College of Sciences
Department of
Statistics and Operations
Research

Functions and Transformations of Random Variables

STAT 415 **2^{nd} Semester, 1444**

Exercise 1 Suppose that $Y \sim U(-1, 1)$.

1) Find the cumulative distribution function F_U of $U = \exp(Y)$.
2) Find the probability distribution function of U.

Exercise 2 X and Y have joint pdf

$$f_{XY}(x, y) = 8xy \quad \text{for} \quad 0 < x < y < 1.$$

1) Find the pdf of $U = (U_1, U_2)$ where $U_1 = \frac{X}{Y}$ and $U_2 = Y$.
2) Find the expected value U_1U_2
3) Find the marginal probability distribution of U_1.
4) Are U_1 and U_2 independent ?

Exercise 3 Let X_1 and X_2 have independent gamma distributions with parameters α, θ and β, respectively. That is, the joint pdf of X_1 and X_2 is

$$f(x_1, x_2) = \frac{1}{\Gamma(\alpha)\Gamma(\beta)\theta^{\alpha+\beta}} x_1^{\alpha-1} x_2^{\beta-1} \exp\left(-\frac{x_1 + x_2}{\theta}\right), \quad 0 < x_1, \infty, \quad 0 < x_2 < \infty.$$

Consider

$$Y_1 = \frac{X_1}{X_1 + X_2}, \qquad Y_2 = X_1 + X_2$$

1) Find the probability distribution function of $Y = (Y_1, Y_2)$.
2) Find the marginal pdf of Y_1

Exercise 4 Given independent random variables Y and X, each with uniform distribution $U(0, 1)$.
Find the joint probability distribution of U and V defined by

$$U = X + Y, \qquad \text{and} \qquad V = X - Y$$

and the marginal pdf of U and V.

mrhaima.c@ksu.edu.sa

Exercise 5 Consider an exponential distribution with pdf

$$f(x) = \lambda e^{-\lambda x}, \qquad x > 0$$

Let $Y = X^2$.

1) Find the pdf of Y.
2) Compute the mgf of X and Y.
3) Find the mean of X.
4) Deduce the variance of X.

Exercise 6 Suppose X has a normal distribution, $N(\mu, \sigma^2)$,

1) Prove that the mgf of X is given by

$$M_X(t) = \exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right)$$

2) Find $M'_X(t)$ and $M"_X(t)$.
3) Deduce the mean and the variance of X.

Exercise 7 Suppose $Z \sim N(0,1)$ and $Y = Z^2$.

1) Find the mgf of Z.
2) Find the mgf of Y.
3) Find the mgf of χ_1^2.
4) Deduce the pdf of Y.

Exercise 8 Let X be a random variable with Characteristic function given by

$$\phi_X(t) = \exp\left(-\frac{t^2}{2}\right)$$

Using the inversion formula, find the pdf of X.

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