Exercise #7

Question(1):

Given the standard normal distribution find :

a) The area under the curve between Z = 0 and Z = 1.43

$P(0 < Z < 1.43) = P(Z < 1.43) - P(Z < 0) = 0.92364 - 0.5 = 0.42364$

b) The probability that a Z picked at random will a value between

Z = 2.87 and Z = 2.64

$P(2.64 < Z < 2.87) = P(Z < 2.87) - P(Z < 2.64) =$

$= 0.99795 - 0.99585$

$= 0.0021$

c) Area to the left of Z = 1.43 equals $P(Z < 1.43) = 0.92364$

d) Area to the right of Z = 2 equals

$P(Z > 2) = 1 - P(Z \leq 2) = 1 - 0.97725 = 0.02275$

Or

$P(Z > 2) = P(Z < -2) = 0.02275$

$$P(Z > a) = 1 - P(Z < a)$$

or $P(Z > a) = P(Z < -a)$

e) P (Z > 0.55) = $1 - P (Z \leq 0.55) = 1 - 0.70884 = 0.29116$

Or

$P(Z > 0.55) = P(Z < -0.55) = 0.29116$

f) $P(Z \geq -0.55) = 1 - P(Z < -0.55) = 1 - 0.29116 = 0.70884$

   Or

   $P(Z \geq -0.55) = P(Z \leq 0.55) = 0.70884$

g) $P(Z < -2.33) = 0.00990$

h) $P(Z < -5) = 0$

i) $P(Z \geq 6.5) = 1 - P(Z < 6.5) = 1 - 1 = 0$

j) $P(Z \leq k) = 0.00554$, then the value of $k = -2.54$

Since $0.00554 < 0.5$, go to Negative table

k) $P(Z \geq k) = 0.03836$, then the value of $k = 1.77$

   $P(Z \geq k) = 0.03836$

   $1 - P(Z < k) = 0.0383$

   $-P(Z < k) = 0.03836 - 1$

   $-P(Z < k) = -0.96164$

   $P(Z < k) = 0.96164$

   $k = 1.77$

Or

   $P(Z \geq k) = 0.03836$

   $P(Z \leq -k) = 0.03836$

   $-k = -1.77$

   $k = 1.77$

l) P(- 2.67 < Z ≤ k) = 0.97179, then the value of k = …..

P(Z< K) – P (Z < -2.67) = 0.97179

P(Z< K) − 0.97179 = 0.00379

P(Z< K) = 0.97179 + 0.00379 = 0.97558

K = 1.97

Home work:

  a) P (Z < 2.33) = **0.99010**

  b) P (- 1.96 < Z ≤ 1.96) = **P(Z < 1.96) – P(Z< -1.96) =**
                                = 0.9750 -0.0250= 0.95

  c) P (- 2.58 < Z < 2.58) = **P(Z < 2.58) – P(Z< - 2.58) =**
                                = 0.99506 - 0.00495 = 0.99012

  d) P (- 1.65 < Z ≤ 1.65) = 0.95053-0.04947=**0.901**

  e) P (Z = 0.74) = **0**

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Question (2):

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For another subject (a 29 – year – old male) in the study Disking et al. (A-10), acetone levels were normally distributed with a mean of 870 and a standard deviation of 211 ppb. Then

$$given\ X\sim Normal\ \ \mu = 870\ \ \ \sigma = 211$$

a) **The probability that on a given day the subject's acetone between 600 and 1000 ppb is………………**

$$P(600 < X < 1000) = P\left(\frac{600-870}{211} < Z < \frac{1000-870}{211}\right)$$

$$= P(-1.28 < Z < 0.62)$$

$$= P(Z < 0.62) - P(Z < -1.28)$$

$$= 0.73237 - 0.10027$$

$$= 0.6321$$

b) **The probability that on a given day the subject's acetone over 900 ppb is …………………..**

$$P(X > 900) = P\left(Z > \frac{900-870}{211}\right) = P(Z > 0.14) = 1-0.55567 = 0.44433$$

c) **The probability that on a given day the subject's acetone under 500 ppb is ………………………..**

$$P(X < 500) = P\left(Z < \frac{500-870}{211}\right) = P(Z < -1.75) = 0.04006$$

d) **Percentage over 900 ppb is …………………..**

$$P(X > 900) = P\left(Z > \frac{900-870}{211}\right) = P(Z > 0.14) = 1-0.55567 = 0.44433$$

$$P(X > 900) \times 100 = 0.44433 \times 100 = 44.433\%$$

e) If we take <mark>population of 10,000</mark> <u>how many</u> would expect be over 900?

$P(X > 900) = P(Z > \frac{900-870}{211}) = P(Z > 0.14) = 1 - 0.55567 = 0.44433$

Number $= P(X > 900) \times 10,000 = 0.44433 \times 10,000 = 4443.3 = 4444$

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Question (3):

In the study of fingerprints an important quantitative characteristic is the total ridge count for the 10 fingers of an individual Suppose that the total ridge counts of individuals in a certain population are approximately normally distributed with a mean of 140 and a standard deviation of 50. Then

$given \ X \sim Normal \ \ \mu = 140 \ \ \ \sigma = 50$

a) The probability that an individual picked at random from this population will have a ridge count of 200 or more is …..

$P(X > 200) = P(Z > \frac{200-140}{50})$

$= P(Z > 1.2) = 1 - P(Z < 1.2) = 1 - 0.88493 = 0.11507$

b) The probability that an individual picked at random from this population will have a ridge count of Less than 100 is……..

$P(X < 100) = P(Z < \frac{100-140}{50}) = P(Z < -0.8) = 0.21186$

c) The probability that an individual picked at random from this population will have a ridge count of between 100 and 200 is …..

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$$P(100 < X < 200)$$

$$=P\left(\frac{100-140}{50} < Z < \frac{200-140}{50}\right)$$

$$=P(-0.8 < Z < 1.2)$$

$$= P(Z < 1.2) - P(Z < -0.8)$$

$$= 0.88493 - 0.21186 = 0.67307$$

d) **The <mark>percentage</mark> that individual picked at random from this population will have a ridge count of Less than 100 is……..**

$$P(X < 100) = P\left(Z < \frac{100-140}{50}\right) = P(Z < -0.8) = 0.21186$$

$$P(X < 100) \times 100 = 0.21186 \times 100 = 21.186\%$$

f) **<mark>In a population of 100,000</mark> people how many would you expect to have a ridge count of 200 or more ?**

$$P(X > 200) = P\left(Z > \frac{200-140}{50}\right) = P(Z > 1.2) = p(Z < -1.2)$$

$$\text{Number} = P(X > 200) \times 100{,}000 = 0.11507 \times 100{,}000 = 11507$$

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Home work:

One of the variables collected in North Carolina Birth Registry data is pounds gained during pregnancy. According to data from the entire registry for 2001. The number of pounds gained during pregnancy was approximately normally distributed with a mean of 30.23 pounds and a standard deviation of 13.84 pounds. Calculate the probability that a randomly selected mother in North Carolina in 2001 gained

a) **Less than 15 pounds during pregnancy is………**

$P(X<15)=P(Z<-1.10)=0.13567$

b) More than 40 pounds is ……………………..

P(X>40)=P(Z>0.71)=1-P(Z<0.71)=1-0.76115=0.23885

c) Between 14 and 40 pounds is ………………..

P(14<X<40)=P(-1.17<Z<0.71)=0.76115-0.12100=0.64015

d) Percentage that mother in North Carolina in 2001 gained between 14 and 40 pounds is ………………..
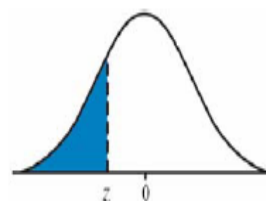   **P(14<X<40)*100=64.015%**

e) If we take population of 10,000 how many would expect mother in North Carolina in 2001 gained between 14 and 40 pounds is ………………..

P(14<X<40)*10,000=6401.5=6402

Standard Normal Table
Areas Under the Standard Normal Curve

z	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.02	-0.01	-0.00	z
-3.50	0.00017	0.00017	0.00018	0.00019	0.00019	0.00020	0.00021	0.00022	0.00022	0.00023	-3.50
-3.40	0.00024	0.00025	0.00026	0.00027	0.00028	0.00029	0.00030	0.00031	0.00032	0.00034	-3.40
-3.30	0.00035	0.00036	0.00038	0.00039	0.00040	0.00042	0.00043	0.00045	0.00047	0.00048	-3.30
-3.20	0.00050	0.00052	0.00054	0.00056	0.00058	0.00060	0.00062	0.00064	0.00066	0.00069	-3.20
-3.10	0.00071	0.00074	0.00076	0.00079	0.00082	0.00084	0.00087	0.00090	0.00094	0.00097	-3.10
-3.00	0.00100	0.00104	0.00107	0.00111	0.00114	0.00118	0.00122	0.00126	0.00131	0.00135	-3.00
-2.90	0.00139	0.00144	0.00149	0.00154	0.00159	0.00164	0.00169	0.00175	0.00181	0.00187	-2.90
-2.80	0.00193	0.00199	0.00205	0.00212	0.00219	0.00226	0.00233	0.00240	0.00248	0.00256	-2.80
-2.70	0.00264	0.00272	0.00280	0.00289	0.00298	0.00307	0.00317	0.00326	0.00336	0.00347	-2.70
-2.60	0.00357	0.00368	0.00379	0.00391	0.00402	0.00415	0.00427	0.00440	0.00453	0.00466	-2.60
-2.50	0.00480	0.00494	0.00508	0.00523	0.00539	0.00554	0.00570	0.00587	0.00604	0.00621	-2.50
-2.40	0.00639	0.00657	0.00676	0.00695	0.00714	0.00734	0.00755	0.00776	0.00798	0.00820	-2.40
-2.30	0.00842	0.00866	0.00889	0.00914	0.00939	0.00964	0.00990	0.01017	0.01044	0.01072	-2.30
-2.20	0.01101	0.01130	0.01160	0.01191	0.01222	0.01255	0.01287	0.01321	0.01355	0.01390	-2.20
-2.10	0.01426	0.01463	0.01500	0.01539	0.01578	0.01618	0.01659	0.01700	0.01743	0.01786	-2.10
-2.00	0.01831	0.01876	0.01923	0.01970	0.02018	0.02068	0.02118	0.02169	0.02222	0.02275	-2.00
-1.90	0.02330	0.02385	0.02442	0.02500	0.02559	0.02619	0.02680	0.02743	0.02807	0.02872	-1.90
-1.80	0.02938	0.03005	0.03074	0.03144	0.03216	0.03288	0.03362	0.03438	0.03515	0.03593	-1.80
-1.70	0.03673	0.03754	0.03836	0.03920	0.04006	0.04093	0.04182	0.04272	0.04363	0.04457	-1.70
-1.60	0.04551	0.04648	0.04746	0.04846	0.04947	0.05050	0.05155	0.05262	0.05370	0.05480	-1.60
-1.50	0.05592	0.05705	0.05821	0.05938	0.06057	0.06178	0.06301	0.06426	0.06552	0.06681	-1.50
-1.40	0.06811	0.06944	0.07078	0.07215	0.07353	0.07493	0.07636	0.07780	0.07927	0.08076	-1.40
-1.30	0.08226	0.08379	0.08534	0.08691	0.08851	0.09012	0.09176	0.09342	0.09510	0.09680	-1.30
-1.20	0.09853	0.10027	0.10204	0.10383	0.10565	0.10749	0.10935	0.11123	0.11314	0.11507	-1.20
-1.10	0.11702	0.11900	0.12100	0.12302	0.12507	0.12714	0.12924	0.13136	0.13350	0.13567	-1.10
-1.00	0.13786	0.14007	0.14231	0.14457	0.14686	0.14917	0.15151	0.15386	0.15625	0.15866	-1.00
-0.90	0.16109	0.16354	0.16602	0.16853	0.17106	0.17361	0.17619	0.17879	0.18141	0.18406	-0.90
-0.80	0.18673	0.18943	0.19215	0.19489	0.19766	0.20045	0.20327	0.20611	0.20897	0.21186	-0.80
-0.70	0.21476	0.21770	0.22065	0.22363	0.22663	0.22965	0.23270	0.23576	0.23885	0.24196	-0.70
-0.60	0.24510	0.24825	0.25143	0.25463	0.25785	0.26109	0.26435	0.26763	0.27093	0.27425	-0.60
-0.50	0.27760	0.28096	0.28434	0.28774	0.29116	0.29460	0.29806	0.30153	0.30503	0.30854	-0.50
-0.40	0.31207	0.31561	0.31918	0.32276	0.32636	0.32997	0.33360	0.33724	0.3409	0.34458	-0.40
-0.30	0.34827	0.35197	0.35569	0.35942	0.36317	0.36693	0.37070	0.37448	0.37828	0.38209	-0.30
-0.20	0.38591	0.38974	0.39358	0.39743	0.40129	0.40517	0.40905	0.41294	0.41683	0.42074	-0.20
-0.10	0.42465	0.42858	0.43251	0.43644	0.44038	0.44433	0.44828	0.45224	0.45620	0.46017	-0.10
-0.00	0.46414	0.46812	0.47210	0.47608	0.48006	0.48405	0.48803	0.49202	0.49601	0.50000	-0.00

Standard Normal Table (continued)
Areas Under the Standard Normal Curve
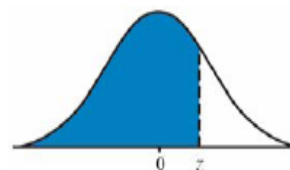
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	z
0.00	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586	0.00
0.10	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535	0.10
0.20	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409	0.20
0.30	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173	0.30
0.40	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793	0.40
0.50	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240	0.50
0.60	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490	0.60
0.70	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524	0.70
0.80	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327	0.80
0.90	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891	0.90
1.00	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214	1.00
1.10	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298	1.10
1.20	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147	1.20
1.30	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774	1.30
1.40	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189	1.40
1.50	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408	1.50
1.60	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449	1.60
1.70	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327	1.70
1.80	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062	1.80
1.90	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670	1.90
2.00	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169	2.00
2.10	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574	2.10
2.20	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899	2.20
2.30	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158	2.30
2.40	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361	2.40
2.50	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520	2.50
2.60	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643	2.60
2.70	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736	2.70
2.80	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807	2.80
2.90	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861	2.90
3.00	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.9990	3.00
3.10	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929	3.10
3.20	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950	3.20
3.30	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965	3.30
3.40	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976	3.40
3.50	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983	3.50