Exercise

Q_1*: *Find the optimum solution for the following problem (using Bisection method):

$$\min f(x) = x^3 - 3x + 1$$

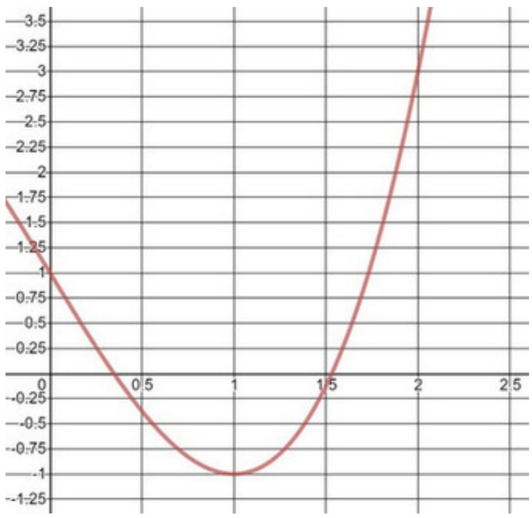
$$s.t \quad 0 \le x \le 3$$

$$\varepsilon = 0.02$$

Solution :

$$f'(x) = 3x^2 - 3 = 0$$

$$x_{mid} = c = \frac{a+b}{2}$$



If :

$f'(x_{mid})\ f'(a) > 0 \quad \rightarrow \quad a = x_{mid}$

$f'(x_{mid})\ f'(b) > 0 \quad \rightarrow \quad b = x_{mid}$

n	a	b	c	$f'(a)$	$f'(b)$	$f'(c)$	$\lvert f'(c)\rvert \le \varepsilon$
1	0	3	1.5	-3	24	3.75	not $\le \varepsilon$
2	0	1.5	0.75	-3	3.75	-1.3125	not $\le \varepsilon$
3	0.75	1.5	1.125	-1.3125	3.75	0.7969	not $\le \varepsilon$
4	0.75	1.125	0.9375	-1.3125	0.7969	-0.3633	not $\le \varepsilon$
5	0.9375	1.125	1.0313	-0.3633	0.7969	0.1907	not $\le \varepsilon$
6	0.9375	1.0313	0.9844	-0.3633	0.1907	-0.0929	not $\le \varepsilon$
7	0.9844	1.0313	1.0079	-0.0929	0.1907	0.0476	not $\le \varepsilon$
8	0.9844	1.0079	0.9962	-0.0929	0.0476	-0.0228	not $\le \varepsilon$
9	0.9962	1.0079	1.0021	-0.0228	0.0476	0.0126	$< \varepsilon$

As $\lvert f'(c)\rvert \le \varepsilon$ Then $x^* = c = 1.0021$

Q_2 : Find the optimum solution for the following problem (using Newton-Raphson method):

$$\min f(x) = x^3 - 3x + 1$$

$$s.t \quad 0 \le x \le 3$$

$$\text{Assume that, } x_0 = 1.5 \text{ , } \varepsilon = 0.01$$

Solution :

$$x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)}$$

$f'(x) = 3x_n^2 - 3$; $f''(x) = 6x_n$

n	x_n	$f'(x_n)$	$f''(x_n)$	x_{n+1}	$\lvert f'(x_{n+1})\rvert$
0	1.5	3.75	9	1.0833	0.5206
1	1.0833	0.5206	6.4998	1.0032	0.0192
2	1.0032	0.0192	6.0192	1.000010207	0

$$x_{n+1} = x_n - \frac{(3x_n^2 - 3)}{(6x_n)}$$

$|f'(x_{n+1})| \le \varepsilon$, then $x^* = x_3 = 1.0000$

Q_3: Find the optimum solution for the following problem (using direct method):

$$\max f(x) = 12x^5 - 45x^4 + 40x^3 + 5$$

$$s.t \quad -0.44 \le x \le 2$$

solution :

1) find $f'(x) \to f'(x) = 60x^2 (x^2 - 3x + 2)$ also can

be write as $f'(x) = 60x^2 (x - 2)(x - 1)$

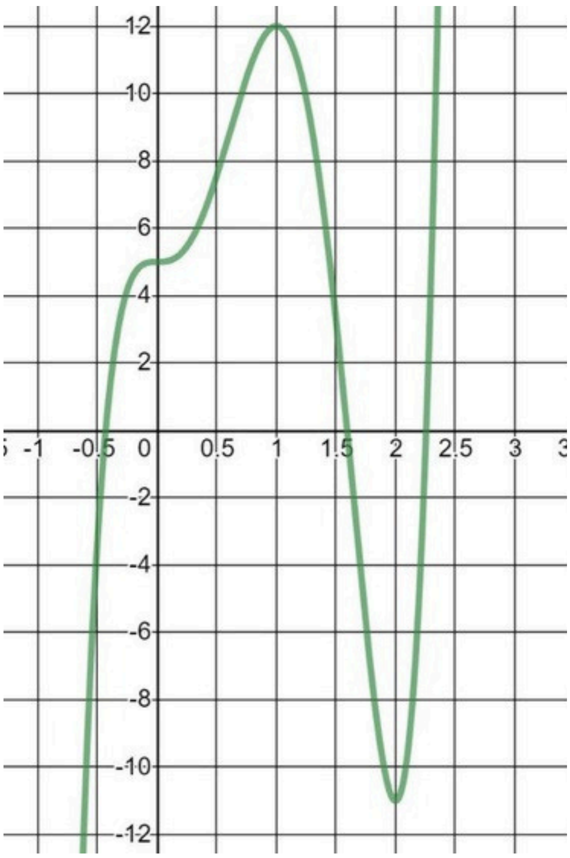
2) stationary point $f'(x) = 0$

$$60x^2(x - 2)(x - 1) = 0$$

Then , the stationary points are x = 0,1,2

f(0) =5 , f(1)=12 , f(2)=-11 ; f(-0.44) = -0.2919

<u>Optimal solution</u> is f_{max} = 12 at x*= 1



Note : if the problem

$\min f(x) = 12x^5 - 45x^4 + 40x^3 + 5$

the optimal solution is f_{min} = at x*=

Q_4 : *Choose the correct answer :*

1. If the function $f(x) = -4x^3+5x^2$, the point x=1 it is :

D Fixed point	C Maximum point	B Root point	A Minimum point

2. If the function $f(x) = -4x^3+5x^2$, the point x=⅚ it is a:

D Minimum point	C Saddle point	B Maximum point	A Inflection point

3. If the function $f(x) = -4x^3+5x^2$, the point x = 0 it is a:

D All of the above	C Fixed point	B Root point	A Minimum point

4. If the program max $f(x) = 4x^3-3x^2+2$ and $-1 \le x \le 1$, the optimal solution at x = :

D x=0.5	C x=0	B x=1	A x=2

H.W

Find the optimum solution for the following nonlinear problem :

$$\max f(x) = \frac{4}{3}x^3 - 5x^2 + 4x$$
$$s.t. \quad 0 \le x \le 1.5$$

a) Using Direct solution method

b) Using Bisection method, assume that ε=0.01

c) Using Newton-Raphson , assume that ε=0.01 , x_0=0