Exercise

Solve the following LPP using simplex method:

1- Max $Z = 3X_1 + 4X_2$

Subject to

$15X_1 + 10X_2 \leq 300$

$2.5X_1 + 5X_2 \leq 110$

$X_1 \geq 0, X_2 \geq 0$

Solution: (we have canonical form)

The standard form of LPP

Max $Z - 3X_1 - 4X_2 = 0$

Subject to

$15X_1 + 10X_2 + S_1 = 300$

$2.5X_1 + 5X_2 + S_2 = 110$

$X_1 \geq 0, X_2 \geq 0, S_1 \geq 0, S_2 \geq 0$

We have m= 2 and n= 4 , thus n-m=2 (Non-basic variable which equal zero)

Initial Basic Feasible Solution = (0,0,300,110)

Iteration 1

Entering Variable (pivot Colum)

Basic Variables	x_1	x_2	S_1	S_2	Solution	Ratio
Z	-3	-4	0	0	0	
S_1	15	10	1	0	300	300/10=30
S_2	2.5	5	0	1	110	110/5=22

Leaving Variable

$\frac{Row\ 3}{pivot\ element}$

Basic Variables	x_1	x_2	S_1	S_2	Solution	Ratio
Z	-3	-4	0	0	0	
S_1	15	10	1	0	300	300/10=30
x_2	0.5	1	0	0.2	22	110/5=22

Row 2 -(10) Row 3 = new Row2

Row 1 -(-4) Row 3 = new Row1

Iteration 2

Basic Variables	x_1	x_2	S_1	S_2	Solution	Ratio
Z	-1	0	0	4/5	88	
S_1	10	0	1	-2	80	80/10=8
x_2	0.5	1	0	0.2	22	22/0.5=44

Basic Variables	x_1	x_2	S_1	S_2	Solution
Z	0	0	0.1	0.6	96
x_1	1	0	0.1	-0.2	8
x_2	0	1	-0.05	0.3	18

Row 1 -(-1) (Row2 /10) = new Row1

Row 3 -(0.5) (Row2/10) = new Row3

The optimal solution: $x_1 = 8$, $x_2 = 18$, $S_1 = S_2 = 0$, $Z = 96$

2- Min $Z = -3X_1 + X_2$

Subject to

$X_1 + X_2 \leq 5$

$2X_1 + X_2 \leq 8$

$X_1 \geq 0, X_2 \geq 0$

Solution:

The standard form of LPP

Min $Z + 3X_1 - X_2 = 0$

Subject to

$X_1 + X_2 + S_1 = 5$

$2X_1 + X_2 + S_2 = 8$

$X_1 \geq 0, X_2 \geq 0, S_1 \geq 0, S_2 \geq 0$

We have m= 2 and n= 4 , thus n-m=2 (Non-basic variable which equal zero)

Initial Basic Feasible Solution = (0,0,5,8)

Basic Variables	x_1	x_2	S_1	S_2	Solution	Ratio
Z	3	-1	0	0	0	
S_1	1	1	1	0	5	5/1=5
S_2	2	1	0	1	8	8/2=4

Basic Variables	x_1	x_2	S_1	S_2	Solution
Z	0	-5/2	0	-3/2	-12
S_1	0	1/2	1	-1/2	1
x_1	1	1/2	0	1/2	4

We note all coefficient of objective function are non-positive values. Thus, the optimal solution is $x_1 = 4$, $S_1 = 1$, $x_2 = 0$, $S_2 = 0$, Z= -12

3- Max $Z = 200X_1 + 140X_2$

Subject to

$3X_1 \leq 6000$

$2.9X_2 \leq 8000$

$2.5X_1 + 2X_2 \leq 7500$

$1.3X_1 + 1.5X_2 \leq 5000$

$X_1 \geq 0, X_2 \geq 0$

Solution: (we have canonical form)

The standard form of LPP

$$\text{Max } Z - 200X_1 - 140X_2 = 0$$

Subject to

$3X_1 + S_1 = 6000$

$2.9X_2 + S_2 = 8000$

$2.5X_1 + 2X_2 + S_3 = 7500$

$1.3X_1 + 1.5X_2 + S_4 = 5000$

$X_1 \geq 0, X_2 \geq 0, S_1 \geq 0, S_2 \geq 0, S_3 \geq 0, S_4 \geq 0$

We have m= 4 and n= 6 , thus n-m=2 (Non-basic variable which equal zero)

Iteration 1								
Basic Variables	x_1	x_2	S_1	S_2	S_3	S_4	**Solution**	**Ratio**
Z	**-200**	**-140**	**0**	**0**	**0**	**0**	**0**	
S_1	**3**	**0**	**1**	**0**	**0**	**0**	**6000**	**6000/3=2000**
S_2	**0**	**2.9**	**0**	**1**	**0**	**0**	**8000**	**-----**
S_3	**2.5**	**2**	**0**	**0**	**1**	**0**	**7500**	**7500/2.5=3000**
S_4	**1.3**	**1.5**	**0**	**0**	**0**	**1**	**5000**	**5000/1.3=3846**

New pivot row= current pivot row / pivot element
All other rows
New row= (current row) - (pivot column coefficient) (New pivot row)

Row 1	Row 3	Row 4	Row 5
[-200 -140 0 0 0 0 0] **- (-200)*** [1 0 1/3 0 0 0 2000] = [0 -140 200/3 0 0 0 400000]	[0 2.9 0 1 0 0 8000] **-(0)*** [1 0 1/3 0 0 0 2000] =[0 2.9 0 1 0 0 8000]	[2.5 2 0 0 1 0 7500] **- (2.5)*** [1 0 1/3 0 0 0 2000] =[0 2 -5/6 0 1 0 2500]	[1.3 1.5 0 0 0 1 5000] **- (1.3)*** [1 0 1/3 0 0 0 2000] =[0 1.5 -13/30 0 0 1 2400]

Iteration 2								
Basic Variables	x_1	x_2	S_1	S_2	S_3	S_4	**Solution**	**Ratio**
Z	**0**	**-140**	**200/3**	**0**	**0**	**0**	**400000**	
x_1	**1**	**0**	**1/3**	**0**	**0**	**0**	**2000**	**----**
S_2	**0**	**2.9**	**0**	**1**	**0**	**0**	**8000**	**8000/2.9=2758.62**
S_3	**0**	**2**	**-5/6**	**0**	**1**	**0**	**2500**	**2500/2=1250**
S_4	**0**	**1.5**	**-13/30**	**0**	**0**	**1**	**2400**	**2400/1.5=1600**

[0 -140 200/3 0 0 0 400000] **-(-140)*** **[0 1 -5/12 0 0.5 0 1250]** = [0 0 25/3 0 70 0 575000]	[1 0 1/3 0 0 0 2000] **-(0)*** **[0 1 -5/12 0 0.5 01250]** = [1 0 1/3 0 0 0 2000]	[0 2.9 0 1 0 0 8000] **-(2.9)*** **[0 1 -5/12 0 0.5 0 1250]** =[0 0 29/24 0 -29/20 0 4375]	[0 1.5 -1.3/3 0 0 1 2400] **-(1.5)*** **[0 1 -5/12 0 0.5 0 1250]** = [0 0 23/120 0 -23/120 1]

Iteration 3							
Basic Variables	x_1	x_2	S_1	S_2	S_3	S_4	**Solution**
Z	**0**	**0**	**25/3**	**0**	**70**	**0**	**575000**
x_1	**1**	**0**	**1/3**	**0**	**0**	**0**	**2000**
S_2	**0**	**0**	**29/24**	**1**	**-29/20**	**0**	**4375**
x_2	**0**	**1**	**-5/12**	**0**	**1/2**	**0**	**1250**
S_4	**0**	**0**	**23/120**	**0**	**-23/120**	**1**	**525**

The optimal solution:

$x_1 = 2000,\ S_2 = 4375\ ,\ x_2 = 1250\ ,\ S_4 = 525\ ,$ **Z=575000**

H.W 3- Max $Z = 30X_1 + 20X_2 + 5\,X_3$

Subject to

$2X_1 + X_2 + X_3 \leq 8$

$X_1 + 3X_2 - 4X_3 \leq 8$

$X_1 \geq 0, X_2 \geq 0\ , X_3 \geq 0$

H.W 4- Max $Z = 2X_1 - X_2 + X_3$

Subject to

$2X_1 + X_2 \le 10$

$X_1 + 2X_2 - 2X_3 \le 20$

$X_2 + 2X_3 \le 5$

$X_1 \ge 0, X_2 \ge 0, X_3 \ge 0$

Solution: (we have canonical form)

The standard form of LPP

Max z

$Z - 2X_1 + X_2 - X_3 = 0$

$2X_1 + X_2 + s_1 = 10$

$X_1 + 2X_2 - 2X_3 + s_2 = 20$

$X_2 + 2X_3 + s_3 = 5$

$X_1 \ge 0, X_2 \ge 0, X_3 \ge 0, s_1, s_2 s_3 \ge 0$

We have m= 3 and n= 6 , thus n-m=3 (Non-basic variable which equal zero)

Iteration 1

Basic Variables	x_1	x_2	x_3	S_1	S_2	S_3	Solution	Ratio
Z	-2	1	-1	0	0	0	0	
S_1	2	1	0	1	0	0	10	10/2= 5
S_2	1	2	-2	0	1	0	20	20/1= 20
S_3	0	1	2	0	0	1	5	---

Iteration 2

Basic Variables	x_1	x_2	x_3	S_1	S_2	S_3	Solution	Ratio
Z	0	2	-1	1	0	0	10	
x_1	1	1/2	0	1/2	0	0	5	---
S_2	0	3/2	-2	-1/2	1	0	15	---
S_3	0	1	2	0	0	1	5	5/2 =2.5

Iteration 3

Basic Variables	x_1	x_2	x_3	S_1	S_2	S_3	Solution
Z	0	5/2	0	1	0	1/2	25/2
x_1	1	1/2	0	1/2	0	0	5
S_2	0	5/2	0	-1/2	1	1	20
x_3	0	1/2	1	0	0	1/2	5/2

The optimal solution: $Z = \frac{25}{2}, x_1 = 5, x_2 = 0, x_3 = \frac{5}{2}, s_2 = 20, s_1 = 0, s_3 = 0$

Q2: For the LP, answer the following questions?

Max $Z = 5X_1 + 4X_2$

Subject to

$6X_1 + 4X_2 \leq 24$

$X_1 + 2X_2 \leq 6$

$X_1 \geq 0, X_2 \geq 0$

a) Express the problem in equation form (standard form).
b) Determine all basic solutions and classify them as feasible and infeasible.
c) Use direct substitution in the objective function to determine the optimum basic feasible solution.
d) Verify graphically that the solution obtained in (c) is the optimum LP solution.

The standard form

Max $Z = 5X_1 + 4X_2$

Subject to

$6X_1 + 4X_2 + \boldsymbol{S_1} = 24$

$X_1 + 2X_2 + \boldsymbol{S_2} = 6$

$X_1 \geq 0, X_2 \geq 0, S_1 \geq 0, S_2 \geq 0$

Note:

(m) linear equations or basic variables; (n−m) non-basic variables

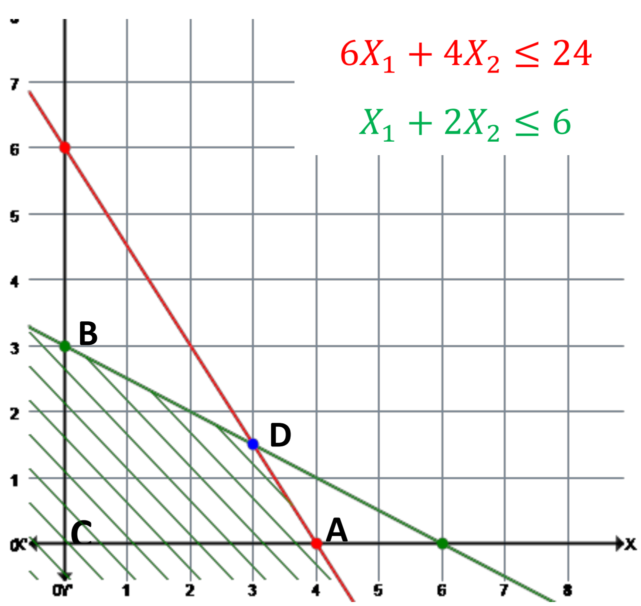
To find total number of basic solutions by use $\binom{n}{m} = nC_m$ "combinations"

All basic solutions are not necessarily feasible.

We have **m=2** constraints and **n=4** variables, thus **n-m=2** Non-basic variables (zero variables).

Total number of Basic solutions are $\binom{4}{2} = 6$

Non-basic Variables	Basic Variables & Basic Solution	Feasibility Status	Extreme point	Objective Value
S_1, S_2	$X_1 = 3, X_2 = 1.5$	Feasible	D	21
S_2, X_2	$X_1 = 6, S_1 = -12$	Infeasible		
S_1, X_2	$X_1 = 4, S_2 = 2$	Feasible	A	20
S_2, X_1	$X_2 = 3, S_1 = 12$	Feasible	B	12
S_1, X_1	$X_2 = 6, S_2 = -6$	Infeasible		
X_1, X_2	$S_1 = 24, S_2 = 6$	Feasible	C	0



1 $S_1 = S_2 = 0$

$6X_1 + 4X_2 = 24$

$-6 * \quad X_1 + 2X_2 = 6$

$(4-12)X_2 = 24 - 36$

$-8X_2 = -12$

$X_2 = 1.5$

$X_1 + 2(1.5) = 6$

$X_1 = 6 - 3 = 3$

$X_1 = 3$

2 $S_2 = X_2 = 0$

$X_1 + 2X_2 + S_2 = 6$

$X_1 = 6$

$6X_1 + 4X_2 + S_1 = 24$

$6(6) + 4(0) + S_1 = 24$

$S_1 = 24 - 36$

$S_1 = -12$

3 $S_1 = X_2 = 0$

$6X_1 + 4X_2 + S_1 = 24$

$6X_1 = 24$

$X_1 = 4$

$X_1 + 2X_2 + S_2 = 6$

$4 + 2(0) + S_2 = 6$

$S_2 = 2$

4 $S_2 = X_1 = 0$

$X_1 + 2X_2 + S_2 = 6$

$2X_2 = 6$

$X_2 = 3$

$6X_1 + 4X_2 + S_1 = 24$

$4(3) + S_1 = 24$

$S_1 = 24 - 12$

$S_1 = 12$

5 $S_1 = X_1 = 0$

$6X_1 + 4X_2 + S_1 = 24$

$4X_2 = 24$

$X_2 = 6$

$X_1 + 2X_2 + S_2 = 6$

$2(6) + S_2 = 6$

$S_2 = 6 - 12$

$S_2 = -6$

6 $X_1 = X_2 = 0$

$6X_1 + 4X_2 + S_1 = 24$

$S_1 = 24$

$X_1 + 2X_2 + S_2 = 6$

$S_2 = 6$

1- Max $Z = 3X_1 + 2X_2$ H.W

Subject to

$2X_1 + 4X_2 \leq 8$

$X_1 + X_2 \leq 2$

$X_1 \geq 0, X_2 \geq 0$

a) Express the problem in equation form.

b) Determine the all basic solutions and classify them as feasible and infeasible.

c) Use direct substitution in the objective function to determin the optimum basic feasible solution.

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