Exercise

Q1: Find the initial feasible solution to the following transportation problem using two methods (1) least cost method (2) vogel's approximation method, then optimize the solution using MODI method.

(A)

Destination / Sources	D_1	D_2	D_3	D_4	Supply
S_1	3	6	8	5	20
S_2	6	1	2	5	28
S_3	7	8	3	9	17
Demand	15	19	13	18	

By using least cost method

Destination / Sources	D_1	D_2	D_3	D_4	Supply		
S_1	3 15	6	8	5 5	20	5	0
S_2	6	(start) 1 19	2 9	5	28	9	0
S_3	7	8	3 4	9 13	17	13	0
Demand	15	19	13	18			
	0	0	4	13			
			0	0			

IBFS: $X_{11}=15$, $X_{14}=5$,$X_{22}=19$,$X_{23}=9$, $X_{33}=4$, $X_{34}=13$

And TTC=15*3+5*5+19*1+9*4+4*3+13*9=236

Optimality test using **MODI** method. (If all $\delta_{kj} \leq 0$, we have optimal solution)

		$V_1= 3$	$V_2= -2$	$V_3= -1$	$V_4= 5$	
	Destination / Sources	D_1	D_2	D_3	D_4	Supply
$U_1=0$	S_1	3 15	6 $\delta_{12}= -8$	8 $\delta_{13}= -9$	5 5	20
$U_2=3$	S_2	6 $\delta_{21}=0$	1 19	- 2 9	+ 5 $\delta_{24}= 3$	28
$U_3=4$	S_3	7 $\delta_{31}=0$	8 $\delta_{32}= -6$	+ 3 4	- 9 13	17
	Demand	15	19	13	18	

		V_1= -2	V_2= -4	V_3= -6	V_4= 0	
	Destination / Sources	D_1	D_2	D_3	D_4	Supply
U_1=5	S_1	3 **15**	6 δ_{12}= -5	8 δ_{13}= -9	5 **5**	20
U_2=5	S_2	6 δ_{21}= -3	1 **19**	2 δ_{23}= -3	5 **9**	28
U_3=9	S_3	7 δ_{31}=0	8 δ_{32}= -3	3 **13**	9 **4**	17
	Demand	15	19	13	18	

Since all $\boldsymbol{\delta_{kj}} \leq 0$, so final optimal solution is arrived.

$$X_{11} = 15,\ X_{22} = 19,\ X_{33} = 13,\ X_{14} = 5,\ X_{24} = 9,\ X_{34} = 4$$

The minimum total transportation cost (TTC) is

$$\boldsymbol{Z = 15 * 3 + 5 * 5 + 19 * 1 + 9 * 5 + 13 * 3 + 4 * 9 = 209\$}$$

By using Voggel's Approximation method (VAM)

Destination / Sources	D_1	D_2	D_3	D_4	Supply			Row penalty		
S_1	3 **15**	6	8	5 **5**	20	5	0	2	2	2
S_2	6	1 **19**	2	5 **9**	28	9	0	1	3	1
S_3	7	8	3 **13**	9 **4**	17	4	0	4	4*	2
Demand	15	19	13	18						
	0	0	0	13						
				4						
				0						
column penalty	3	5*	1	0						
	3	-	1	0						
	3*	-	-	0						

Penalty = next smallest cost - smallest cost

IBFS : $X_{11} = 15,\ X_{22} = 19,\ X_{33} = 13,\ X_{14} = 5,\ X_{24} = 9,\ X_{34} = 4$

TTC : $\boldsymbol{Z = 15 * 3 + 5 * 5 + 19 * 1 + 9 * 5 + 13 * 3 + 4 * 9 = 209\$}$

(B)

Destination / Sources	D_1	D_2	D_3	Supply
S_1	10	7	8	45
S_2	15	12	9	15
S_3	7	8	12	40
Demand	25	55	20	

By least cost method

Destination / Sources	D_1	D_2	D_3	Supply	
S_1	10	7 45	8	45	0
S_2	15	12	9 15	15	0
S_3	7 25	8 10	12 5	40	15 5
Demand	25	55	20		
	0	10 0	5		

$IBFS: X_{12} = 45, X_{23} = 15, X_{31} = 25, X_{32} = 10, X_{33} = 5$

And TTC=45*7+15*9+25*7+8*10+5*12=**765**

		$V_1= 7$	$V_2= 8$	$V_3= 12$	
	Destination / Sources	D_1	D_2	D_3	Supply
$U_1= -1$	S_1	10 $\delta_{12}= -4$	- 7 45	+ 8 $\delta_{13}= 3$	45
$U_2= -3$	S_2	15 $\delta_{21}= -11$	12 $\delta_{22}= -7$	9 15	15
$U_3=0$	S_3	7 25	+ 8 10	- 12 5	40
	Demand	25	55	20	

		$V_1= 6$	$V_2= 7$	$V_3= 8$	
	Destination / Sources	D_1	D_2	D_3	Supply
$U_1= 0$	S_1	10 $\delta_{12}= -4$	7 40	8 5	45
$U_2= 1$	S_2	15 $\delta_{21}= -8$	12 $\delta_{22}= - 4$	9 15	15
$U_3= 1$	S_3	7 25	8 15	12 $\delta_{33}= -3$	40
	Demand	25	55	20	

Since all $\boldsymbol{\delta_{kj}} \leq 0$, so final optimal solution is arrived.

$$X_{12} = 40,\ X_{23} = 5,\ X_{23} = 15,\ X_{31} = 25,\ X_{32} = 15$$

The minimum total transportation cost (TTC) is

$$\boldsymbol{Z = 25 * 7 + 40 * 7 + 15 * 8 + 5 * 8 + 15 * 9 = 750\$}$$

By VAM

Destination / Sources	D1	D2	D3	Supply			Row penalty		
S1	10	7 40	8 5	45	5	0	1	1	1
S2	15	12	9 15	15			3	3	3
S3	7 25	8 15	12	40	15	0	1	4*	-
Demand	25	55	20						
	0	40	15						
		0							
column penalty	3*	1	1						
	-	1	1						
	-	5*	1						

IBFS: $X_{12} = 40,\ X_{23} = 5,\ X_{23} = 15,\ X_{31} = 25,\ X_{32} = 15$

TTC: $\boldsymbol{Z = 25 * 7 + 40 * 7 + 15 * 8 + 5 * 8 + 15 * 9 = 750\$}$

H.W **Q2:** A Company has 3 production facilities S1, S2 and S3 with production capacity of 7, 9 and 18 units (in 100's) per week of a product, respectively. These units are tobe shipped to 4 warehouses D1, D2, D3 and D4 with requirement of 5,8,7 and 14 units (in 100's) per week, respectively. The transportation costs (in rupees) per unit between factories to warehouses are given in the table below.

	D_1	D_2	D_3	D_4	Capacity
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	34

Find initial basic feasible solution for given problem by using
(a) North-West corner method
(b) Least cost method
(c) Vogel's approximation method
(d) obtain an optimal solution by MODI method if the object is to minimize the total transportation cost.