4.1

a) Plot → [ ]

b) $y_i \sim \text{Poisson}(\lambda_i) \rightarrow$

$\mu_i = E(y_i) = \lambda_i$

$\sigma^2_i = V(y_i) = \lambda_i$

$\lambda_i = i^\theta \rightarrow \ln(\lambda_i) = \theta \ln(i)$

$\ln(\lambda_i) = \theta x_i$

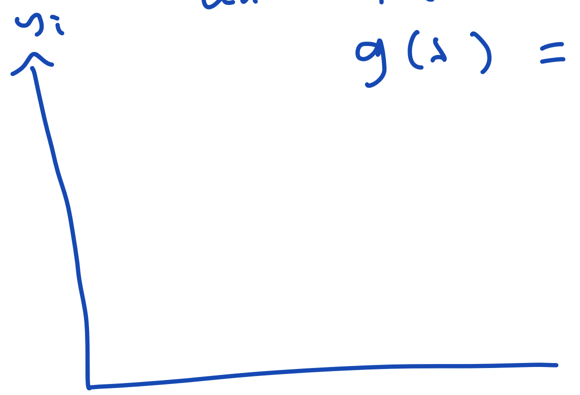
$= \eta_i$

$(\eta_i = \theta x_i)$

link function

$g(\lambda) = \ln(\lambda_i)$

i	y_i
1	1
2	6
⋮	⋮
20	159



c) fit

$g(\lambda_i) = \log \lambda_i = \beta_1 + \beta_2 x_i$

$\lambda_i = e^{\beta_1 + \beta_2 x_i}$

$\eta_i = \beta_1 + \beta_2 x_i$

$x_i = \ln(i)$

$\mu_i = E(y_i) = \lambda_i$

$\mu_i = e^{\eta_i}$

$\mu_i = e^{\beta_1 + \beta_2 \ln i}$

$\mu_i = e^{\beta_1} e^{\ln i^{\beta_2}} = e^{\beta_1} i^{\beta_2}$

$\mu_i = e^{\beta_1} i^{\beta_2}$

$\beta_0, \beta_1, \mu_i \leftarrow$ [illegible]

1) $\mu_i = e^{\eta_i} \longrightarrow \frac{\partial \mu_i}{\partial \eta_i} = e^{\eta_i}$ =

2) $W = diag\left[\frac{1}{V(y_i)} \cdot \left(\frac{\partial \mu_i}{\partial \eta_i}\right)^2\right]$

بدولة η_i

$v(y_i) = E(y_i)$

$=$

$= diag\left[\cdot \frac{1}{e^{\eta_i}} \left(e^{\eta_i}\right)^2\right]$

$= diag\left[e^{\eta_i}\right]$ نرجع نعوض

$= diag\left[e^{\beta_1} i^{\beta_2}\right]$ بدولة β_1, β_2

3) The information Matrix :-

$J = X^t W X$ -

$$X = \begin{bmatrix} 1 & \ln(1) \\ 1 & \ln(2) \\ \vdots & \vdots \\ 1 & \ln(20) \end{bmatrix}_{N\times 2,\ 20\times 2}$$

- $$X^t = \begin{bmatrix} 1 & \cdots & 1 \\ \ln(1) & \cdots & \ln(20) \end{bmatrix}_{2\times 20}$$

- $$W = \begin{bmatrix} e^{\beta_1} 1^{\beta_2} & & & 0 \\ & e^{\beta_1} 2^{\beta_2} & & \\ 0 & & \ddots & \\ & & & e^{\beta_1} (20)^{\beta_2} \end{bmatrix}_{N\times N,\ 20\times 20}$$

$$J = \begin{bmatrix} \sum e^{\beta_1} i^{\beta_2} & \sum e^{\beta_1} i^{\beta_2} \ln(i) \\ \sum e^{\beta_1} i^{\beta_2} \ln(i) & \sum e^{\beta_1} i^{\beta_2} (\ln(i))^2 \end{bmatrix}_{2\times 2}$$

$$= \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \ln(1) & \ln(2) & \cdots & \ln(20) \end{bmatrix}_{2\times 20} \times \begin{bmatrix} e^{\beta_1} 1^{\beta_2} & 0 & & \\ \vdots & e^{\beta_1} 2^{\beta_2} & & 0 \\ 0 & & \ddots & \\ & & & e^{\beta_1} N^{\beta_2} \end{bmatrix}_{20\times 20}$$

$$\times \begin{bmatrix} 1 & \ln(1) \\ 1 & \vdots \\ 1 & \\ \vdots & \ln(N) \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} e^{\beta_1} 1^{\beta_2} & e^{\beta_1} 2^{\beta_2} & \cdots & e^{\beta_1} i^{\beta_2} \\ \ln(1)\, e^{\beta_1} 1^{\beta_2} & \ln(2)\, e^{\beta_1} 2^{\beta_2} & \cdots & \ln(20)\, e^{\beta_1} (20)^{\beta_2} \end{bmatrix}_{2\times 20}$$

$$\times \begin{bmatrix} 1 & \ln(1) \\ 1 & \ln(2) \\ \vdots & \vdots \\ 1 & \ln(N) \end{bmatrix}_{20\times 2}$$

$$= \begin{bmatrix} \sum_{i=1}^{N} e^{\beta_1} i^{\beta_2} & \sum e^{\beta_1} i^{\beta_2} \ln(i) \\ \sum \ln(i)\, e^{\beta_1} i^{\beta_2} & \sum (\ln(i))^2 e^{\beta_1} i^{\beta_2} \end{bmatrix}_{2\times 2}$$

4 U Score Statistics:

$P(\lambda_i)$

$$f(y) = \frac{e^{-\lambda} \lambda^{y}}{y!}$$

$$* \quad L(\beta) = \sum_{i=1}^{N} \left[y_i \ln(\lambda_i) - \lambda - \ln(y_i) \right]$$

$$\lambda_i = \mu_i = e^{\beta_1 + \beta_2 \ln i}$$

دالة β_1, β_2

$$= \sum_{i=1}^{N} \left[y_i \ln\left(e^{\beta_1 + \beta_2 \ln i}\right) - e^{\beta_1 + \beta_2 \ln i} - \ln y_i \right]$$

$$= \sum_{i=1}^{N} \left[y_i \left(\beta_1 + \beta_2 \ln i\right) - e^{\beta_1 + \beta_2 \ln i} - \ln y_i \right]$$

$$U_1 = \frac{\partial L}{\partial \beta_1} = \sum_{i=1}^{N} \left[y_i - e^{\beta_1 + \beta_2 \ln i} \right]$$

$$U_2 \; \frac{\partial L}{\partial \beta_2} = \sum_{i=1}^{N} \left[y_i \ln(i) - e^{\beta_1 + \beta_2 \ln i} (\ln i) \right]$$

$$U = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \begin{bmatrix} \sum \left(y_i - e^{\beta_1 + \beta_2 \ln i}\right) \\ \sum \left(y_i \ln(i) - \ln(i) \, e^{\beta_1 + \beta_2 \ln i}\right) \end{bmatrix}$$

** $\mathcal{J}$ طريقتان لايجاد

Another way for find information Matrix:

$$(1) \quad V(U_1) = V\left(\sum \left(y_i - e^{\beta_1 + \beta_2 \ln i}\right)\right)$$

التباين y_i

$$= \sum_{i=1}^{20} V(y_i) = \sum e^{\beta_1 + \beta_2 \ln i}$$

$$= \sum e^{\beta_1} i^{\beta_2}$$

$$\mu = e^{\beta_1 + \beta_2 \ln i}$$

$$② \; V(u_2) = Var\left(\sum \left[y_i \, \ln(i) - \ln(i)\, e^{\beta_1 + \beta_2 \ln(i)} \right] \right)$$

$$= \sum var\left(y_i \ln(i) \right)$$

$$= \sum (\ln i)^2 \, v(y_i)$$

$$= \sum (\ln i)^2 \left[e^{\beta_1 + \beta_2 \ln i} \right]$$

$$= \sum (\ln i)^2 \, e^{\beta_1} \, i^{\beta_2}$$

$$\mathcal{J} = \begin{bmatrix} v & cov \\ cov & v \end{bmatrix}$$

$$cov(u_1, u_2) = cov\left[\sum (y_i - e^{\beta_1 + \beta_2 \ln i}), \; \sum (y_i \ln i - \ln(i)\, e^{\beta_1 + \beta_2 \ln(i)}) \right]$$

$$= \sum cov\left(y_i, \; y_i \ln(i) \right)$$

$$cov(ay, bx) = ab\, cov(y, x)$$

$$= \sum \ln(i) \; cov(y_i, y_i)$$

$$= \sum \ln(i) \; var(y_i)$$

$$= \sum \ln(i) \; e^{\beta_1 + \beta_2 \ln(i)}$$

$$= \sum \ln(i) \; e^{\beta_1} \, i^{\beta_2}$$

$$J = \begin{bmatrix} V(u_1) & cov(u_1, u_2) \\ cov(u_1, u_2) & V(u_2) \end{bmatrix}$$

$$= \begin{bmatrix} \sum e^{\beta_1} i^{\beta_2} & \sum e^{\beta_1} i^{\beta_2} \ln(i) \\ \sum e^{\beta_1} i^{\beta_2} \ln(i) & \sum e^{\beta_1} i^{\beta_2} (\ln i)^2 \end{bmatrix}_{2\times 2}$$

(5) we use the following iterative equation to find an approximate estimate of $\beta = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

- let use initial value of

$$b^{(0)} = \begin{bmatrix} b_1^{(0)} \\ b_2^{(0)} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$b^{(m+1)} = b^{(m)} + (J^{m})^{-1} U^{(m)}$$

Ste (0), $m = 0$

$\beta_1^{(0)} = 1$

$\beta_2^{(0)} = 1$

$$J^{(0)} = \begin{bmatrix} \sum_{i=1}^{20} e^{\prime} i^{\prime} & \sum e^{\prime} i^{\prime} \ln(i) \\ \sum e^{\prime} i^{\prime} \ln(i) & \sum e^{\prime} i^{(1)} [\ln(i)]^2 \end{bmatrix}$$

$i: 1 \rightarrow 20$

$$= \begin{bmatrix} 570.839 & 1439.61 \\ 1439.61 & 3768.84 \end{bmatrix}$$

$$[J]^{-1} = \begin{bmatrix} 0.04775 & -0.01824 \\ -0.01824 & 0.00723 \end{bmatrix}$$

$$U^{(0)} = \begin{bmatrix} \sum (y_i - e^{\prime} i^{\prime}) \\ \sum (y_i \ln i - \ln i \, e^{(1)} i^{(1)}) \end{bmatrix} = \begin{bmatrix} 740.1608 \\ 1956.721 \end{bmatrix}$$

$$b^{(1)} = b^{(0)} + (J^{(0)})^{-1} U^{(0)}$$

$$b_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0.04775 & -0.01824 \\ -0.01824 & 0.00723 \end{bmatrix} \begin{bmatrix} 740.1608 \\ 1956.771 \end{bmatrix}$$

$$b^{(1)} = \begin{bmatrix} 0.652354 \\ 1.651991 \end{bmatrix}$$

* Step $m = 1$)

We repet step (0) using $b_1^{(1)} = 0.652354$, $b_2^{(1)} = 1.651991$

$$b_1^{(2)} = 0.841856$$

$$b_2^{(2)} = 1.42954$$

m	0	1	2	3	4	5	6
b_1^{m}	1	0.652354	0.841856	0.98454	0.995952	0.995998	0.999929
$b_2^{(m)}$	1	1.651991	1.429548	1.33373	1.326639	1.32661	1.32661

$$b = \begin{bmatrix} 0.995998 \\ 1.32661 \end{bmatrix}$$ same

R

الاسبوع لعراهيد

Minitab

4.2 ⟷ + البرنج

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