}{n_1} + \frac{1}{n_5} \right)}$

$$LSD = t_{\frac{\alpha}{2}, N-a} \sqrt{MS_E \left(\frac{1}{n_1} + \frac{1}{n_5} \right)} = 2.09 \sqrt{8.06 \left(\frac{2}{5} \right)} = 3.75$$

$$|\bar{Y}_1 - \bar{Y}_5| = |9.8 - 10.8| = 1 < 3.75 = LSD$$

\Rightarrow We cannot reject $H_0: \mu_1 = \mu_5$, that means there is no significant difference between them.

Construct a 95 percent confidence interval on the difference between μ_1 and μ_5

$$\mu_1 - \mu_5 \in \bar{Y}_1 - \bar{Y}_5 \pm t_{\frac{\alpha}{2}, N-a} \sqrt{MS_E \left(\frac{1}{n_1} + \frac{1}{n_5} \right)}$$

$$\bar{Y}_1 - \bar{Y}_5 = 9.8 - 10.8 = -1$$

$$t_{\frac{\alpha}{2}, N-a} \sqrt{MS_E \left(\frac{1}{n_1} + \frac{1}{n_5} \right)} = 2.09 \sqrt{8.06 \left(\frac{2}{5} \right)} = 3.75$$

$$\mu_1 - \mu_5 \in -1 \pm 3.75 \Rightarrow \mu_1 - \mu_5 \in (-4.75, 2.75)$$

3.12: A pharmaceutical manufacturer wants to investigate the bioactivity of a new drug. A completely randomized single-factor experiment was conducted with three dosage levels, and the following results were obtained.

Dosage		Observations			
20 g	24	28	37	30	
30 g	37	44	31	35	
40 g	42	47	52	38	

- (a) Is there evidence to indicate that dosage level affects bioactivity? Use $\alpha = 0.05$.
- (b) If it is appropriate to do so, make comparisons between the pairs of means. What conclusions can you draw?

Dosage	observations
20	24
20	28
20	37
20	30
30	37
30	44
30	31
30	35
40	42
40	47
40	52
40	38

a=3
n= 4
N= 12

Perform an ANOVA

- Choose Stat > ANOVA > One-Way.
- Choose Response data are in one column for all factor levels.
- In Response, enter. **observations**
- In Factors, enter **Dosage**
- Click **Comparisons**.
- Under Comparison procedures assuming equal variances, check **Tukey**. Click **OK**

One-way ANOVA: observations versus dosage

Source	DF	SS	MS	F	P
dosage	2	450.7	225.3	7.04	0.014
Error	9	288.3	32.0		
Total	11	738.9			

S = 5.659 R-Sq = 60.99% R-Sq(adj) = 52.32%

$H_0: \mu_1 = \mu_2 = \mu_3$ VS H_1 : at least one of the means
different to others means

Since p-value = 0.014 < 0.05 = α

⇒ We reject $H_0: \mu_1 = \mu_2 = \mu_3$, there appears to be a different in the dosages.

Individual 95% CIs For Mean Based on Pooled StDev			
Level	N	Mean	StDev
20	4	29.750	5.439
30	4	36.750	5.439
40	4	44.750	6.076

Pooled StDev = 5.659

Grouping Information Using Tukey Method

dosage	N	Mean	Grouping
40	4	44.750	A
30	4	36.750	A B
20	4	29.750	B

Means that do not share a letter are significantly different.

Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons among Levels of dosage

Individual confidence level = 97.91%

dosage = 20 subtracted from:

dosage	Lower	Center	Upper
30	-4.177	7.000	18.177
40	3.823	15.000	26.177

dosage = 30 subtracted from:

dosage	Lower	Center	Upper
40	-3.177	8.000	19.177

Manually: for example

If we want to compare μ_1 and μ_2 (Tukey's Test)

$$H_0: \mu_1 = \mu_2 \text{ VS } H_1: \mu_1 \neq \mu_2$$

We reject $H_0: \mu_1 = \mu_2$ if $|\bar{Y}_{1.} - \bar{Y}_{2.}| > T_\alpha = q_{\alpha(a,f)} \sqrt{MS_E \left(\frac{1}{n}\right)}$

$q_{\alpha(a,f)}$ where $a = 3$ and $N = 12 \Rightarrow f = N - a = 12 - 3 = 9$ (table VII pdf 718)

$$T_{0.05} = q_{0.05}(3,9) \sqrt{MS_E \left(\frac{1}{n}\right)} = 3.948 \sqrt{32 \left(\frac{1}{4}\right)} = 11.16$$

$$|\bar{Y}_{1.} - \bar{Y}_{2.}| = |29.75 - 36.75| = 7 < 11.16 = T_\alpha$$

\Rightarrow We cannot reject $H_0: \mu_1 = \mu_2$, that means there is no significant difference between them.

Construct a 95 percent confidence interval on the difference between μ_1 and μ_2

$$\mu_i - \mu_j \in \bar{Y}_{i.} - \bar{Y}_{j.} \pm q_{\alpha(a,f)} \sqrt{\frac{MS_E}{n}}$$

$$\mu_1 - \mu_2 \in \bar{Y}_{1.} - \bar{Y}_{2.} \pm q_{\alpha(a,f)} \sqrt{\frac{MS_E}{n}}$$

$$\bar{Y}_{1.} - \bar{Y}_{2.} = 29.75 - 36.75 = -7$$

$$q_{0.05}(3,9) \sqrt{\frac{MS_E}{n}} = 3.948 \sqrt{32 \left(\frac{1}{4}\right)} = 11.16$$

$$\mu_1 - \mu_2 \in -7 \pm 11.16 \Rightarrow \mu_1 - \mu_2 \in (-18.16, 4.16)$$

If we want to compare μ_1 and μ_3 (Tukey's Test)

$$H_0: \mu_1 = \mu_3 \text{ VS } H_1: \mu_1 \neq \mu_3$$

We reject $H_0: \mu_1 = \mu_3$ if $|\bar{Y}_{1.} - \bar{Y}_{3.}| > T_\alpha = q_{\alpha(a,f)} \sqrt{MS_E \left(\frac{1}{n}\right)}$

$$T_\alpha = q_{0.05(3,9)} \sqrt{MS_E \left(\frac{1}{n}\right)} = 3.948 \sqrt{8.06 \left(\frac{2}{5}\right)} = 11.16$$

$$|\bar{Y}_{1.} - \bar{Y}_{3.}| = |29.75 - 44.75| = 15 > 12.49 = T_\alpha$$

\Rightarrow We reject $H_0: \mu_1 = \mu_3$, that means there is a difference between them.

Construct a 95 percent confidence interval on the difference between μ_1 and μ_3

$$\mu_1 - \mu_3 \in \bar{Y}_{1.} - \bar{Y}_{3.} \pm q_{\alpha(a,f)} \sqrt{\frac{MS_E}{n}}$$

$$\bar{Y}_{1.} - \bar{Y}_{3.} = 29.75 - 44.75 = -15$$

$$q_{0.05(3,9)} \sqrt{\frac{MS_E}{n}} = 3.948 \sqrt{32 \left(\frac{1}{4}\right)} = 11.16$$

$$\mu_1 - \mu_3 \in -15 \pm 11.16 \Rightarrow \mu_1 - \mu_3 \in (-26.16, -3.84)$$

If we want to compare μ_1 and μ_2 (Scheffé's Method)

suppose that the contrast of interests $\Gamma_1 = \mu_1 - \mu_2$

$$H_0: \Gamma_1 = 0 \text{ VS } H_1: \Gamma_1 \neq 0 \quad \equiv \quad H_0: \mu_1 = \mu_2 \text{ VS } H_1: \mu_1 \neq \mu_2$$

The numerical value of this contrast is $C_1 = \bar{Y}_1 - \bar{Y}_2 = 29.75 - 36.75 = -7$

The standard error is $S_{C_1} = \sqrt{MS_E \sum_{i=1}^3 \frac{c_{i1}^2}{n_i}} = \sqrt{32 \left(\frac{1}{4} + \frac{1}{4} \right)} = 4 \quad (n_i = 4)$

The Critical value against C_1 which should be compared is

$$S_{\alpha,1} = S_{C_1} \sqrt{(a-1)F_{\alpha,a-1,N-a}} = 4\sqrt{2F_{0.05,2,9}} = 4\sqrt{2(4.26)} = 11.68 \quad (a=3)$$

$F_{0.05,2,9}$ (by using minitab $F_{0.95,2,9} = 4.25649$)

Minitab: $F_{0.95,2,9}$

calc → probability distribution → F

→ inverse cumulative distribution

→ Numerator degree of freedom = 2 Denominator degree of freedom = 9 input constant=0.95

→OK

If $|C_1| = |\bar{Y}_1 - \bar{Y}_2| > S_{\alpha,1}$ the hypothesis that the contrast $\Gamma_1 = 0$ is rejected

Here, $|C_1| = 7$ and $S_{\alpha,1} = 11.68 \Rightarrow |C_1| = 7 < 11.68 = S_{\alpha,1}$

Then, we cannot reject the hypothesis that contrast $\Gamma_1 = 0$, that means there is no significant difference between μ_1 and μ_2 .

3.13: A rental car company wants to investigate whether the type of car rented affects the length of the rental period. An experiment is run for one week at a particular location, and 10 rental contracts are selected at random for each car type. The results are shown in the following table.

Type of Car	Observations									
Subcompact	3	5	3	7	6	5	3	2	1	6
Compact	1	3	4	7	5	6	3	2	1	7
Midsize	4	1	3	5	7	1	2	4	2	7
Full size	3	5	7	5	10	3	4	7	2	7

Is there evidence to support a claim that the type of car rented affects the length of the rental contract? Use $\alpha = 0.05$. If so, which types of cars are responsible for the difference?

Type of car	observations
subcompact	3
subcompact	5
subcompact	3
subcompact	7
subcompact	6
subcompact	5
subcompact	3
subcompact	2
subcompact	1
subcompact	6
compact	1
compact	3
compact	4
compact	7
compact	5
compact	6
compact	3
compact	2
compact	1
compact	7
midsize	4
midsize	1
midsize	3

a=4
n=10
N=40

midsize	5
midsize	7
midsize	1
midsize	2
midsize	4
midsize	2
midsize	7
fullsize	3
fullsize	5
fullsize	7
fullsize	5
fullsize	10
fullsize	3
fullsize	4
fullsize	7
fullsize	2
fullsize	7

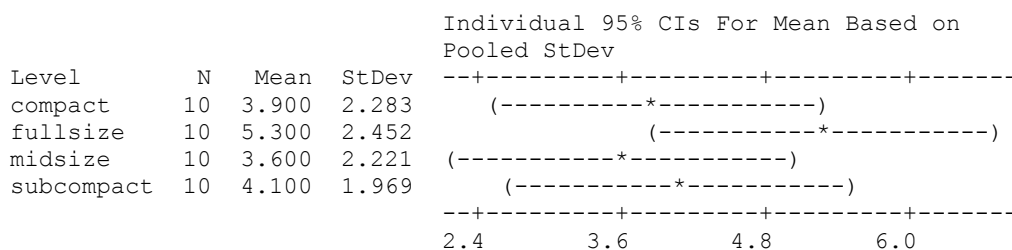
Perform an ANOVA

- Choose Stat > ANOVA > One-Way.
- Choose Response data are in one column for all factor levels.
- In Response, enter. **Observations**
- In Factors, enter **Type of car**
- Click **OK**

One-way ANOVA: observations versus Type of car

Source	DF	SS	MS	F	P
Type of car	3	16.67	5.56	1.11	0.358
Error	36	180.30	5.01		
Total	39	196.98			

S = 2.238 R-Sq = 8.47% R-Sq(adj) = 0.84%



Pooled StDev = 2.238

(a) We are testing here

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$ VS H_1 : at least one of the means different to others means

Since $p\text{-value} = 0.358 > 0.05 = \alpha$

\Rightarrow We cannot reject $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

which means the type of car rented does not affects the length of the rental contract .

HW 3.15

3.15: A regional opera company has tried three approaches to solicit donations from 24 potential sponsors. The 24 potential sponsors were randomly divided into three groups of eight, and one approach was used for each group. The dollar amounts of the resulting contributions are shown in the following table.

Approach	Contributions (in \$)							
1	1000	1500	1200	1800	1600	1100	1000	1250
2	1500	1800	2000	1200	2000	1700	1800	1900
3	900	1000	1200	1500	1200	1550	1000	1100

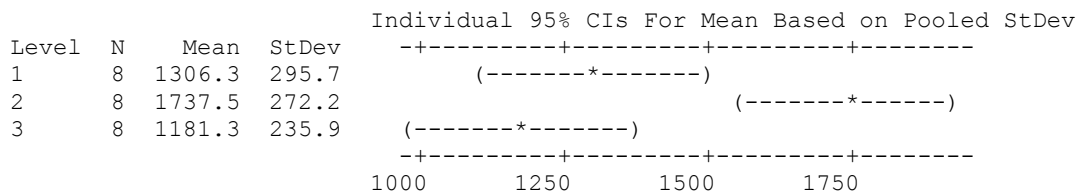
approach	contributions
1	1000
1	1500
1	1200
1	1800
1	1600
1	1100
1	1000
1	1250
2	1500

2	1800
2	2000
2	1200
2	2000
2	1700
2	1800
2	1900
3	900
3	1000
3	1200
3	1500
3	1200
3	1550
3	1000
3	1100

One-way ANOVA: contributions versus approach

Source	DF	SS	MS	F	P
approach	2	1362708	681354	9.41	0.001
Error	21	1520625	72411		
Total	23	2883333			

S = 269.1 R-Sq = 47.26% R-Sq(adj) = 42.24%



Pooled StDev = 269.1

One-way ANOVA

$H_0: \mu_1 = \mu_2 = \mu_3$ VS H_1 : at least one of the means different to others
means

Since p-value = 0.001 < 0.05 = α

\Rightarrow We reject $H_0: \mu_1 = \mu_2 = \mu_3$,

That means there is a difference between the three approaches.

Grouping Information Using Tukey Method

approach	N	Mean	Grouping
2	8	1737.5	A
1	8	1306.3	B
3	8	1181.3	B

Means that do not share a letter are significantly different.

Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons among Levels of approach

Individual confidence level = 98.00%

approach = 1 subtracted from:

approach	Lower	Center	Upper
2	92.6	431.3	769.9
3	-463.7	-125.0	213.7

-----+-----+-----+-----+-----+
 (-----*-----)
 (-----*-----)
 -----+-----+-----+-----+-----+
 -500 0 500 1000

approach = 2 subtracted from:

approach	Lower	Center	Upper
3	-894.9	-556.3	-217.6

-----+-----+-----+-----+-----+
 (-----*-----)
 -----+-----+-----+-----+-----+
 -500 0 500 1000

Manually: for example

If we want to compare μ_1 and μ_2 (Tukey's Test)

$$H_0: \mu_1 = \mu_2 \text{ VS } H_0: \mu_1 \neq \mu_2$$

We reject $H_0: \mu_1 = \mu_2$ if $|\bar{Y}_1 - \bar{Y}_2| > T_\alpha = q_{\alpha(a,f)} \sqrt{MS_E \left(\frac{1}{n} \right)}$

$$T_{0.05} = q_{0.05}(3,21) \sqrt{MS_E \left(\frac{1}{n} \right)} = 3.565 \sqrt{72411 \left(\frac{1}{8} \right)} = 339.17$$

$$|\bar{Y}_1 - \bar{Y}_2| = |1306.3 - 1737.5| = 431.2 > 339.17 = T_\alpha$$

\Rightarrow We reject $H_0: \mu_1 = \mu_2$, that means there a difference between them.

Construct a 95 percent confidence interval on the difference between μ_1 and μ_2

$$\mu_i - \mu_j \in \bar{Y}_i - \bar{Y}_j \pm q_\alpha(a, f) \sqrt{\frac{MS_E}{n}}$$

$$\mu_1 - \mu_2 \in \bar{Y}_1 - \bar{Y}_2 \pm q_\alpha(a, f) \sqrt{\frac{MS_E}{n}}$$

$$\bar{Y}_1 - \bar{Y}_2 = 1306.3 - 1737.5 = -431.2$$

$$q_{0.05}(3, 21) \sqrt{\frac{MS_E}{n}} = 3.565 \sqrt{72411 \left(\frac{1}{8}\right)} = 339.17$$

$$\mu_1 - \mu_2 \in -431.2 \pm 339.17 \Rightarrow \mu_1 - \mu_2 \in (-770.37, -92.03)$$

If we want to compare μ_1 and μ_3 (Tukey's Test)

$$H_0: \mu_1 = \mu_3 \text{ VS } H_0: \mu_1 \neq \mu_3$$

We reject $H_0: \mu_1 = \mu_3$ if $|\bar{Y}_1 - \bar{Y}_3| > T_\alpha = q_{\alpha(a, f)} \sqrt{MS_E \left(\frac{1}{n}\right)}$

$$T_\alpha = q_{0.05}(3, 21) \sqrt{\frac{MS_E}{n}} = 3.565 \sqrt{72411 \left(\frac{1}{8}\right)} = 339.17 = T_\alpha$$

$$|\bar{Y}_1 - \bar{Y}_3| = |1306.3 - 1181.3| = 125 > 339.17 = T_\alpha$$

\Rightarrow We reject $H_0: \mu_1 = \mu_3$, that means there is a difference between them.

Construct a 95 percent confidence interval on the difference between μ_1 and μ_3

$$\mu_1 - \mu_3 \in \bar{Y}_1 - \bar{Y}_3 \pm q_{\alpha}(a, f) \sqrt{\frac{MS_E}{n}}$$

$$\bar{Y}_1 - \bar{Y}_3 = 1306.3 - 1181.3 = 125$$

$$q_{0.05}(3,21) \sqrt{\frac{MS_E}{n}} = 3.565 \sqrt{72411 \left(\frac{1}{8}\right)} = 339.17$$

$$\mu_1 - \mu_3 \in 125 \pm 339.17 \Rightarrow \mu_1 - \mu_3 \in (-214.17, 464.17)$$

Grouping Information Using Fisher Method

approach	N	Mean	Grouping
2	8	1737.5	A
1	8	1306.3	B
3	8	1181.3	B

Means that do not share a letter are significantly different.

Fisher 95% Individual Confidence Intervals
All Pairwise Comparisons among Levels of approach

Simultaneous confidence level = 88.16%

approach = 1 subtracted from:

approach	Lower	Center	Upper	
2	151.4	431.3	711.1	(-----*-----)
3	-404.8	-125.0	154.8	(-----*-----)

-----+-----+-----+-----+-----
-500 0 500 1000

approach = 2 subtracted from:

approach	Lower	Center	Upper	
3	-836.1	-556.3	-276.4	(-----*-----)

-----+-----+-----+-----+-----
-500 0 500 1000

Manually: for example

If we want to compare μ_1 and μ_2 (LSD method)

$$H_0: \mu_1 = \mu_2 \text{ VS } H_0: \mu_1 \neq \mu_2$$

We reject $H_0: \mu_1 = \mu_2$ if $|\bar{Y}_1 - \bar{Y}_2| > LSD = t_{\frac{\alpha}{2}, N-a} \sqrt{MS_E \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$

$$\begin{aligned} LSD &= t_{\frac{\alpha}{2}, N-a} \sqrt{MS_E \left(\frac{1}{n_1} + \frac{1}{n_2} \right)} = t_{0.025, 21} \sqrt{72411 \left(\frac{2}{8} \right)} \\ &= 2.08 \sqrt{72411 \left(\frac{2}{8} \right)} = 279.86 \end{aligned}$$

$$|\bar{Y}_1 - \bar{Y}_2| = |1306.3 - 1737.5| = 431.2 > 279.86 = LSD$$

\Rightarrow We reject $H_0: \mu_1 = \mu_2$, that means there is a difference between them.

Construct a 95 percent confidence interval on the difference between μ_1 and μ_2

$$\mu_i - \mu_j \in \bar{Y}_i - \bar{Y}_j \pm t_{\frac{\alpha}{2}, N-a} \sqrt{MS_E \left(\frac{1}{n_i} + \frac{1}{n_j} \right)}$$

$$\mu_1 - \mu_2 \in \bar{Y}_1 - \bar{Y}_2 \pm t_{\frac{\alpha}{2}, N-a} \sqrt{MS_E \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$$

$$\bar{Y}_1 - \bar{Y}_2 = 1306.3 - 1737.5 = -431.2$$

$$t_{\frac{\alpha}{2}, N-a} \sqrt{MS_E \left(\frac{1}{n_1} + \frac{1}{n_2} \right)} = t_{0.025, 21} \sqrt{72411 \left(\frac{2}{8} \right)}$$

$$= 2.08 \sqrt{72411 \left(\frac{2}{8}\right)} = 279.86$$

$$\mu_1 - \mu_2 \in -431.2 \pm 279.86 \Rightarrow \mu_1 - \mu_2 \in (-711.06, -151.34)$$

If we want to compare μ_1 and μ_3 (LSD method)

$$H_0: \mu_1 = \mu_3 \text{ VS } H_0: \mu_1 \neq \mu_3$$

We reject $H_0: \mu_1 = \mu_3$ if $|\bar{Y}_{1.} - \bar{Y}_{3.}| > LSD = t_{\frac{\alpha}{2}, N-a} \sqrt{MS_E \left(\frac{1}{n_1} + \frac{1}{n_3}\right)}$

$$LSD = t_{\frac{\alpha}{2}, N-a} \sqrt{MS_E \left(\frac{1}{n_1} + \frac{1}{n_3}\right)} = t_{0.025, 21} \sqrt{72411 \left(\frac{2}{8}\right)} =$$

$$2.08 \sqrt{72411 \left(\frac{2}{8}\right)} = 279.86$$

$|\bar{Y}_{1.} - \bar{Y}_{3.}| = |1306.3 - 1181.3| = 125 < 279.86 = LSD \Rightarrow$ We cannot reject $H_0: \mu_1 = \mu_3$, that means there is no significant difference between them.

Construct a 95 percent confidence interval on the difference between μ_1 and μ_3

$$\mu_1 - \mu_3 \in \bar{Y}_{1.} - \bar{Y}_{3.} \pm t_{\frac{\alpha}{2}, N-a} \sqrt{MS_E \left(\frac{1}{n_1} + \frac{1}{n_3}\right)}$$

$$\bar{Y}_{1.} - \bar{Y}_{3.} = 1306.3 - 1181.3 = 125$$

$$t_{\frac{\alpha}{2}, N-a} \sqrt{MS_E \left(\frac{1}{n_1} + \frac{1}{n_3}\right)} = t_{0.025, 21} \sqrt{72411 \left(\frac{2}{8}\right)} = 2.08 \sqrt{72411 \left(\frac{2}{8}\right)} = 279.86$$

$$\mu_1 - \mu_3 \in 125 \pm 279.86 \Rightarrow \mu_1 - \mu_3 \in (-154.86, 404.86).$$