

KING SAUD UNIVERSITY
COLLEGE OF SCIENCES
DEPARTMENT OF MATHEMATICS

Mid-term Exam I / MATH-244 (Linear Algebra) / Semester 451

Max. Marks: 25

Max. Time: 1.5 hrs

Note: Scientific calculators are not allowed.

Question 1: [Marks: 4+2+3]:

- a) Find the reduced row echelon form of the matrix $A = \begin{bmatrix} 1 & 0 & 3 & 0 \\ -1 & 2 & -3 & -1 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$ and use it to find non-trivial solutions of the linear system $AX = O$, where $O = [0 \ 0 \ 0 \ 0]^T$.
- b) Let B be a 3×3 matrix with $\det(B) = 2$. Compute $\det(B^{-1} + \text{adj}(B))$.
- c) Let $P = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 1 & 3 & -1 \end{bmatrix}$. Compute $\text{adj}(P)$ and use it to find P^{-1} .

Question 2: [Marks: 2+3+3]:

- a) Give example of an invertible matrix A with $\text{tr}(A) = 0$.
- b) Find the values of λ for which the matrix $C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & \lambda \\ 1 & -1 & 3 - 2\lambda \end{bmatrix}$ is not invertible.
- c) Solve the matrix equation $AZ = X + Y$ for Z , where A is an invertible matrix of size 3,
 $X = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$, $Y = \begin{bmatrix} 5 \\ 0 \\ -4 \end{bmatrix}$, $AX = \frac{1}{3}X$ and $AY = \frac{1}{2}Y$.

Question 3: [Marks: 4+4]

- a) Find the values of δ for which the following linear system of equations

$$\begin{aligned} x + y + z + t &= 4 \\ x + \delta y + z + t &= 4 \\ x + y + \delta z + (3 - \delta)t &= 6 \\ 2x + 2y + 2z + (\delta - 5)t &= 6 \end{aligned}$$

has: (i) no solution (ii) infinitely many solutions.

- b) Use Cramer's rule to solve the following linear system of equations:

$$\begin{aligned} x + y &= 1 \\ x + 2y + z &= -1 \\ x + 3y - z &= 2 \end{aligned}$$