Question

Compute the number of permutations of the word "MISSISSIPPI" where no two "I"s are neighbors.

Solution

To solve this problem, we will use the gap method. This method involves two main steps: first, arranging the letters that are not "I", and second, placing the "I"s into the gaps created by those letters.

Step 1: Arrange the non-"I" letters

1. The word "MISSISSIPPI" has the following non-"I" letters:

   - 1 M
   - 4 S's
   - 2 P's

   The total number of non-"I" letters is $1 + 4 + 2 = 7$.

2. The number of permutations of these 7 letters is given by the multiset permutation formula:

$$\frac{7!}{4!2!} = \frac{5040}{(24)(2)} = \frac{5040}{48} = 105$$

   There are 105 ways to arrange the non-"I" letters.

Step 2: Place the "I"s into the gaps

1. Arranging the 7 non-"I" letters creates 8 possible gaps (including the beginning and end) where the "I"s can be placed so that they are not adjacent.

$$\_\, M \,\_\, S \,\_\, S \,\_\, S \,\_\, S \,\_\, P \,\_\, P \,\_$$

2. We have 8 gaps and need to place 4 identical "I"s. This is a combination problem, as the order of placing the identical "I"s does not matter. The number of ways to choose the locations for the "I"s is:

$$C(8,4) = \binom{8}{4} = \frac{8!}{4!(8-4)!} = \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} = 70$$

   There are 70 ways to choose the locations for the "I"s.

Step 3: Compute the total number of permutations

To find the total number of permutations where no two "I"s are neighbors, we multiply the number of ways to arrange the non-"I" letters by the number of ways to place the "I"s in the gaps.

$$\text{Total permutations} = 105 \times 70 = 7350$$

Final Answer

The number of permutations of the word "MISSISSIPPI" where no two "I"s are neighbors is 7,350.