<u>Exercises #9</u>

<u>Question 1:</u>

We wish to estimate the average number of heart beats per minute for a certain population**. The average number of heart beats per minute for a sample of 49 subjects was found to be 90**. Assume that these 49 patients constitute a random sample, and that the **population is normally distributed with a standard deviation of 10.**

Given, n=49 $\bar{X} = 90$, $\sigma = 10$ (population)

Normal , σ known

(a) The point estimate of the mean (μ) is

$$\bar{X} = 90$$

(b) The standard error of sample mean $\bar{X}$ is

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{49}} = 1.4286$$

(c) The reliability coefficient is (at 95% C.I)

$$\alpha = 0.05 \ then \ 1 - \frac{\alpha}{2} = 0.975$$

$$Z_{1-\frac{\alpha}{2}} = Z_{.975} = 1.96$$

(D) Construct the 95% confidence interval for the mean (μ) of the heart beats per minute?

$$\bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

$$90 \pm Z_{0.975} \frac{10}{\sqrt{49}}$$

$$90 \pm (1.96) \frac{10}{\sqrt{49}} = (87.2, 92.8)$$

Question 2:

The concern of a study by beynnon et al. (A-4) were nine subjects with chronic anterior cruciate ligament (ACL) tears. One of the variables of interest was the laxity of the anteroposterior, where higher values indicate more knee instability. The researchers found that among subjects with ACL-deficient knees, the **mean laxity value was 17.4 mm with a standard deviation of 4.3 mm**. Assuming **normally distributed:**

Given, n=9 $\bar{X} = 17.4$, $S = 4.3$ **(sample)**

Normal , n small , σ unknown

(a) The point estimate of the population mean is

$$\bar{X} = 17.4$$

(b) The reliability coefficient is (at 90% C.I)

$\alpha = 0.1$ *then* $1 - \frac{\alpha}{2} = 0.95$ **df=n-1=9-1=8**

$$t_{1-\frac{\alpha}{2}} = t_{0.95} = 1.860$$

(C) Construct the 90 percent confidence interval for the mean of the population from which the 9 subjects may be presumed to be random sample?

$$\bar{X} \pm t_{1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

$$17.4 \pm t_{0.95} \frac{4.3}{\sqrt{9}}$$

$$17.4 \pm (1.860) \frac{4.3}{\sqrt{9}} = (14.734, 20.066)$$

The **upper limit** is:

$$\bar{X} + t_{1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}} = 17.4 + t_{0.95} \frac{4.3}{\sqrt{9}} = 17.4 + (1.860) \frac{4.3}{\sqrt{9}} = 20.066$$

(d) What is the precision of the estimate (margin of error) ?

$$\text{precision of the estimate} = t_{1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}} = t_{0.95} \frac{4.3}{\sqrt{9}} = (1.860) \frac{4.3}{\sqrt{9}} = 2.666$$

(e) What assumptions are necessary for the validity of the confidence interval you constructed?

Assumption : Normal , n small , σ unknown

H.W. 1:

We wish to estimate the mean serum indirect bilirubin level of 4-day-old infants. The mean for a sample of 16 infants was found to be 5.98 mg/100 cc. assume that bilirubin levels in 4-day-old infants are approximately normally distributed with a standard deviation of 3.5 mg/100 cc.

(a) What is the point estimate of the mean (μ)? (Answer = 5.98)

(b) The reliability coefficient is (at 98% C.I) (Answer = $\boldsymbol{Z_{0.99} = 2.325}$)

(C) Construct the 98% confidence interval for the mean (μ) of the serum indirect bilirubin level?

(1) The value of α is …**0.02**……

(2) Lower limit is

$$(Answer: \bar{X} - \boldsymbol{Z_{0.975}} \frac{\boldsymbol{\sigma}}{\sqrt{\boldsymbol{n}}} = 5.98 - \boldsymbol{2.325} \frac{\boldsymbol{3.5}}{\sqrt{\boldsymbol{16}}} = 5.98 - \boldsymbol{2.0344} = \boldsymbol{3.9456})$$

H.W. 2:

A sample of 16 ten-year-old girls had a mean weight of 71.5 and a standard deviation of 12 pounds, respectively. Assuming normally:

(a) What is the point estimate of the mean? (Answer = 71.5)

(b) Construct the 99 percent confidence interval for the mean of the population?

$$Answer: \bar{X} \pm \boldsymbol{t_{0.995}} \frac{\boldsymbol{S}}{\sqrt{\boldsymbol{n}}} = 71.5 \pm \boldsymbol{2.947} \frac{\boldsymbol{12}}{\sqrt{\boldsymbol{16}}} = 71.5 \pm \boldsymbol{8.841}$$
$$= (\boldsymbol{62.659}, \boldsymbol{80.341})$$

(c) What is the precision of the estimate? (Answer = $\boldsymbol{8.841}$)

(d) What assumptions are necessary for the validity of the confidence interval you constructed?

(Answer: normal , n small , σ unknown)

Question 3:

Chan et al. (A-9) developed a questionnaire to assess knowledge of prostate cancer. There was a total of 36 questions to which respondents could answer "agree", "disagree", or "don't know". Scores could range from 0 to 36. The number of Caucasian study participants was 185, and the number of African-American was 86. The mean scores for Caucasian study participants was 20.6, while the mean scores for African-American men was 17.4. **The population standard deviation for Caucasian study participants and African-American men equal of 5.8.**

Given $\boldsymbol{\sigma_1, \sigma_2}$ are known , $\boldsymbol{n_1 = 185 \geq 30}$; $\boldsymbol{n_2 = 86 \geq 30}$

(a) What is the point estimate of $(\mu_{Caucasian} - \mu_{African\_American})$?

$\bar{x}_{Caucasian} - \bar{x}_{African\_American} = \bar{x}_1 - \bar{x}_2 = 20.6 - 17.4 = 3.2$

(b) Construct the 99 percent confidence interval for the difference between the population mean scores for Caucasian study participants and the population mean scores for African-American men?

(1) The value of α is…. 0.01

(2) The reliability coefficient is

$$\boldsymbol{Z_{1-\frac{\alpha}{2}} = Z_{1-\frac{0.01}{2}} = Z_{0.995} = \frac{(2.57 + 2.58)}{2} = 2.575}$$

(3) The precision of the estimate is

$$\boldsymbol{Z_{1-\frac{\alpha}{2}}}\sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}} = \boldsymbol{Z_{0.995}}\sqrt{\frac{5.8^2}{185} + \frac{5.8^2}{86}} = (2.575)\sqrt{\frac{5.8^2}{185} + \frac{5.8^2}{86}} = 1.9492$$

(4) The upper limit is …….

$$\boldsymbol{\bar{x}_1 - \bar{x}_2 + Z_{1-\frac{\alpha}{2}}}\sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}}$$

$$(3.2) + \boldsymbol{Z_{0.995}}\sqrt{\frac{5.8^2}{185} + \frac{5.8^2}{86}} = (3.2) + (2.575)\sqrt{\frac{5.8^2}{185} + \frac{5.8^2}{86}} = 5.1492$$

(c) What assumptions are necessary for the validity of the confidence interval you constructed?

Non- normal and n_1, n_2 are large

σ_1, σ_2 known

Question 4:

Transverse diameter measurements on the hearts of adult males and females gave the following results:

Group	Sample size	$\bar{x}(cm)$	$s\ (cm)$
1 Males	12	13.21	1.05
2 Females	9	11.00	1.01

Assume normally distributed populations with equal variances.

(a) What is the point estimate of $(\mu_{Males} - \mu_{Females})$?

$\bar{x}_1 - \bar{x}_2 = 13.21 - 11 = 2.21$

(b) What is the value of S_P^2 ?

$$S_P{}^2 = \frac{(n_1 - 1)S_1{}^2 + (n_2 - 1)S_2{}^2}{n_1 + n_2 - 2} = \frac{(12-1)(1.05^2) + (9-1)(1.01^2)}{12+9-2} = 1.0678$$

(c) the value of degree of freedom is …

df= $n_1 + n_2 - 2 = 12 + 9 - 2 = 19$

(d) Construct the 90 percent confidence interval for the difference between the population mean of diameter the hearts for males and the population mean of diameter the hearts for females?

(1) The reliability coefficient is $\boldsymbol{\alpha = 0.1}$

$$\boldsymbol{t_{1-\frac{\alpha}{2}} = t_{1-\frac{0.1}{2}} = t_{0.95} = 1.729}$$

(2) What is the precision of the estimate?

$$\boldsymbol{t_{1-\frac{\alpha}{2}}} \sqrt{\frac{S_p{}^2}{n_1} + \frac{S_p{}^2}{n_2}} = \textbf{(1.729)} \sqrt{\frac{1.0678}{12} + \frac{1.0678}{9}} = \textbf{0.78784}$$

(3) The lower limit is

$$(\bar{x}_1 - \bar{x}_2) - t_{1-\frac{\alpha}{2}} \sqrt{\frac{S_p{}^2}{n_1} + \frac{S_p{}^2}{n_2}}$$

$$= (2.21) - (1.729) \sqrt{\frac{1.0679}{12} + \frac{1.0679}{9}}$$

$$= \boldsymbol{1.4221}$$

H.W. 3:

Twenty-four experimental animals with vitamin D deficiency were divided equally into two groups. Group 1 received treatment consisting of a diet provided vitamin D. the second group was not treated. At the end of the experimental period, serum calcium determinations were made with the following results:

	$\bar{x}(mg/100\ ml)$	$s\ (mg/100\ ml)$
Treated group	11.1	1.5
Untreated group	7.8	2.0

Assume normally distributed populations with equal variances.

(a) What is the point estimate of ($\mu_{Treated} - \mu_{Untreated}$)?

(Answer = **3.3**)

(b) What is the precision of the point estimate (at 90% C.I)

? (Answer = 1.239)

Df = (12+12)-2 =22 ; $t_{0.95} = 1.717$

$$t_{0.95} \times S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 1.717 \times \sqrt{3.125} \times \sqrt{\frac{1}{12} + \frac{1}{12}} = 1.239$$

(c) What is the value of S^2?(Answer = 3.125)

$$S_P^2 = \frac{11x1.5^2 + 11x2^2}{12 + 9 - 2} = 3.125$$

(d) Construct the 90 percent confidence interval for the difference between the population mean of vitamin D deficiency for males and the population mean of vitamin D deficiency for females?

(Answer = (2.061, 4.539)

$$\bar{X}_1 - \bar{X}_2 \mp t_{0.95} \times S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 3.3 \mp 1.239 = (2.061, 4.539)$$

Question 5:

In a study by von zur Muhlenet al. (A-16), 136 subjects with syncope or near syncope were studied. Syncope is the temporary loss of consciousness due to a sudden decline in blood flow to the brain. Of these subjects, 75 also reported having cardiovascular disease.

(a) What is the point estimate of p ?

$$\hat{P} = \frac{75}{136} = 0.55$$

(b) What is the estimated standard error of the sample proportions?

$$\sqrt{\frac{\hat{P}\hat{q}}{n}} = \sqrt{\frac{0.55(0.45)}{136}} = 0.0427$$

(c) Construct a 99 percent confidence interval for the population proportion of subjects with syncope or near syncope who also have cardiovascular disease?

$$\boldsymbol{\alpha = 0.01 \text{ then } Z_{1-\frac{\alpha}{2}} = Z_{1-\frac{0.01}{2}} = Z_{0.995} = 2.575}$$

$$\hat{P} \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{P}\hat{q}}{n}}$$

$$0.55 \pm (2.575) \sqrt{\frac{0.55(0.45)}{136}} = (\, 0.4402,\ 0.6598)$$

H.W. 4:

In a simple random sample of 125 unemployed male high-school dropouts between the ages of 16 and 21, inclusive, 88 stated that they were regular consumers of alcoholic beverages.

(a) What is the point estimate of p ? (Answer = 0.704)

$$\boldsymbol{\hat{P} = 88/125 = 0.704 \quad , \quad \hat{q} = 1 - \hat{P} = 0.296}$$

(b)Construct a 95 percent confidence interval for the population proportion?

$$\sqrt{\frac{\hat{P}\hat{q}}{n}} = \sqrt{\frac{0.704x0.296}{125}} = \mathbf{0.0408}$$

$$Z_{0.975} = 1.96$$

$$\hat{P} \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{P}\hat{q}}{n}} = 0.704 \pm \mathbf{1.96 \times 0.0408}$$

$$\mathbf{(\, 0.6240\, ,\ 0.7839)}$$

Question 6:

Horwitz et al. (A-18) studied 637 persons who were identified by court records from 1967 to 1971 as having experienced abuse or neglect. For a control group, they located 510 subjects who as children attended the same elementary school and lived within a five-block radius of those in the abused/neglected group. In the abused/neglected group, and control group, 114 and 57 subjects, respectively, had developed antisocial personality disorders over their lifetimes.

(a) What is the value of $\hat{p}_{\text{abused/neglected}}$?

$$\hat{p}_{abused/neglected} = \hat{p}_1 = \frac{114}{637} = 0.179$$

(b) What is the value of $\hat{p}_{\text{control}}$?

$$\hat{p}_{control} = \hat{p}_2 = \frac{57}{510} = 0.112$$

(c) What the point estimate of $p_{\text{abused/neglected}} - p_{\text{control}}$?

$$\hat{p}_1 - \hat{p}_2 = 0.179 - 0.112 = 0.067$$

(d) What is the estimated standard error of the difference between sample proportions $\hat{p}_{\text{abused/neglected}} - \hat{p}_{\text{control}}$?

$$\text{S.E} = \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} = \sqrt{\frac{(0.179)(0.821)}{637} + \frac{(0.112)(0.888)}{510}} = 0.0206$$

(e) Construct a 95 percent confidence interval (C.I.) for the difference between the preparations of subjects developed antisocial personaliy disorders one might expect to find in the populations of subjects from which the subjects of this study may be presumed to have been drawn ($p_{\text{abused/neglected}} - p_{\text{control}}$)?

$$\boldsymbol{\alpha = 0.05 \ \text{ then } \ Z_{1-\frac{\alpha}{2}} = Z_{1-\frac{0.05}{2}} = Z_{0.975} = 1.96}$$

$$\hat{p}_1 - \hat{p}_2 \pm \boldsymbol{Z}_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

$$= 0.067 \pm (1.96)\sqrt{\frac{(0.179)(0.821)}{637} + \frac{(0.112)(0.888)}{510}}$$

$$p_1 - p_2 \in (0.02656, 0.10744)$$

H.W. 5:

In a study on patients with cancer, we have the following data:

Group	n	Smoking	Not smoking
Men	50	41	9
Women	40	22	18

(a) The point estimate of the difference between smoking proportions of men and women $p_{\text{Men}} - p_{\text{Women}}$? (Answer = 41/50 – 22/40= 0.82 - 0.55 = 0.27)

(b) What is the estimated standard error of the difference between sample proportions $\hat{p}_{\text{Men}} - \hat{p}_{\text{Women}}$? (Answer = 0.0956)

$$\hat{p}_1 = \frac{41}{50} = 0.82 \quad , \hat{q}_1 = 1 - 0.82 = 0.18$$

$$\hat{p}_2 = \frac{22}{40} = 0.55 \; ; \; \hat{q}_1 = 1 - 0.55 = 0.45$$

$$\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} = \sqrt{\frac{(0.82)(0.18)}{50} + \frac{(0.55)(0.45)}{40}} = 0.0956$$

(c) What is 99% C.I. for $p_{Men} - p_{Women}$?

1) The reliability coefficient is

(Answer = $Z_{0.995} = 2.575$)

2) What is the precision of the estimate?
(Answer = 0.24617)

3) The upper limit is
(Answer = 0.51617)