Exercise

Q1: Put the following LP in the standard form:

1- Max $Z = X_1 - X_2 - 3X_3$

Subject to

$2X_1 + X_2 - X_3 \leq 2$

$X_1 - 3X_2 + 2X_3 \leq 3$

$X_1 + X_2 - X_3 \geq -2$ *(-1)

$X_1 \geq 0, X_2 \leq 0, X_3 \ URS$

The standard form

Replace X_2 with $(-X_2^{-})$, and Replace X_3 with $(X_3^{+} - X_3^{-})$

Max $Z = X_1 + X_2^{-} - 3X_3^{+} + 3X_3^{-}$

Subject to:

$2X_1 - X_2^{-} - X_3^{+} + X_3^{-} + S_1 = 2$

$X_1 + 3X_2^{-} + 2X_3^{+} - 2X_3^{-} + S_2 = 3$

$-X_1 + X_2^{-} + X_3^{+} - X_3^{-} + S_3 = 2$

$X_1, X_2^{-}, X_3^{+}, X_3^{-}, S_1, S_2, S_3 \geq 0$

2- Min $Z = 80X_1 + X_2$

Subject to
$0.2X_1 + 0.32X_2 \leq 0.25$
$X_1 + X_2 = 1$
$X_1 \geq 0, X_2 \geq 0$

Min $Z = 80X_1 + X_2$
Subject to
$0.2X_1 + 0.32X_2 + S_1 = 0.25$
$X_1 + X_2 = 1$

$$X_1, X_2, S_1 \geq 0$$

3- Min $Z = 3X_1 + 8X_2 + 4X_3$

Subject to

$$X_1 + X_2 \geq 8$$

$$2X_1 - 3X_2 \leq 0$$

$$X_2 \geq 9$$

$$X_1 \geq 0, X_2 \geq 0, X_3 \; URS$$

Replace X_3 with $(X_3^+ - X_3^-)$

The standard form

Min $Z = 3X_1 + 8X_2 + 4(X_3^+ - X_3^-)$

Subject to

$$X_1 + X_2 - S_1 = 8$$

$$2X_1 - 3X_2 + S_2 = 0$$

$$X_2 - S_3 = 9$$

$$X_1, X_2, X_3^+, X_3^-, S_1, S_2, S_3 \geq 0$$

Q2: For the LP, answer the following questions?

Max $Z = 5X_1 + 4X_2$

Subject to

$$6X_1 + 4X_2 \leq 24$$

$$X_1 + 2X_2 \leq 6$$

$$X_1 \geq 0, X_2 \geq 0$$

   a) Express the problem in equation form.
   b) Determine the all basic solutions and classify them as feasible and infeasible.
   c) Use direct substitution in the objective function to determin the optimum basic feasible solution.
   d) Verify graphically that the solution obtained in (c) is the optimum LP solution.

<u>The standard form</u>
Max $Z = 5X_1 + 4X_2$
Subject to
$6X_1 + 4X_2 + S_1 = 24$
$X_1 + 2X_2 + S_2 = 6$
$X_1 \geq 0, X_2 \geq 0, S_1 \geq 0, S_2 \geq 0$

Note:
(m) basic variables; (n−m) non-basic variables
To find total number of basic solutions by use $\binom{n}{m} = nC_m$ "combinations"
All basic solutions are not necessarily feasible.

We have **m=2** constraints and **n=4** variables, thus **n-m=2** Non-basic variables (zero variables).
Total number of Basic solutions are $\binom{4}{2} = 6$

Non-basic Variables	Basic Variables & Basic Solution	Feasibility Status	Extreme point	Objective Value
S_1, S_2	$X_1 = 3, X_2 = 1.5$	Feasible	D	21
S_2, X_2	$X_1 = 6, S_1 = -12$	Infeasible		
S_1, X_2	$X_1 = 4, S_2 = 2$	Feasible	A	20
S_2, X_1	$X_2 = 3, S_1 = 12$	Feasible	B	12
S_1, X_1	$X_2 = 6, S_2 = -6$	Infeasible		
X_1, X_2	$S_1 = 24, S_2 = 6$	Feasible	C	0
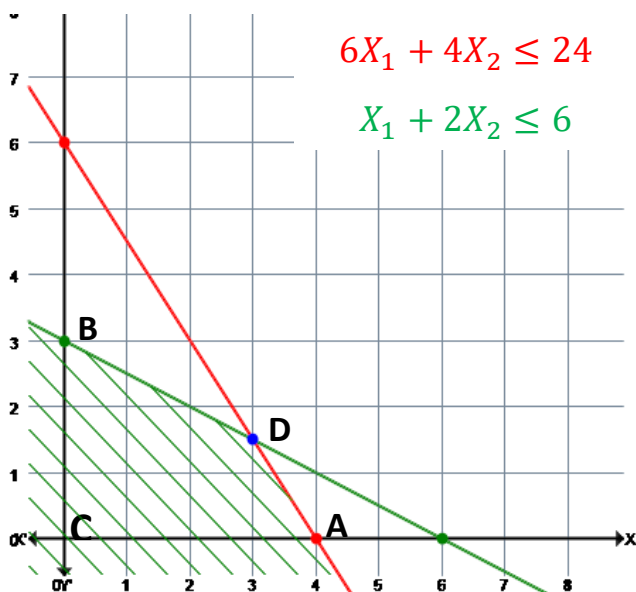
$6X_1 + 4X_2 \leq 24$

$X_1 + 2X_2 \leq 6$

1 $S_1 = S_2 = 0$

$$6X_1 + 4X_2 = 24$$
$$-6 * \quad X_1 + 2X_2 = 6$$
$$\overline{\qquad\qquad\qquad}$$
$$(4-12)X_2 = 24 - 36$$
$$-8X_2 = -12$$
$$\boxed{X_2 = 1.5}$$

$$X_1 + 2(1.5) = 6$$
$$X_1 = 6 - 3 = 3$$

$$\boxed{X_1 = 3}$$

2 $S_2 = X_2 = 0$

$$X_1 + 2X_2 + S_2 = 6$$
$$\boxed{X_1 = 6}$$
$$6X_1 + 4X_2 + S_1 = 24$$
$$6(6) + 4(0) + S_1 = 24$$
$$S_1 = 24 - 36$$
$$\boxed{S_1 = -12}$$

3 $S_1 = X_2 = 0$

$$6X_1 + 4X_2 + S_1 = 24$$
$$6X_1 = 24$$
$$\boxed{X_1 = 4}$$
$$X_1 + 2X_2 + S_2 = 6$$
$$4 + 2(0) + S_2 = 6$$
$$\boxed{S_2 = 2}$$

4 $S_2 = X_1 = 0$

$$X_1 + 2X_2 + S_2 = 6$$
$$2X_2 = 6$$
$$\boxed{X_2 = 3}$$

$$6X_1 + 4X_2 + S_1 = 24$$
$$4(3) + S_1 = 24$$
$$S_1 = 24 - 12$$
$$\boxed{S_1 = 12}$$

5 $S_1 = X_1 = 0$

$$6X_1 + 4X_2 + S_1 = 24$$
$$4X_2 = 24$$
$$\boxed{X_2 = 6}$$

$$X_1 + 2X_2 + S_2 = 6$$
$$2(6) + S_2 = 6$$
$$S_2 = 6 - 12$$
$$\boxed{S_2 = -6}$$

6 $X_1 = X_2 = 0$

$$6X_1 + 4X_2 + S_1 = 24$$
$$\boxed{S_1 = 24}$$
$$X_1 + 2X_2 + S_2 = 6$$
$$\boxed{S_2 = 6}$$

1- Max $Z = 3X_1 + 2X_2$ **H.W**

Subject to

$2X_1 + 4X_2 \leq 8$

$X_1 + X_2 \leq 2$

$X_1 \geq 0, X_2 \geq 0$

a) Express the problem in equation form.

b) Determine the all basic solutions and classify them as feasible and infeasible.

c) Use direct substitution in the objective function to determin the optimum basic feasible solution.

d) Verify graphically that the solution obtained in (c) is the optimum LP solution.