

Basic probability theory

STAT 415

2nd Semester, 1444

Exercise 1 A fair coin is tossed three times, and the sequence of heads (H) and tails (T) is observed. Let events A , B and C be given by

$$A = \{\text{at least 2 heads}\}, \quad B = \{\text{at most 2 heads}\}, \quad C = \{\text{head on the second toss}\}$$

- 1) Describe the sample space Ω .
- 2) Find $\mathbb{P}(A)$, $\mathbb{P}(A \cap B)$ and $\mathbb{P}(A \cup C)$.

Exercise 2 Given $\mathbb{E}(X + 4) = 10$ and $\mathbb{E}((X + 4)^2) = 116$. Find

- 1) $\mathbb{E}(X)$.
- 2) $V(X)$.
- 3) $V(X + 4)$.

Exercise 3 Assume $\mathbb{P}(A) = 0.4$, $\mathbb{P}(B) = 0.5$, and $\mathbb{P}(A \cap B) = 0.3$. Find

- 1) $\mathbb{P}(A \cup B)$
- 2) $\mathbb{P}(A \cap B^c)$
- 3) $\mathbb{P}(A^c \cup B^c)$

Exercise 4 A survey is made to determine the number of households having electric appliances in a certain city. It is found that 75% have radios (R), 65% have irons (I), 55% have electric toasters (T), 50% have (IR), 40% have (RI), 30% have (IT), and 20% have all three.

- 1) Find the probability that a household has at least one of these appliances ($\mathbb{P}(R \cup I \cup T)$).
- 2) Find $\mathbb{P}(R^C \cup T^C)$.

Exercise 5 Let X = number turning up when tossing one fair die.

- 1) Find the probability function f of X .
- 2) Find the cumulative function F of X .

Exercise 6 Let Y be a continuous random variable on the interval $(0, 1)$ with density function

$$f(x) = \begin{cases} 3x^2, & \text{for } 0 < x < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the cumulative function F of Y .

Exercise 7 If the mean and the variance of a binomial distribution are 10 and 5 respectively, then :

- 1) Determine the probability mass function.
- 2) Calculate the probability $\mathbb{P}(X = 0)$, $\mathbb{P}(X = 1)$ and $\mathbb{P}(X = 2)$.
- 3) Calculate the probability $\mathbb{P}(X \geq 0)$.

Exercise 8 Suppose that the probability that a person dies when he or she contracts a certain disease is 0.4. A sample of 10 persons who contracted this disease is randomly chosen. Find the following

- 1) The probability that exactly 4 persons will die among this sample.
- 2) The probability that less than 3 persons will die among this sample.
- 3) The probability that more than 8 persons will die among this sample.
- 4) The expected number of persons who will die in this sample.
- 5) The variance of the number of persons who will die in this sample.

Exercise 9 The proportion of time per day that all checkout counters in a supermarket are busy follows a distribution

$$f(x) = \begin{cases} kx^2(1-x)^9, & \text{for } 0 < x < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

What is the value of the constant k so that $f(x)$ is a valid probability density function?

Exercise 10 A local fraternity is conducting a raffle where 50 tickets are to be sold—one per customer. There are three prizes to be awarded. If the four organizers of the raffle each buy one ticket, what is the probability that the four organizers win

- 1) all of the prizes?
- 2) exactly two of the prizes?
- 3) exactly one of the prizes?
- 4) none of the prizes?