Exercises 8

The criteria of Decision-Making Under Uncertainty is summarized as follows:

(i) To find Optimism (Maximax) criterion.

1. Locate the maximum payoff values corresponding to each decision alternative.
2. Select a decision alternative with best payoff value (maximum for profit).

(ii) Pessimism (Maximin) criterion

1. Locate the minimum payoff value corresponding to each decision alternative.
2. Select a decision alternative with the best payoff value (maximum for profit).

(iii) Equal probabilities (Laplace) criterion.

1. Assign equal probability value to each state of nature by using the formula: *1 ÷ (number of states of nature).*
2. Compute the expected (or average) payoff for each alternative by adding all the payoffs and dividing by the number of possible states of nature.
3. Select the best expected payoff value (maximum for profit).

(iv) Hurwiez criterion

1. Decide the coefficient of optimism a (alpha) and then coefficient of pessimism (1 – α).
2. For each decision alternative select the largest and lowest payoff value and multiply these with α and (1 – α) values, respectively. Then calculate the weighted average,

D= α (Maximum in column) + (1 – α)(Minimum in column)

3. Select an alternative with best weighted average payoff value.

(v) Minmax (savage or Regret) criterion

1. from the given payoff matrix, develop an opportunity-loss (or regret) matrix as follows:

- Find the best payoff corresponding to each state of nature
- Subtract all other payoff values in that row from this value.

2. For each decision alternative identify the worst (maximum) payoff value. Record this value in the new row.
3. Select a decision alternative resulting in a smallest anticipated opportunity-loss value.

The criteria of DECISION-MAKING UNDER RISK is summarized as follows:

(i) Expected Monetary Value (EMV)

1. Construct a payoff matrix listing all possible courses of action and states of nature.
2. Calculate the EMV for each course of action by multiplying the conditional payoffs by the associated probabilities and adding these weighted values for each course of action
$EMV = \sum_{i=1}^{m} r_{ij}\, p_i$
3. Select the course of action that yields the optimal EMV.

(ii) Expected Opportunity Loss (EOL)

1. Prepare a conditional payoff values matrix for each combination of course of action and state of nature along with the associated probabilities.
2. For each state of nature calculate the conditional opportunity loss (COL) values by subtracting each payoff from the maximum payoff.
3. Calculate the EOL for each course of action by multiplying the probability of each state of nature with the COL value and then adding the values.

$EOL = \sum_{i=1}^{m} L_{ij}\, p_i$, L_{ij}: opportunity loss due to state of nature i and course of action j

4. Select a course of action for which the EOL is minimum.

(iii) Maximum Likelihood (ML):

(iv) Expected Value of Perfect Information (EVPI)

EVPI = (Expected profit with perfect information)– (Expected profit without perfect information)
$EVPI = \sum_{i=1}^{m} MAX(r_{ij}) p_i - EMV$

Example: An agricultural company wants to decide which commodity should stock to get maximum profit. It was supplied with the following information. The probability that the monsoon will be excess, normal and deficient is 0.40,0.30 and 0.30. The estimated profit or loss three commodities in respect of these different kinds of monsoon are:

Profit per 1 ton			
Monsoon	Excess	Normal	Deficient
Rice	10,000	-4,000	15,000
Wheat	4,000	-3,000	8,000
Maize	4,000	1,000	-1,000

Determine the optimal decision under each of the following decision criteria and show how you arrived at it:

(a) Maximax, (b) Maximin, (c) Minimax regret (savage criterion), (d) Equal likelihood (Laplace), (e) Hurwicz Alpha criterion α=0.8,

(f) EMV, (g) EOL, (h) ML, EVPI

- Decision making under uncertainty.

1. Maximax criterion (OPTIMISM)

Alternative / Monsoon	Rice	Wheat	Maize
Excess	10,000	4,000	4,000
Normal	-4,000	-3,000	1,000
Deficient	15,000	8,000	-1,000
MAX	**15,000**	**8,000**	**4,000**

The maximum of column maxima is 15,000. Hence, the company should adopt Rice commodity.

2. Maximin criterion (pessimism).

Alternative / Monsoon	Rice	Wheat	Maize
Excess	10,000	4,000	4,000
Normal	-4,000	-3,000	1,000
Deficient	15,000	8,000	-1,000
Min	**-4,000**	**-3,000**	**-1,000**

The maximum of column minima is -1,000. Hence, the company should adopt Maize commodity.

3. Minimax (savage) criterion

Alternative / Monsoon	Rice	Wheat	Maize
Excess	**10,000**	4,000	4,000
Normal	-4,000	-3,000	1,000
Deficient	15,000	8,000	-1,000

Note: From the given payoff matrix, develop an opportunity-loss (or regret) matrix as follows:
(i) Find the best payoff corresponding to each state of nature
(ii) Subtract all other payoff values in that row from this value.

Opportunity Loss Table

Alternative / Monsoon	Rice	Wheat	Maize
Excess	10,000-10,000= 0	10,000-4,000= 6,000	10,000-4,000= 6,000
Normal	1,000-(-4,000)= 5,000	1,000-(-3,000) = 4,000	1,000-1,000= 0
Deficient	15,000-15,000= 0	15,000-8,000 = 7,000	15,000-(-1,000)= 16,000
MAX	**5,000**	**7,000**	**16,000**

Decision: Rice

4. Equal likelihood (Laplace).

Alternative / Monsoon	Rice	Wheat	Maize
Excess	10,000	4,000	4,000
Normal	-4,000	-3,000	1,000
Deficient	15,000	8,000	-1,000
Expected (average)	$\frac{1}{3}(10,000-4,000+15,000) = 7000$	$\frac{1}{3}(4,000-3,000+8,000) = 3000$	$\frac{1}{3}(4,000-1,000+1,000) = 1333.33$

Decision: Rice

5. Hurwicz Alpha criterion α=0.8.

Alternative / Monsoon	Rice	Wheat	Maize
Excess	10,000	4,000	4,000
Normal	-4,000	-3,000	1,000
Deficient	15,000	8,000	-1,000
weighted average: D_i	0.8(15,000)+0.2(-4,000)= 11,200	0.8(8,0000+0.2(-3,000)= 5,800	0.8*4,000+0.2(-1,000)= 3000

$D = \alpha\,(\textit{Maximum in column}) + (1 - \alpha)(\textit{Minimum in column})$

Decision: Rice

- Decision making under Risk.

1. Expected monetary value (EMV).

Alternative / Monsoon	Rice	Wheat	Maize	probability
Excess	10,000	4,000	4,000	**0.4**
Normal	-4,000	-3,000	1,000	0.30
Deficient	15,000	8,000	-1,000	0.30
EMV	0.4(10,000)+0.3(-4,000)+0.3(15,000)= 7300	0.4(4,000)+0.3(-3,000)+0.3(8,000)= 3100	0.4(4,000)+0.3(-1,000)+0.3(1,000)= 1600	

Decision: Rice

2. Expected Opportunity Loss (EOL).

Opportunity Loss Table

Alternative / Monsoon	Rice	Wheat	Maize	probability
Excess	0	6,000	6,000	0.4
Normal	5,000	4,000	0	0.30
Deficient	0	7,000	16,000	0.30
EOL	0.3(5,000) =1,500	0.4(6,000)+0.3(4,000)+0.3(7,000) =5,700	0.4(6,000)+0.3(16,000) =7,200	

Decision: Rice

3. Maximum Likelihood (ML).

Alternative / Monsoon	Rice	Wheat	Maize	probability
Excess	**10,000**	4,000	4,000	**0.4**

The best choice is Rice

4. Expected Value of Perfect Information (EVPI).

Alternative / Monsoon	Rice	Wheat	Maize	probability
Excess	**10,000**	4,000	4,000	0.4
Normal	-4,000	-3,000	**1,000**	0.30
Deficient	**15,000**	8,000	-1,000	0.30
EMV	0.4(10,000)+0.3(-4,000)+0.3(15,000)= 7300	0.4(4,000)+0.3(-3,000)+0.3(8,000)= 3100	0.4(4,000)+0.3(-1,000)+0.3(1,000)= 1600	

$EVPI = Expected\ profit\ with\ perfect\ information\ - \max EMV$

$= \sum_{i=1}^{m} p_i \max(r_{ij}) - \max EMV$; (i: represent stat of nature)

$EVPI = 10{,}000(0.4) + 0.3(1{,}000) + 0.3(15{,}000) - 7{,}800 = 8{,}800 - 7{,}300 = 1{,}500$

Example: The management of a company is faced with the problem of choosing one of three products that it wants to manufacture. The potential demand for each product may turn out to be good, moderate or poor. The probabilities for each of the states of nature were estimated as follows.

Nature of demand			
product	Good	Moderate	Poor
X	0.70	0.20	0.10
Y	0.50	0.30	0.20
Z	0.40	0.50	0.10

The Estimated profit or loss in Dollar $ under the three states may be taken as:

product	Good	Moderate	Poor
X	300,000	200,000	100,000
Y	600,000	300,000	200,000
Z	400,000	100,000	-150,000

Prepare the Expected value table, and advise the management about the choice of product.

Expected payoff			
	X	Y	Z
Good	3(0.7)= 2.1	6(0.5)= 3	4(0.4)= 1.6
Moderate	2(0.2)= 0.4	3(0.3)= 0.9	1(0.5)= 0.5
Poor	1(0.1)= 0.1	2(0.2)= 0.4	-1.5 (0.1)= -0.15
EMV	2.1+0.4+0.1=2.6	4.3	1.95

The best Act is Y

Or by use EOL

Opportunity Loss Table			
	X	Y	Z
Good	6-3=3	6-6=0	6-4=2
Moderate	3-2=1	3-3=0	3-1=2
Poor	2-1=1	2-2=0	2-(-1.5)= 3.5

Expected Opportunity Loss			
	X	Y	Z
Good	3(0.7)= 2.1	0	2(0.4) = 0.8
Moderate	1(0.2)= 0.2	0	2(0.5) = 1
Poor	1(0.1)= 0.1	0	3.5(0.1) = 0.35
EOL	2.1+0.2+0.1=2.4	0	2.15

The best choice is product Y

H.W

Example 19.2: A food products' company is contemplating the introduction of a revolutionary new product with new packaging or replacing the existing product at much higher price (S1). It may even make a moderate change in the composition of the existing product, with a new packaging at a small increase in price (S2), or may mall a small change in the composition of the existing product, backing it with the word 'New' and a negligible increase in price (S3). The three possible states of nature or events are:
(i) high increase in sales (N1) with probability **0.3** , (ii) no change in sales (N2) with probability **0.5** and (iii) decrease in sales (N3) with probability **0.2**.
The marketing department of the company worked out the payoffs in terms of yearly net profits for each of the strategies of three events (expected sales). This is represented in the following table:

Strategies	States of Nature		
	N_1	N_2	N_3
S_1	700,000	300,000	150,000
S_2	500,000	450,000	0
S_3	300,000	300,000	300,000

Which strategy should the concerned executive choose on the basis of

a. Pessimism (Maximin) criterion
b. Optimism (Maximax) criterion
c. Minmax (salvage or Regret) criterion
d. Laplace criterion,
e. Hurwicz Alpha criterion α=0.6

f. Expected Monetary Value (EMV)
g. Expected Opportunity Loss (EOL)
h. ML
i. Expected Value of Perfect Information (EVPI)