STAT 337

Design and Analysis of Experiments

Exercises

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Chapter 2

2.2. Computer output for a random sample of data is shown below. Some of the quantities are missing. Compute the values of the missing quantities.

Variable	N	Mean	SE Mean	Std. Dev.	Sum
Y	16	?	0.159	?	399.851

Mean=$\frac{\sum_{i=1}^{n} x_i}{n} = \frac{399.851}{16}$ =24.99

S.E=$\frac{SD}{\sqrt{n}}$ $\Rightarrow SD = S.E\ \sqrt{n}$ =0.591$\sqrt{16}$=2.364

2.3. Suppose that we are testing $H_0: \mu = \mu_0$ versus $H_1: \mu \neq \mu_0$. Calculate the P-value for the following observed values of the test statistic:

(a) $Z_0 = 2.25$

p-value=$2P(Z > |Z_0|) = 2P(Z > 2.25) = 2P(Z < -2.25) = 2(0.01222) = 0.02444$

2.5. Consider the computer output shown below.

One-Sample Z

```
Test of mu = 30 vs not = 30
The assumed standard deviation = 1.2
```

N	Mean	SE Mean	95% CI	Z	P
16	31.2000	0.3000	(30.6120, 31.7880)	?	?

(a) Fill in the missing values in the output. What conclusion would you draw?

$z = \frac{\bar{X}-30}{SE} = \frac{31.2-30}{0.3} = 3.466 \approx 3.47$

$p - value = 2P(Z > |Z_0|) = 2P(Z > 3.47) = 5.2 \times 10^{-4}$

Since $p - value = 5.2 \times 10^{-4} < 0.05 = \alpha$, we reject $H_0: \mu = 30$

(b) Is this a one-sided or two-sided test?

yes it's

(c) Use the output and the normal table to find a 99 percent CI on the mean.

$\mu \in \bar{X} \pm Z_{1-\frac{\alpha}{2}} S.E$

$\bar{X} = 31.2 \qquad Z_{1-\frac{\alpha}{2}} = Z_{1-\frac{0.01}{2}} = Z_{0.995} = \mathbf{2.58} \quad S.E = 0.3$

Then , $\mu \in 31.2 \pm 2.58\ (0.3)$

$\mu \in (30.43, 31.97)$

(d)What is the *P*-value if the alternative hypothesis is $H1 : \mu > 30$?

$p - value = P(Z > Z_0) = P(Z > 3.47) = P(Z < -3.47) = 0.000260229$

2.15. Consider the computer output shown below.

Two-Sample T-Test and CI: Y1, Y2

```
Two-sample T for Y1 vs Y2

       N    Mean    Std. Dev.   SE Mean
Y1    20   50.19      1.71       0.38
Y2    20   52.52      2.48       0.55

Difference = mu (X1) - mu (X2)
Estimate for difference: -2.33341
95% CI for difference: (-3.69547, -0.97135)
T-Test of difference = 0 (vs not =) : T-Value = -3.47
P-Value = 0.001 DF = 38
Both use Pooled Std. Dev. = 2.1277
```

$T = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{sp\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1+n_2-2}$

(a) Can the null hypothesis be rejected at the 0.05 level? Why?

since p-value=0.001< $0.05 = \alpha$, we reject $H_0: \mu_1 = \mu_2$

(b) Is this a one-sided or a two-sided test?

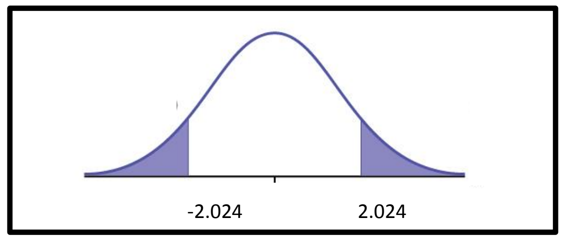
two-sided test

(c) If the hypotheses had been $\boldsymbol{H_0: \mu_1 - \mu_2 = 2}$ versus $\boldsymbol{H_0: \mu_1 - \mu_2 \neq 2}$ would you reject the null hypothesis

at the 0.05 level?

$H_0: \mu_1 - \mu_2 = 2 \quad \text{vs } H_1: \mu_1 - \mu_2 \neq 2$

test statistics: $\boldsymbol{T} = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{sp\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{-2.33341 - 2}{2.1277\sqrt{\frac{1}{20} + \frac{1}{20}}} = \boldsymbol{-6.44}$



The critical values:

$\alpha = 0.05 \rightarrow 1 - \frac{\alpha}{2} = 1 - 0.025 = 0.975$

$t_{0.975,38} = 2.024 \qquad t_{0.025,38} = -2.024$

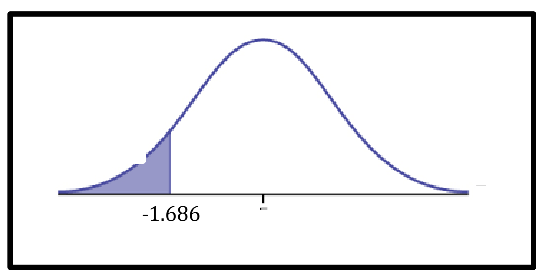
We reject $H_0: \mu_1 - \mu_2 = 2$, if $T > t_{0.975,38} = 2.024$ or $T < t_{0.025,38} = -2.024$

Since $T = -6.44 < -2.024$ we reject $H_0: \mu_1 - \mu_2 = 2$

(d) If the hypotheses had been $\boldsymbol{H_0: \mu_1 - \mu_2 = 2}$ versus $\boldsymbol{H_1: \mu_1 - \mu_2 < 2}$ would you reject the null hypothesis
at the 0.05 level? Can you answer this question without doing any additional calculations? Why?

$H_0: \mu_1 - \mu_2 = 2 \quad \text{vs} \quad H_1: \mu_1 - \mu_2 < 2$

From (c) we got $T = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{sp\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{-2.33341 - 2}{2.1277\sqrt{\frac{1}{20} + \frac{1}{20}}} = -6.44$



The critical value:

$\alpha = 0.05 \quad \rightarrow \quad t_{0.05,38} = -1.686$

We reject $H_0: \mu_1 - \mu_2 = 2$ if $T < -t_{0.95,38} = -1.686$

Since $T = -6.44 < -1.686$ we reject $H_0: \mu_1 - \mu_2 = 2$

(e) Use the output and the t table to find a 95 percent upper confidence bound on the difference in means.

$$\mu_1-\mu_2 \in \bar{X}_1-\bar{X}_2 \pm t_{1-\frac{\alpha}{2},n_1+n_2-2}\, sp\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$\mu_1-\mu_2 \in -2.33341 \pm 2.024\left(2.1277\sqrt{\frac{1}{20}+\frac{1}{20}}\right)$$

Upper bound $= -2.33341 + 2.024\left(2.1277\sqrt{\frac{1}{20}+\frac{1}{20}}\right)$=-0.972

2.20. The shelf life of a carbonated beverage is of interest. Ten bottles are randomly selected and tested, and the following results are obtained:

Days	
108	138
124	163
124	159
106	134
115	139

(a) We would like to demonstrate that the mean shelf life exceeds 120 days. Set up appropriate hypotheses for investigating this claim.

$H_0: \mu \le 120$ vs $H_1: \mu > 120$

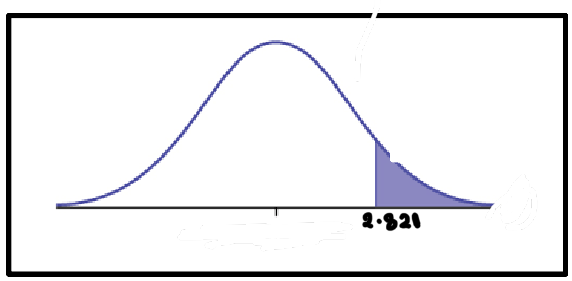
(b) Test these hypotheses using $\alpha = 0.01$. What are your conclusions?

test statistics: $T = \frac{\bar{X}-\mu_0}{s/\sqrt{n}} = \frac{131-120}{19.54/\sqrt{10}}$=1.78

The critical value $t_{1-\alpha,n-1} = t_{0.99,9} = 2.821$

We reject $H_0: \mu = 120$ if $T > t_{0.99,9} = 2.821$

Since $T = 1.78 < 2.821 = t_{0.99,9}$ we cannot reject $H_0: \mu = 120$



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Using Minitab to perform One-Sample T-test:

Days
108
124
124
106
115
138
163
159
134
139

- Choose Stat > Basic Statistic > 1t-1-Sample t.
- Check Sample in one column.
- In the Box, enter **Days** .
- Check Perform hypothesis
- Enter **120** in hypothesized mean
- Click **options.**
- Enter **99** in confidence level
- Chose **greater than** in alternative
- Click OK

One-Sample T: Days

Test of $\mu = 120$ vs > 120

Variable	N	Mean	StDev	SE Mean	99% Lower Bound	T	P
Days	10	131.00	19.54	6.18	113.56	1.78	0.054

2.26. The following are the burning times (in minutes) of chemical flares of two different formulations. The design engineers are interested in both the mean and variance of the burning times.

Type 1		Type 2	
65	82	64	56
81	67	71	69
57	59	83	74
66	75	59	82
82	70	65	79

(a) Test the hypothesis that the two variances are equal. Use $\alpha = 0.05$.

$H_0: \sigma_1{}^2 = \sigma_2{}^2$ vs $H_1: \sigma_1{}^2 \neq \sigma_2{}^2$

test statistics: $F_0 = \frac{S_1{}^2}{S_2{}^2} = \frac{85.82}{87.73} = \mathbf{0.98}$

The critical values: $\alpha = 0.05,\ n_1 = 10\ ,\ n_2 = 10$

$F_{\frac{\alpha}{2},n_1-1,n_2-1} = F_{0.025,9,9} = 4.03$

$F_{1-\frac{\alpha}{2},n_1-1,n_2-1} = F_{0.975,9,9} = \frac{1}{F_{0.025,9,9}} = \frac{1}{4.03} = 0.248$

We reject H_0 if $F_0 > F_{\frac{\alpha}{2},n_1-1,n_2-1} = 4.03$ or $F_0 < F_{1-\frac{\alpha}{2},n_1-1,n_2-1} = 0.248$

We cannot reject $H_0: \sigma_1{}^2 = \sigma_2{}^2$ that means there is no significant difference between $\sigma_1{}^2 and\ \sigma_2{}^2$

Using Minitab to perform the test of two variances:

type 1	type 2
65	64
81	71
57	83
66	59
82	65
82	56
67	69
59	74
75	82
70	79

- Choose Stat > Basic Statistic > 2 variances
- Choose each Sample is in own column.
- In Sample1 enter **Type 1**
- In Sample2 enter **Type 2.**
- Click **options.**
- Enter **95** in confidence level
- Chose ≠ in alternative
- Click OK

Test and CI for Two Variances: type 1; type 2

Method

Null hypothesis $\sigma(\text{type 1}) / \sigma(\text{type 2}) = 1$
Alternative hypothesis $\sigma(\text{type 1}) / \sigma(\text{type 2}) \neq 1$
Significance level **$\alpha = 0.05$**

Statistics

Variable	N	StDev	Variance	95% CI for StDevs
type 1	10	**9.264**	**85.822**	(6.920; 15.425)
type 2	10	**9.367**	**87.733**	(6.844; 15.945)

Ratio of standard deviations = 0.989
Ratio of variances = **0.978**

95% Confidence Intervals

Method	CI for StDev Ratio	CI for Variance Ratio
Bonett	(0.572; 1.758)	(0.328; 3.092)
Levene	(0.508; 1.933)	(0.258; 3.735)

Tests

Method	DF1	DF2	Test Statistic	P-Value
Bonett	1	—	0.00	0.963
Levene	1	18	0.00	1.000

(b) Using the results of (a), test the hypothesis that the mean burning times are equal. Use $\alpha = 0.05$. What is the *P*-value for this test?

From (a) we got that there is no significant difference between $\sigma_1{}^2 and\ \sigma_2{}^2$

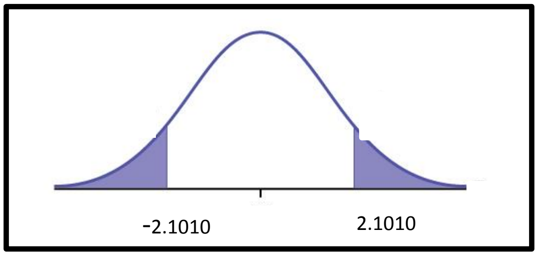
Then the variance unknown and equals

$H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$

test statistics: $t_0 = \frac{\bar{X}_1 - \bar{X}_2}{sp\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \sim t_{n_1+n_2-2}$

$$sp^2 = \frac{(n_1-1)S_1{}^2 + (n_2-1)S_2{}^2}{n_1+n_2-2} = \frac{(9)(9.264)^2+(9)(9.367)^2}{18} = 86.775$$

$$t_0 = \frac{\bar{X}_1 - \bar{X}_2}{sp\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{70.4-70.2}{9.32\sqrt{\frac{1}{10}+\frac{1}{10}}} = 0.048$$



The critical value: $\alpha = 0.05,\ n_1 = 10\ , n_2 = 10$

$t_{\frac{\alpha}{2}, n_1+n_2-2} = t_{0.025,18} = 2.101$

We reject H_0 if $t_0 > 2.101$ or $t_0 < -2.101$

We cannot reject $H_0: \mu_1 = \mu_2$ that means there is no significant difference between $\mu_1 and\ \mu_2$.

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Using Minitab to perform the test of two means :

- Choose Stat > Basic Statistic > **2 Sample t**
- Choose each Sample is in own column.
- In Sample1 enter **Type 1**
- In Sample2 enter **Type 2**.
- Check **assume equal variance**
- Click **options.**
- Enter **95** in confidence level
- Chose ≠ in alternative
- Click OK.

Two-Sample T-Test and CI: type 1; type 2

Two-sample T for type 1 vs type 2

	N	Mean	StDev	SE Mean
type 1	10	70.40	9.26	2.9
type 2	10	70.20	9.37	3.0

Difference = μ (type 1) - μ (type 2)
Estimate for difference: 0.20
95% CI for difference: (-8.55; 8.95)
T-Test of difference = 0 (vs ≠): T-Value = **0.05** P-Value = 0.962 **DF = 18**
Both use Pooled StDev = **9.3155**

2.30. Front housings for cell phones are manufactured in an injection molding process. The time the part is allowed to cool in the mold before removal is thought to influence the occurrence of a particularly troublesome cosmetic defect, flow lines, in the finished housing. After manufacturing, the housings are inspected visually and assigned a score between 1 and 10 based on their appearance, with 10 corresponding to a perfect part and 1 corresponding to a completely defective part. An experiment was conducted using two cool-down times, 10 and 20 seconds, and 20 housings were evaluated at each level of cool-down time. All 40 observations in this experiment were run in random order. The data are as follows.

10 seconds		20 seconds	
1	3	7	6
2	6	8	9
1	5	5	5
3	3	9	7
5	2	5	4
1	1	8	6
5	6	6	8
2	8	4	5
3	2	6	8
5	3	7	7

10 sec	20 sec
1	7
2	8
1	5
3	9
5	5
1	8
5	6
2	4
3	6
5	7
3	6

6	9
5	5
3	7
2	4
1	6
6	8
8	5
2	8
3	7

Using Minitab to perform the test of two means :

- Choose Stat > Basic Statistic > **2 Sample t**
- Choose each Sample is in own column.
- In Sample1 enter **10 sec**
- In Sample2 enter **20 sec**.
- Check **assume not equal variance**
- Click **options.**
- Enter **95** in confidence level
- Chose ≠ in alternative
- Click OK.

Two-Sample T-Test and CI: 10 sec; 20 sec

Two-sample T for 10 sec vs 20 sec

	N	Mean	StDev	SE Mean
10 sec	20	3.35	2.01	0.45
20 sec	20	6.50	1.54	0.34

Difference = μ (10 sec) - μ (20 sec)
Estimate for difference: -3.150
95% CI for difference: (-4.295; -2.005)
T-Test of difference = 0 (vs ≠): T-Value = -5.57 P-Value = 0.000 DF = 38
Both use Pooled StDev = 1.7885

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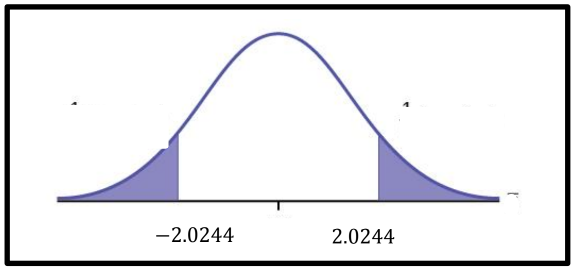
(a) Is there evidence to support the claim that the longer cool-down time results in fewer appearance defects? Use $\alpha = 0.05$

$H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$

test statistics: $t_0 = -5.57$



The critical value: $\alpha = 0.05,\ n_1 = 20\ , n_2 = 20$

$t_{\frac{\alpha}{2}, n_1+n_2-2} = t_{0.025,38} = 2.0244$

We reject H_0 if $t_0 > 2.0244$ or $t_0 < -2.0244$

We reject $H_0: \mu_1 = \mu_2$ that means there is a difference between $\mu_1 and\ \mu_2$, which means there is a difference between cool-down time results.

(b) What is the *P*-value for the test conducted in part (a)? P-Value = **0.000**

(c) Find a 95 percent confidence interval on the difference in means. Provide a practical interpretation of this interval.

95% CI for difference: **(-4.295; -2.005)**

2.34. An article in the *Journal of Strain Analysis* (vol. 18, no. 2, 1983) compares several procedures for predicting the shear strength for steel plate girders. Data for nine girders in the form of the ratio of predicted to observed load for two of these procedures, the Karlsruhe and Lehigh methods, are as follows:

Girder	Karlsruhe Method	Lehigh Method
S1/1	1.186	1.061
S2/1	1.151	0.992
S3/1	1.322	1.063
S4/1	1.339	1.062
S5/1	1.200	1.065
S2/1	1.402	1.178
S2/2	1.365	1.037
S2/3	1.537	1.086
S2/4	1.559	1.052

(a) Is there any evidence to support a claim that there is a difference in mean performance between the two methods? Use $\alpha = 0.05$.

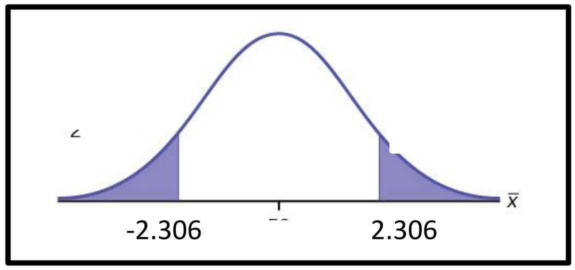
$\boldsymbol{H_0: \mu_1 = \mu_2}$ **vs** $\boldsymbol{H_1: \mu_1 \neq \mu_2}$ **or equivalently** $\boldsymbol{H_0: \mu_d = 0}$ **vs** $\boldsymbol{H_1: \mu_d \neq 0}$

test statistics: $t_0 = \dfrac{\bar{d}}{S_d\sqrt{\frac{1}{n}}}$

$\bar{d} = 0.274$

$$S_d = \left[\frac{\sum_{i=1}^{n} d_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} d_i\right)^2}{n-1}\right]^{\frac{1}{2}} = \left[\frac{0.821151 - \frac{1}{9}(2.465)^2}{9-1}\right]^{\frac{1}{2}} = 0.135$$

$$t_0 = \frac{\bar{d}}{S_d\sqrt{\frac{1}{n}}} = \frac{0.274}{\frac{0.135}{\sqrt{9}}} = 6.08$$



The critical value: $\alpha = 0.05,\ n = 10$

$t_{\frac{\alpha}{2}, n-1} = t_{0.025,9} = 2.306$

We reject H_0 if $t_0 > 2.306$ or $t_0 < -2.306$

We reject $H_0: \mu_d = 0$ that means there is a difference between $\mu_1 and\ \mu_2$. **which means there is a difference in mean performance between the two methods**

(c) Construct a 95 percent confidence interval for the difference in mean predicted to observed load

$$\mu_d \in \bar{d} \pm t_{\frac{\alpha}{2}, n-1} \frac{S_d}{\sqrt{n}}$$

$$\mu_d \in 0.274 \pm (2.306)\ \frac{0.135}{\sqrt{9}}$$

$$\mu_d \in (0.17023, 0.37777)$$

Using Minitab to perform the Paired test:

Karlsruhe method	Lehigh method
1.186	1.061
1.151	0.992
1.322	1.063
1.339	1.062

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1.2	1.065
1.402	1.178
1.365	1.037
1.537	1.086
1.559	1.052

- Choose Stat > Basic Statistic > **Paired t**
- Choose each Sample is in own column.
  In Sample1 enter **Karlsruhe method.**
- In Sample2 enter **Lehigh method.**
- Click **options.**
- Enter **95** in confidence level
- Enter **0** in Hypothesized difference
- Chose ≠ in alternative
- Click OK.

Paired T-Test and CI: Karlsruhe method; Lehigh method

Paired T for Karlsruhe method - Lehigh method

	N	Mean	StDev	SE Mean
Karlsruhe method	9	1.3401	0.1460	0.0487
Lehigh method	9	1.0662	0.0494	0.0165
Difference	9	0.2739	0.1351	0.0450

95% CI for mean difference: **(0.1700; 0.3777)**
T-Test of mean difference = 0 (vs ≠ 0): T-Value = **6.08** P-Value = 0.000

2.37. In semiconductor manufacturing wet chemical etching is often used to remove silicon from the backs of wafers prior to metalization. The etch rate is an important characteristic of this process. Two different etching solutions are being evaluated. Eight randomly selected wafers have been etched in each solution, and the observed etch rates (in mils/min) are as follows.

Solution 1		Solution 2	
9.9	10.6	10.2	10.6
9.4	10.3	10.0	10.2
10.0	9.3	10.7	10.4
10.3	9.8	10.5	10.3

Using Minitab to perform the test of two means :

solution1	solution 2
9.9	10.2
9.4	10
10	10.7
10.3	10.5
10.6	10.6
10.3	10.2
9.3	10.4
9.8	10.3

- Choose Stat > Basic Statistic > **2 Sample t**
- Choose each Sample is in own column.
- In Sample1 enter **Solution 1**
- In Sample2 enter **Solution 2.**
- Check **assume equal variance**
- Click **options.**
- Enter **95** in confidence level
- Chose ≠ in alternative
- Click OK.

Two-Sample T-Test and CI: solution1; solution 2

Two-sample T for solution1 vs solution 2

	N	Mean	StDev	SE Mean
solution1	8	9.950	0.450	0.16
solution 2	8	10.363	0.233	0.082

Difference = μ (solution1) - μ (solution 2)
Estimate for difference: -0.413
95% CI for difference: **(-0.797; -0.028)**
T-Test of difference = 0 (vs ≠): T-Value = **-2.30** P-Value **= 0.037** DF = 14
Both use Pooled StDev = 0.3584

(**a**) Do the data indicate that the claim that both solutions have the same mean etch rate is valid? Use $\boldsymbol{\alpha} = \mathbf{0.05}$ and assume equal variances.

$H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$

test statistics: $t_0 = \mathbf{-2.30}$

The critical value: $\alpha = 0.05,\ n_1 = 8\ , n_2 = 8$

$t_{\frac{\alpha}{2}, n_1+n_2-2} = t_{0.025,14}$ =2.14479

We reject H_0 if $t_0 > 2.14479$or $t_0 < -2.14479$

We reject $H_0: \mu_1 = \mu_2$ that means there is a difference between $\mu_1 and\ \mu_2$. which means that both solutions do not have the same mean etch rate is valid.

(**b**) Find a 95 percent confidence interval on the difference in mean etch rates.
95% CI for difference: **(-0.797; -0.028)**