College of Sciences
Department of
Statistics and Operations
Research

Random Vectors and Joint Probability Distributions

STAT 415 **2^{nd} Semester, 1444**

Exercice 1 Suppose that (X,Y) has probability density function f_{XY} given by

$$f_{XY}(x,y) = 6x^2y \quad \text{for} \quad 0 \le x \le 1, \quad 0 \le y \le 1.$$

1) Find the cumulative distribution function F_{XY} of (X,Y).
2) Find the marginal cumulative distribution function of X and Y.
3) Find $\mathbb{P}(X+Y>1)$.
4) Are X and Y independent ?
5) Find the expected value of $2X+3Y$.
6) Find the probability density function of X.
7) Find the probability density function of Y.
8) Find the expected value of X.
9) Find the expected value of Y.

Exercice 2 Let f be a probability density function of (X,Y) given by

$$f_{XY}(x,y) = ae^{-x}e^{-y} \quad \text{for} \quad 0 < x < y < +\infty.$$

1) Prove that the constant $a=2$..
2) Find the probability density function of X.
3) Find the probability density function of Y.
4) Are X and Y independent ?

Exercice 3 Suppose that (X,Y) has probability density function f_{XY} given by

$$f_{XY}(x,y) = \begin{cases} k\left(\frac{1}{x^2} + y^2\right), & \text{if } \ 1 \le x \le 5 \ -1 \le y \le 1 \\ 0, & \text{elsewhere.} \end{cases}$$

1) Find the constant K.

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2) Find the probability density functions of X and Y.
3) Are X and Y independent ?
4) Compute the variances of X and Y.

Exercice 4 Find the joint probability density of the two random variables X and Y whose joint distribution is given by

$$F(x,y) = \begin{cases} \left(1 - e^{-x^2}\right)\left(1 - e^{-y^2}\right), & \text{if } x > 0, \quad y > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

Exercice 5 Let f be a function defined by with joint (pdf)

$$f_{XY}(x,y) = \begin{cases} 6xy^2, & \text{if} \quad 0 < x < 1,\ 0 < y < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

1) Prove that f is a joint (pdf) for two continuous random variables X and Y.
2) Find the cumulative distribution function of X and Y.
3) Find the marginal cumulative distribution of X and Y.
4) Are X and Y independent ?
5) Compute $\mathbb{P}(X < 0.6, Y < 0.6)$ and $\mathbb{P}(X + Y > 1.)$.

Exercice 6 Random variables X and Y have joint PDF

$$f_{XY}(x,y) = \begin{cases} \frac{5x^2}{2}, & \text{if} \quad -1 \leq x \leq 1,\ 0 \leq y \leq x^2 \\ 0, & \text{elsewhere.} \end{cases}$$

1) What are $\mathbb{E}(X)$ and $\mathbb{E}(Y)$.
2) Calculate the moments of X and Y.
3) Find $\mathbb{E}(X + Y)$

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Exercice 7 Let X and Y be two discrete random variables with the joint probability distribution

$$p(x,y) = \frac{1}{21}(x+y) \quad x = 1,2,3 \quad y = 1,2$$

Find

1) the marginal distribution of X
2) the marginal distribution of Y
3) the joint cumulative distribution function of (X,Y)
4) the expected value of X
5) the expected value of Y
6) the moments of X and Y

For y=1,2 find

1) the conditional probability mass $p_{X\backslash Y}$
2) the conditional mean $\mathbb{E}(X\backslash Y)$ and $\mathbb{E}(X^2\backslash Y = y)$ and $V(X\backslash Y = y)$
3) $\mathbb{E}(4X^3\backslash Y = 1)$
4) $\mathbb{P}(X < 3\backslash Y = 2)$

Exercice 8 Let X and Y have the pdf

$$f_{XY}(x,y) = \begin{cases} axy, & \text{if } \quad 0 < x < y < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

1) Prove that $a = 8$
2) Compute $\mathbb{E}(X^2Y)$
3) Find $f_{X\backslash Y}$
4) Compute $\mathbb{E}(X^2\backslash Y = 0.5)$

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