Exercise 6

Question 1:

The same survey data base cited shows that 32 % of U.S. adults indicated that they have tested for HIV at some point in their life. Consider a simple random sample of 15 adults selected at that time. Find the probability that the number of adults who have been tested for HIV in the sample would be:

$n = 15 \quad p = 0.32 \quad then \quad q = 1 - p = 1 - 0.32 = 0.68$

$X \sim Binomial\ (n, p) \qquad X \sim Binomial\ (15, 0.32)$

$\boldsymbol{P(X = x)} = {}_{n}C_{x}\ (p)^{x}(q)^{n-x} \qquad x = 0,1,2,3, \dots, n$

$\boldsymbol{P(X = x)} = {}_{15}C_{x}\ (0.32)^{x}(0.68)^{15-x} \qquad x = 0,1,2,3, \dots, 15$

(a) Three.

$P(X = \mathbf{3}) = {}_{15}C_{3}(0.32)^{3}(0.68)^{15-3} = \mathbf{0.14574}$

(b) Less than five.

$$P(X < 5) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$$

$$= 0.2057 + 0.1457 + 0.0715 + 0.0217 + 0.00307 = \mathbf{0.4477}$$

By calculator : $\sum_{x=0}^{4}(({}_{15}C_{x})(0.32)^{x}(0.68)^{15-x}) = \mathbf{0.4477}$

(c) Between five and nine. Inclusive.

$$P(5 \leq X \leq 9) = P(X = 5) + P(X = 6) + P(X = 7) + P(X = 8) + P(X = 9)$$

$$= 0.2130 + 0.1671 + 0.1011 + 0.0476 + 0.0174 = \mathbf{0.5462}$$

By calculator : $\sum_{x=5}^{9}(({}_{15}C_{x})(0.32)^{x}(0.68)^{15-x}) = \mathbf{0.5462}$

(d) More than five , but less than 10.

$P(5 < X < 10) = P(X = 6) + P(X = 7) + P(X = 8) + P(X = 9)$

$= 0.1671 + 0.1011 + 0.0476 + 0.0174 = \mathbf{0.3332}$

By calculator : $\sum_{x=6}^{9}((_{15}C_x)(0.32)^x(0.68)^{15-x}) = \mathbf{0.3331}$

(e) Six or more.

$P(X \geq 6) = 1 - P(X < 6)$

$= 1 - [P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)]$

$= 1 - 0.6607 = \mathbf{0.3393}$

by calculator : $\sum_{x=6}^{15}((_{15}C_x)(0.32)^x(0.68)^{15-x}) = \mathbf{0.3393}$

(f) Mean equals …$\mu = np = (15)(0.32) = \mathbf{4.8}$……..

(g) Variance equals …$\sigma^2 = n\,p\,q = (15)(0.32)\,(0.68) =$**3.264**….

(h) Standard deviation $\sigma = \sqrt{n\,p\,q} = \sqrt{(15)(0.32)\,(0.68)} = \mathbf{1.81}$

Question 2:

Coughlin et al. estimated the percentage of woman living in border counties along the southern United States with Mexico (designated counties in California, Arizona, New Mexico, and Texas) who have less than a high school education to be 19. Suppose we select three women at random.

Then

$n = 3 \quad p = 0.19 \quad then \quad q = 1 - p = 1 - 0.19 = 0.81$

$X \sim binomial\,(n, p) \quad X \sim binomial\,(3, 0.19)$

$\boldsymbol{P(X = x)} = {}_nC_x\,(p)^x(q)^{n-x} \qquad x = 0,1,2,3,\dots,n$

$\boldsymbol{P(X = x)} = {}_3C_x\,(0.19)^x(0.81)^{3-x} \qquad x = 0,1,2,3$

(a) The probability that the number of women with less than a high-school education is zero …………………

$P(X = 0) = {}_{3}C_{0}(0.19)^{0}(0.81)^{3-0} = \mathbf{0.5314}$

(b) The probability that the number of women with less than a high-school education is one……………………….

$P(X = 1) = \mathbf{0.3739}$

(c) The probability that the number of women with less than a high-school education is two or fewer…………………

$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$= 0.5314 + 0.3740 + 0.0877 = \mathbf{0.9931}$

by calculator : $\sum_{x=0}^{2}(({}_{3}C_{x})(0.19)^{x}(0.81)^{3-x}) = \mathbf{0.9931}$

(d) The probability that the number of women with less than a high-school education is two or three……………………..

$P(X = 2) + P(X = 3) = 0.0877 + 0.0069 = \mathbf{0.0946}$

by calculator : $\sum_{x=2}^{3}(({}_{3}C_{x})(0.19)^{x}(0.81)^{3-x}) = \mathbf{0.0946}$

(e) Mean equal $\mu = np = (3)(0.19) = \mathbf{0.57}$

(f) Standard deviation equal $\sqrt{\sigma^2} = \sqrt{npq} = \sqrt{(3)(0.19)(0.81)} = \mathbf{0.6795}$

Home work:

In a survey of nursing students pursuing a master's degree, 75 percent stated that they expect to be promoted to a higher position within one month after receiving the degree. If this percentage holds for the entire population, find, for a sample of 15, the probability that the number expecting a promotion within a month after receiving their degree is:

$n=15$, $p=0.75$, $q=0.25$

(a) Six. = 0.0034

(b) At last seven. = 0.996

(c) No more than five. 0.0008

(d) Between six and nine, inclusive.

Question 3:

Singh et al. (A-7) Looked at the occurrence of retinal capillary hemangiona patients with von hippel - Lindau (VHL) disease. RCH is a benign vascular tumor of the retina. Using a retrospective consecutive case series review, the researchers found that the number of RCH tumor incidents followed a Poisson distribution with $\lambda = 4$ tumors per eye for patients with VHL. Using this model.

The probability that in a randomly patient with VHL

(a) There will be exactly five occurrences of tumors per eye equals…

$$\lambda = 4 \text{ per eye}$$

$$P(X = x) = f(x) = \frac{e^{-4}\, 4^x}{x!}, \quad x = 0, 1, 2, \ldots$$

$$P(X = 5) = \frac{e^{-4} 4^5}{5!} = 0.1563$$

(b) There are more than five occurrences of tumors per 2-eyes equals…

$$\lambda^* = 2\lambda = (2)(4) = 8$$

$$P(X = x) = f(x) = \frac{e^{-8}\, 8^x}{x!}, \quad x = 0, 1, 2, \ldots$$

$$P(X > 5) = 1 - P(X \leq 5)$$

$$= 1 - [P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)]$$

$$= 1 - [0.0003 + 0.0027 + 0.0107 + 0.0286 + 0.0573 + 0.0916]$$

$$= 1 - 0.1912 = 0.8088$$

By use summation: $P(X > 5) = 1 - 0.19124 = \mathbf{0.80876}$

by calculator : $P(X > 5) = 1 - \sum_0^5 \left(\frac{e^{-8} \times 8^x}{x!}\right) = 0.80876$

(c) There are between five and seven occurrences tumors per eye, inclusive……..

$$\lambda = 4 \text{ per eye}$$

$$P(X = x) = f(x) = \frac{e^{-4} 4^x}{x!}, \quad x = 0, 1, 2, \ldots$$

$$P(5 \le X \le 7) = P(X = 5) + P(X = 6) + P(X = 7) = \mathbf{0.320}$$

by calculator : $\boldsymbol{P(5 \le X \le 7)} = \sum_{5}^{7}\left(\frac{e^{-4} \times 4^{x}}{x!}\right) = 0.320$

(d) The mean(per eye) $\mu = \lambda = 4$

(e) The variance (Per 2-eyes) $\sigma^2 = \lambda^* = 2\lambda = (2)(4) = 8$

Question 4:

In a certain population an average of 13 new cases of esophageal cancer are diagnosed each year. If the annual of esophageal cancer follows a Poisson distribution, find the probability that the number of newly diagnosed cases of esophageal cancer will be.

(a) Exactly 10 in year

$$\boldsymbol{\lambda = 13\ per\ year}$$

$$\boldsymbol{P(X = x) = f(x) = \frac{e^{-13}13^{x}}{x!}, \quad x = 0, 1, 2, \dots}$$

$$\boldsymbol{P(X = 10) = \frac{e^{-13}13^{10}}{10!} = 0.0859}$$

(b) At last eight in a month ……………….

$$\boldsymbol{\lambda^* = \frac{\lambda}{12} = \frac{13}{12} = 1.083}$$

$$\boldsymbol{P(X = x) = f(x) = \frac{e^{-1.083}\ 1.083^{x}}{x!}, \quad x = 0, 1, 2, \dots}$$

$P(X \ge 8) = 1 - P(X < 8)$

$= 1 - [P(X = 0) + P(X = 2) + \cdots + P(X = 7)]$

$= 2\text{x}10^{-5} = 0.00002$

by calculator : $\boldsymbol{P(X \ge 8)} = 1 - \sum_{0}^{7}\left(\frac{e^{-1.083} \times 1.083^{x}}{x!}\right) = 2 \times 10^{-5}$

(c) Between two and five, inclusive in a week …….

$$\boldsymbol{\lambda^* = \frac{\lambda}{48} = \frac{13}{48} = 0.27}$$

$$P(X = x) = f(x) = \frac{e^{-0.27}\ 0.27^x}{x!}, \qquad x = 0, 1, 2, \ldots$$

$P(2 \leq X \leq 5)$

$= P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) = \mathbf{0.03051}$

by calculator: $\sum_2^5 \left(\frac{e^{-0.27} \times 0.27^x}{x!}\right) = 0.03051$

(d) Fewer than one in 2 years ....

$\lambda^* = 2\lambda = 2(13) = 26$

$$P(X = x) = f(x) = \frac{e^{-26}\ 26^x}{x!}, \qquad x = 0, 1, 2, \ldots$$

$$P(X < 1) = P(X = 0) = \frac{e^{-26}\ 26^0}{0!} = 5.1091 \times 10^{-12} = 0$$

(e) Mean(in week) $\mu = \lambda^* = \frac{\lambda}{48} = \frac{13}{48} = 0.27$

(f) Standard deviation(in year) $\sqrt{\sigma^2} = \sqrt{\lambda} = \sqrt{13} = 3.61$

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Home work:

If the mean number of serious accident per month in a large factory (where the number of employs remains constant) is 2 ,

(a) The probability that in the current year there will be exactly seven accidents is ....

(b) The probability that in the current week there will be two or more accident is......................

(c) The probability that in the current day there will be no accidents is

(d) Mean(In a week) ....

(e) Variance(In a 3 years)..................................

<u>How to calculate probability in Binomail distribution</u>

<u>(By Calculater):</u>

<u>Example 1</u>: Binomail distribution n=5 ,p=0.3 ,q=0.7

X ~ Binomail(5,0.3)

Find

1) P(X ≤ 2) =?

$$\sum_0^2 (\ 5Cx * (0.3)^x * (0.7)^{(5-x)})$$

$$\sum_0^2 \Big(\big(\underbrace{5\ \text{Shift} \div}_{5C}\ \underbrace{\text{Alpha}\ )}_{x} \ * \ \big(\underbrace{0.3^{\text{Alpha}}}_{x}\ \big) \ * \ \big(0.7^{(5-\underbrace{\text{Alpha}\ )}_{x}}\big)\Big)$$

<u>To write the following in calculator :</u>

$\sum$	→	*Shift log*
X	→	**Alpha**)
5C2	→	5 shift ÷ 2

2) P(X ≥ 3) =?

$$\sum_3^5 (\ 5Cx * (0.3)^x * (0.7)^{(5-x)}) =$$

$$\sum_3^5 \Big(\big(\underbrace{5\ \text{Shift} \div}_{5C}\ \underbrace{\text{Alpha})}_{x} \ * \ \big(\underbrace{0.3^{\text{Alpha}}\ )}_{x} \ * \ (\ 0.7^{(5-\underbrace{\text{Alpha}\ )}_{x}}\big)\Big)$$