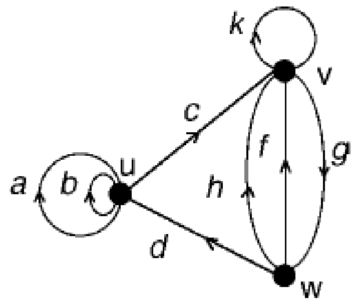
Exercises 3 – chapter 1+2

chapter 1:

Directed graphs (or digraphs) :

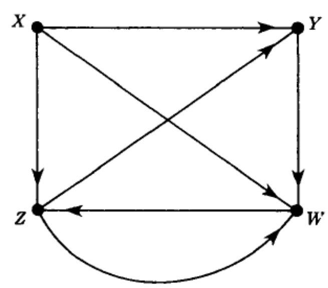
Q1: Find the **indegrees** and **outdegrees** of a digraph .



Solution:

Vertex	**u**	**v**	**w**
Indegree	3	4	1
Outdegree	3	2	3

Q2 : Let G be the directed graph:



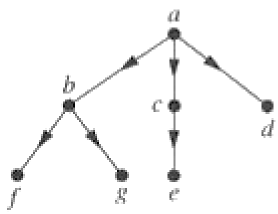
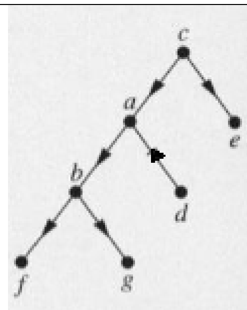
Find all simple paths from **X** to **Z**.

Solution:

There are three simple paths from X to Z, which are :

(X→ Z), (X→ W→ Z), and (X→ Y→ W→Z).

Q3: Which of these graphs are rooted tree

Graphs	Type
	☑ Rooted tree (with root a) ☐Not Rooted tree
	☐ Rooted tree ☑Not Rooted tree

Graph isomorphism:

Ex. 1.27: show whether the following two graphs **G** and **H** are isomorphic.

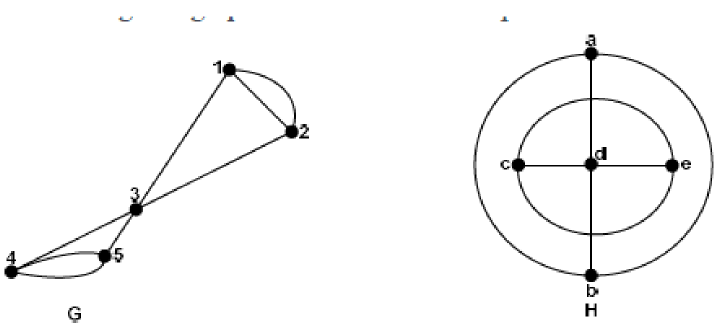


Figure 1.77: Two isomorphic graphs.

Solution:

The adjacent matrices for the two graphs are,

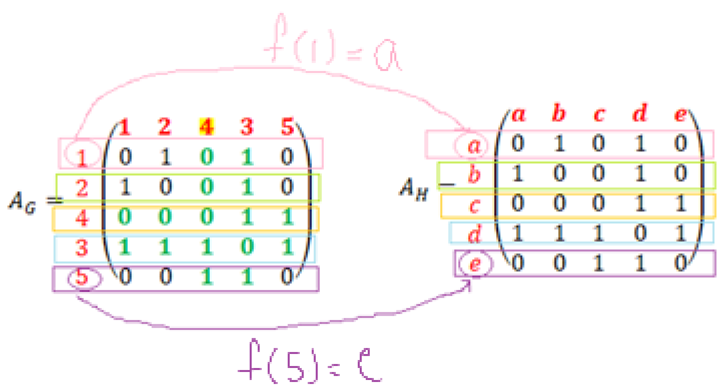
$$A_G = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix} \quad , \quad A_H = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{pmatrix} \boldsymbol{a} & \boldsymbol{b} & \boldsymbol{c} & \boldsymbol{d} & \boldsymbol{e} \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

We observe that <u>transposing rows</u> **3** and **4** and also <u>transposing columns</u> **3** and **4** transforms the matrix $\boldsymbol{A_G}$ into matrix $\boldsymbol{A_H}$.

$$A_G = \begin{array}{c} \\ 1 \\ 2 \\ 4 \\ 3 \\ 5 \end{array} \begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{1} \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$A_G = \begin{array}{c} \\ 1 \\ 2 \\ 4 \\ 3 \\ 5 \end{array} \begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{4} & \mathbf{3} & \mathbf{5} \\ 0 & 1 & \mathbf{0} & \mathbf{1} & 0 \\ 1 & 0 & \mathbf{0} & \mathbf{1} & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ 0 & 0 & \mathbf{1} & \mathbf{1} & 0 \end{pmatrix}$$

So the bijection make becomes as follows.



$$f(1) = a \ , f(2) = b \, , f(3) = d \, , f(4) = c \ \ and \ f(5) = e$$

Hence the two graphs are isomorphic.

* The same bijection map can be obtained by <u>transposing rows</u> **c** and **d** and also <u>transposing columns</u> **c** and **d** transforms the matrix $\boldsymbol{A_H}$ into matrix $\boldsymbol{A_G}$.

However the drawings of the above two graphs look quite different but they are isomorphic.

chapter 2 :

Shortest path algorithms

Ex. 2.5: Find the shortest path from vertex **u** to vertex **w** for the following network **using pruning algorithm**. And draw the rooted tree from the vertex **u**.

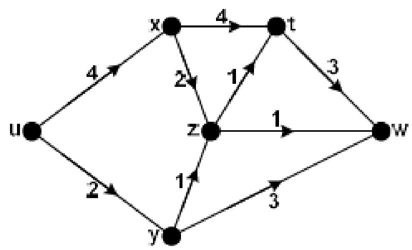


Figure 2.13: A weighted directed cycle-free graph.

Solution: using pruning algorithm:

From u: let d(u) = 0, p(u) = u, [d(v)=∞ and p(v)= Φ ; v≠u]
d(x) = d(u) + 4 = 0+4= 4 , p(x) = ux,
d(y) = d(u) + 2= 0+2= 2 , p(y) = uy,

From x: let d(x) = 4, p(x)= ux
d(t) = d(x) + 4 = 4+4 = 8 , p(t) = p(x) t = uxt
d(z) = d(x) + 2 = 4+2 = 6 , p(z) = p(x) z = uxz

From y: let d(y) = 2, p(y)= uy
d(z) = d(y) + 1 = 2 +1 = 3 , p(z) = p(y) t = uyz
d(w) = d(y) + 3 = 2 + 3 = 5 , p(w) = p(y) z = uyw

From z: let d(z) = 3, p(z)= uyz (note: uyz is shortest bath then uxz)
d(t) = d(z) + 1 = 3+1 = 4 , p(t) = p(z) t = uyzt
d(w) = d(z) + 1 = 3 +1 = 4 , p(w) = p(z) w = uyzw

From t: let d(t) = 4, p(t) = uyzt (note: uyzt is shortest bath the uxt)
d(w) = d(t) + 3 = 4 + 3 =7 , p(w) = p(t) w = uyztw

p(w) to read:

d(w) = 4 and p(w) = **uyzw**.

Since d(w) has been determined, then the algorithm is finished and thus the shortest path from u to w is p(w) = **uyzw** and d(w) = 4.

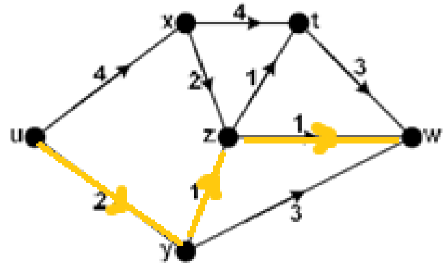


Figure 2.2: The shortest path from u to w for the graph 2.13:

Draw the rooted tree :

One can solve the above example by drawing the rooted tree from u as follows.