College of Sciences
Department of
Statistics and Operations
Research

Probability inequalities
&
Order Statistics

STAT 415 **2^{nd} Semester, 1444**

Exercice 1 Seismic data indicate that California suffers a major earthquake on average once every 10 years. What can we say about the probability that there will be an earthquake in the next 30 years ?

Exercice 2 A factory that produces batches of 1,000 laptops each finds that on average, two laptops per batch are defective. Estimate the probability that fewer than five laptops in the next batch will be defective.

Exercice 3 Let us flip a fair coin n times. Let X_i be the indicator random variable for the event that the i-th coin flip is heads. Let $X = \sum_{i=1}^{n} X_i$ be the number of heads in the sequence of n coin flips. What can we say about the probability to obtain 80% or more heads in such a sequence of coin flips.

Exercice 4 Let $X \sim Binomial(n, p)$. Using Chebyshev's inequality, find an upper bound on $\mathbb{P}(X \geq \alpha n)$, where $p < \alpha < 1$. Evaluate the bound for $p = \frac{1}{2}$ and $\alpha = \frac{3}{4}$.

Exercice 5 Suppose that Y has the geometric distribution with parameter $p = \frac{3}{4}$. Compute the exact value and the Chebyshev bound for the probability that Y is at least 2 standard deviations away from the mean.
<u>**Note**</u> : If Y has geometric distribution, $\mathbb{E}(Y) = \frac{1}{p}$ and $V(Y) = \frac{1-p}{p^2}$.

Exercice 6 Let $Y_1 < Y_2 < \cdots < Y_n$ be the order statistics of n independent observations from a $U(0, 1)$ distribution.

1) Find the pdf of Y_1 and that of Y_n.
2) Use the results of (1) to verify that $\mathbb{E}(Y_1) = \frac{1}{n+1}$ and $\mathbb{E}(Y_n) = \frac{n}{n+1}$.
3) Show that the pdf of W_i is beta.

Exercice 7 Let $Y_1 < Y_2 < \cdots < Y_19$ be the order statistics of $n = 19$ independent observations from the exponential distribution with mean θ.

1) What is the pdf of Y_1 ?
2) sing integration, find the value of $\mathbb{E}(F(Y_1))$, where F is the cdf of the exponential distribution