Exercise

Example 1: An office has five workers, and five tasks have to be performed. Workers differ in efficiency and tasks differ in their intrinsic difficulty. Time each worker would take to complete each task is given in the effectiveness matrix.

How the tasks should be allocated to each worker so as to minimize the total man-hours?

workers / tasks	**1**	**2**	**3**	**4**	**5**	**P_i=min**
A	9	11	14	11	7	**7**
B	6	15	13	13	10	**6**
C	12	13	6	8	8	**6**
D	11	9	10	12	9	**9**
E	7	12	14	10	14	**7**

We note #Row = # column, we can solve this problem.

	1	**2**	**3**	**4**	**5**
A	**2**	**4**	**7**	**4**	**0**
B	**0**	**9**	**7**	**7**	**4**
C	**6**	**7**	**0**	**2**	**2**
D	**2**	**0**	**1**	**3**	**0**
E	**0**	**5**	**7**	**3**	**7**
q_j=min	**0**	**0**	**0**	**2**	**0**

	1	**2**	**3**	**4**	**5**
A	**2**	**4**	**7**	**2**	**0**
B	**0**	**9**	**7**	**5**	**4**
C	**6**	**7**	**0**	**0**	**2**
D	**2**	**0**	**1**	**1**	**0**
E	**0**	**5**	**7**	**1**	**7**

$since\ \#raw\ or\ colum \neq k;$

$so\ determain\ h = \min\{cell(i,j) is\ not\ coverd\} = 1$

	1	**2**	**3**	**4**	**5**
A	**2**	**4**	**6**	**1**	**0** step 1
B	**0** step 2	**9**	**6**	**4**	**4**
C	**7**	**8**	**0** step 4	**0**	**3**
D	**2**	**0** step 5	**0**	**0**	**0**
E	**0**	**5**	**6**	**0** step 3	**7**

$x_{15} = x_{21} = x_{54} = x_{33} = x_{42} = 1$

Optimum solution

Man	1	2	3	4	5
Job	B	D	C	E	A
Man hours	6	9	6	10	7

$Total\ time = Z^* = 38\ hours$

Step 1: In a given problem, if the number of rows is not equal to the number of columns and vice versa, then add a dummy row or a dummy column. The assignment costs for dummy cells are always assigned as zero.

Step 2: Row reduction: Subtract the smallest number in each row from every number in that row .

Step 3: Column reduction.

Step 4: Draw minimum number of straight lines to cover all zero elements in the table.

Step 5: If the number of lines (k) = order of matrix then the solution is optimum, and we proceed to step 7. Otherwise, we go to step 6.

Step 6: subtract the smallest element which is NOT COVERED by lines from every uncovered element . Add the smallest element to any element lying at the intersection of any two lines. Return to **step 4** and continue until an optimum is reached.

Step 7: Find allocation: start the assignment from the row or column that has minimum number of Zeros. Then, eliminate row and column related to that assignment.

Step 8: Write down the assignment results and find the minimum cost/time.

Example 2: A workshop contains four persons available for work on the four jobs. Only one person can work on any one job. The following table shows the cost of assigning each person to each job. The objective is to assign person to jobs such that the total assignment cost is a minimum.

Jobs / persons	**1**	**2**	**3**	**4**
I	20	25	22	28
II	15	18	23	17
III	19	17	21	24
IV	25	23	24	24

Solution:

Find the First Reduced Cost Table:

Jobs	**1**	**2**	**3**	**4**	**P_i=min**
I	20	25	22	28	20
II	15	18	23	17	15
III	19	17	21	24	17
IV	25	23	24	24	23

Find the Second Reduced Cost Table:

Jobs / Persons	**1**	**2**	**3**	**4**
I	0	5	2	8
II	0	3	8	2
III	2	0	4	7
IV	2	0	1	1
q_j=min	0	0	1	1

Draw minimum number of lines to cover all zeros:

Jobs / Persons	**1**	**2**	**3**	**4**
I	0	5	1	7
II	0	3	7	1
III	2	0	3	6
IV	2	0	0	0

$since\ \#raw\ or\ colum \neq k = 4;$
$so\ determain\ h = \min\{cell(i,j) is\ not\ coverd\} = 1$

Jobs / Persons	**1**	**2**	**3**	**4**
I	0	4	0	6
II	0	2	6	0
III	3	0	3	6
IV	3	0	0	0

$since\ \#raw\ or\ colum = k = 4$ so, we arrive at an optimal solution (assignment).
Determine an assignment:

Jobs / Persons	**1**	**2**	**3**	**4**
I	0 $\boxed{20}$	4	0 **step 3** $\boxed{22}$	6
II	0 **step 2** $\boxed{15}$	2	6	0 $\boxed{17}$
III	3	0 **step 1**	3	6
IV	3	0	0 $\boxed{24}$	0 **step 4** $\boxed{24}$

Optimum solution

Persons	I	II	III	IV
Job	3	1	2	4
Man hours	22	15	17	24

$Total\ time = Z^* = 78$

Example 3: There are five jobs to be assigned to the machines. Only one job could be assigned to one machine are given in following matrix.

Machines / Jobs	**A**	**B**	**C**	**D**
1	1	6	4	3
2	0	7	2	1
3	3	7	2	4
4	4	6	5	7
5	3	2	4	6

1- Find an optimum assignment of jobs to the machines to minimize the total processing time. (using Hungarian method)
2- Find for which job no machine is assigned.
3- What is the total processing time to complete all the jobs.

Solution:

We note #Row ≠ # column. we have an unbalanced assignment problem, so we need to add dummy column (machine) with zero cost .

	A	B	C	D	Dummy
1	1	6	4	3	0
2	0	7	2	1	0
3	3	7	2	4	0
4	4	6	5	7	0
5	3	2	4	6	0
q_k	**0**	**2**	**2**	**1**	**0**

	A	B	C	D	Dummy
1	**1**	**4**	**2**	**2**	**0**
2	**0**	**5**	**0**	**0**	**0**
3	**3**	**5**	**0**	**3**	**0**
4	**4**	**4**	**3**	**6**	**0**
5	**3**	**0**	**2**	**5**	**0**

$h = 1$

	A	B	C	D	Dummy
1	**0**	**4**	**2**	**1**	**0**
2	**0**	**6**	**1**	**0**	**1**
3	**2**	**5**	**0**	**2**	**0**
4	**3**	**4**	**3**	**5**	**0**
5	**2**	**0**	**2**	**4**	**0**

$since\ \#raw\ or\ colum = k = 5$ so, we arrive at an optimal solution (assignment).
Determine an assignment:

	A	B	C	D	Dummy
1	**0** step 4	**4**	**2**	**1**	**0**
2	**0**	**6**	**1**	**0** step 5	**1**
3	**2**	**5**	**0** step 3	**2**	**0**
4	**3**	**4**	**3**	**5**	**0** step 1
5	**2**	**0** step 2	**2**	**4**	**0**

$x_{45} = x_{52} = x_{33} = x_{24} = x_{11} = 1$

Optimum solution

Jobs	1	2	3	4	5
Machines	A	D	C	dummy	B
hours	1	1	2	-	2

For job 4 no machines is assigned
$Total\ time = Z^* = 6\ hours$

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Example 4: There are four jobs to be assigned to the machines. Only one job could be assigned to one machine are given in following matrix.

Jobs	Machines				
	A	*B*	*C*	*D*	*E*
1	4	3	6	2	7
2	10	12	11	14	16
3	4	3	2	1	5
4	8	7	6	9	6

1- Find an optimum assignment of jobs to the machines to minimize the total processing time.(using Hungarian method)
2- find for which machine no job is assigned.
3- What is the total processing time to complete all the jobs?

Solution:

Job	*Machine*
1	B
2	A
3	D
4	C

For machines E no job is assigned

Total time = 10 + 3 + 6 + 1= 20

Example 5 : Assign the three tasks (A,B,C) to three operators(using Hungarian method). The assigning costs are given in Table

	A	B	C
1	20	15	31
2	17	16	33
3	18	19	27

Solution:

Example 6:

A dairy plant has five milk tankers I, II, III, IV & V. These milk tankers are to be used on five delivery routes A, B, C, D, and E. The distances (in kms) between dairy plant and the delivery routes are given in the following distance matrix

	I	II	III	IV	V
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

How the milk tankers should be assigned to the chilling centers so as to minimize the distance travelled?