Program 4.4
MATLAB m-file for the Linear Spline Functions
function LS=LSpline(X,Y,x)
n=length(X); for i=n-1:-1:1
$D = x - X(i)$; if ($D >= 0$); break; end; end
$D = x - X(i)$; if ($D < 0$); $i = 0$; $D = x - X(1)$; end
$M = (Y(i+1) - Y(i))/(X(i+1) - X(i))$; $LS = Y(i) + M * D$; end

4.4 Exercises

1. Use the Lagrange interpolation formula based on the points $x_0 = 0, x_1 = 1, x_2 = 2.5$ to find the equation of the quadratic polynomial to approximate $f(x) = \dfrac{2}{x+2}$ at $x = 2.3$.

2. Let $f(x) = \cos(x\pi/4)$, where x is in radian. Use the quadratic Lagrange interpolation formula based on the points $x_0 = 0, x_1 = 1, x_2 = 2$ and $x_3 = 4$ to find the polynomial $p_2(x)$ to approximate the function $f(x)$ at $x = 0.5$ and $x = 3.5$.

3. Let $f(x) = x + 2\ln(x+2)$. Use the quadratic Lagrange interpolation formula based on the points $x_0 = 0, x_1 = 1, x_2 = 2$ and $x_3 = 3$ to approximate $f(0.5)$ and $f(2.8)$. Also, compute the error bounds for your approximations.

4. Consider the function $f(x) = e^{x^2}$ and $x = 0, 0.25, 0.5, 1$. Then use the suitable Lagrange interpolating polynomial to approximate $f(0.75)$. Also, compute an error bound for your approximation.

5. Let $f(x) = x^4 - 2x + 1$. Use cubic Lagrange interpolation formula based on the points $x_0 = -1, x_1 = 0, x_2 = 2$ and $x_3 = 3$ to find the polynomial $p_3(x)$ to approximate the function $f(x)$ at $x = 1.1$. Also, compute an error bound for your approximation.

6. Construct the Lagrange interpolation polynomials for the following functions and compute the error bounds for the approximations:

   (a) $f(x) = x + 2^{x+1}$, $x_0 = 0, x_1 = 1, x_2 = 2.5, x_3 = 3$.
   (b) $f(x) = 3x^3 + 2x^2 + 1$, $x_0 = 1, x_1 = 2, x_2 = 3$.
   (c) $f(x) = \cos x - \sin x$, $x_0 = 0, x_1 = 0.25, x_2 = 0.5, x_3 = 1$.

7. Consider the following table:

x	0	1	2	3
$f(x)$	2.0	3.72	8.39	21.06

   (a) Construct divided difference table for the tabulated function.
   (b) Compute the Newton interpolating polynomials $p_2(x)$ and $p_3(x)$ at $x = 2.2$.

8. Consider the following table:

x	1	2	3	4	5
$f(x)$	3.60	1.80	1.20	0.90	0.72

(a) Construct divided difference table for the tabulated function.
(b) Compute the Newton interpolating polynomials $p_3(x)$ and $p_4(x)$ at $x = 2.5, 3.5$.

9. Consider the following table of the $f(x) = \sqrt{x}$:

x	4	5	6	7	8
$f(x)$	2.0000	2.2361	2.4495	2.6458	2.8284

(a) Construct the divided difference table for the tabulated function.
(b) Find the Newton interpolating polynomials $p_3(x)$ and $p_4(x)$ at $x = 5.9$.
(c) Compute error bounds for your approximations in part *(b)*.

10. Let $f(x) = e^x sinx$, with $x_0 = 0, x_1 = 2, x_2 = 2.5, x_3 = 4, x_4 = 4.5$. Then
(a) Construct the divided-difference table for the given data points.
(b) Find the Newton divided difference polynomials $p_2(x), p_3(x)$ and $p_4(x)$ at $x = 2.4$.
(c) Compute error bounds for your approximations in part *(b)*.
(d) Compute the actual error.

11. Show that if x_0, x_1 and x_2 are distinct then

$$f[x_0, x_1, x_2] = f[x_1, x_2, x_0] = f[x_2, x_0, x_1]$$

12. The divided difference form of the interpolating polynomial $p_3(x)$ is

$$\begin{aligned} p_3(x) &= f[x_0] + (x - x_0)f[x_0, x_1] + (x - x_0)(x - x_1)f[x_0, x_1, x_2, x_0] \\ &+ (x - x_0)(x - x_1)(x - x_2)f[x_0, x_1, x_2, x_3] \end{aligned}$$

By expressing these divided differences in terms of the function values $f(x_i)(i = 0, 1, 2, 3)$, verify that $p_3(x)$ does pass through the points $(x_i, f(x_i))(i = 0, 1, 2, 3)$.

13. Let $f(x) = x^2 + e^x$ and $x_0 = 0, x_1 = 1$. Use the divided differences to find the value of the second divided difference $f[x_0, x_1, x_0]$.

14. Which of the following functions are linear splines ?

$$(a) \qquad s(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2x - 1, & 1 \le x \le 2 \\ x + 2, & 2 \le x \le 4 \end{cases}$$

$$(b) \qquad s(x) = \begin{cases} 2 - x, & 0 \le x \le 1 \\ 2x - 1, & 1 \le x \le 2 \\ x + 1, & 2 \le x \le 4 \end{cases}$$

15. Find the linear spline which interpolates the data:

$$(0, 3.5), (1, 3.9), (2, 4.7), (3, 5.8)$$

What are its values at $x = 0.55, 1.15$ and 2.5 ?

16. Find the linear spline which interpolates the data:

$$(0, 0), (0.2, 0.18), (0.3, 0.26), (0.5, 0.41)$$

What are its values at $x = 0.15, 0.25$, and 0.45 ?

17. Find the linear splines which interpolate the following data:

$$(0, 0), (1, 1), (16, 2), (81, 3)$$

Compare interpolated values at $x = 0.5, 11.5$, and 30.5 to $f(x) = \sqrt[4]{x}$.

18. Find the linear splines which interpolate the following data:

$$(0, 1), (2, 0.9976), (3, 0.9945), (4, 0.9903)$$

Compare interpolated values at $x = 1.5, 2.5$, and 3.5 to $f(x) = \cos(2x)$.

19. Find the linear splines which interpolate the following data:

$$(0, 1), (3, 2), (8, 3), (15, 4)$$

Compare interpolated values at $x = 2.5, 5.5$, and 10.5 to $f(x) = \sqrt{x + 1}$.