

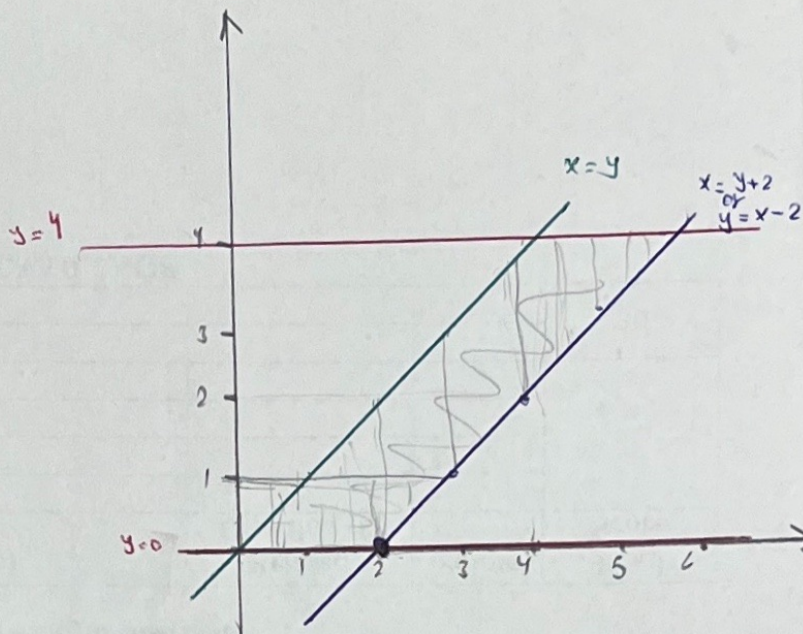
$$f(x,y) = \frac{1}{8} \quad 0 \leq y \leq 4 \quad ; \quad y \leq x \leq y+2$$

$$* f(x) = \int f(x,y) dy$$

$$= \int_0^x \frac{1}{8} dy = \frac{x}{8} \quad ; \quad 0 \leq x < 2$$

$$\int_{x-2}^x \frac{1}{8} dy = \frac{1}{4} \quad ; \quad 2 \leq x < 4$$

$$\int_{x-2}^4 \frac{1}{8} dy = \frac{6-x}{8} \quad ; \quad 4 \leq x < 6$$



$$* f(y) = \int f(x,y) dx = \int_y^{y+2} \frac{1}{8} dx = \frac{1}{4} \quad ; \quad 0 \leq y \leq 4$$

	$x = y+2$				
y	0	1	2	3	4
$x = y+2$	2	3	4	5	6

$$* h(y|x) = \frac{f(x,y)}{f(x)} = \begin{cases} \text{if } f(x) = \frac{x}{8} \Rightarrow h(y|x) = \frac{1/8}{x/8} = \frac{1}{x} \quad ; \quad 0 \leq y \leq x \\ \text{if } f(x) = \frac{1}{4} \Rightarrow h(y|x) = \frac{1/8}{1/4} = \frac{1}{2} \quad ; \quad x \leq y \leq x-2 \\ \text{if } f(x) = \frac{6-x}{8} \Rightarrow h(y|x) = \frac{1/8}{(6-x)/8} = \frac{1}{6-x} \quad ; \quad x-2 \leq y \leq 4 \end{cases}$$

$$* g(x|y) = \frac{f(x,y)}{f(y)} = \frac{1/8}{1/4} = \frac{1}{2} \quad ; \quad y \leq x \leq y+2$$

$$E[Y|x] = \int g \cdot f(y|x) dy = \begin{cases} \int_0^x \frac{1}{x} dy = \frac{y^2}{2x} \Big|_0^x = \frac{x^2}{2x} = \frac{x}{2} \quad ; \quad 0 \leq x < 2 \\ \int_x^{x-2} \frac{1}{2} dy = \frac{y^2}{4} \Big|_x^{x-2} = \frac{(x-2)^2 - x^2}{4} = 1-x \quad ; \quad 2 \leq x < 4 \\ \int_{x-2}^4 \frac{1}{6-x} dy = \frac{y^2}{2(6-x)} \Big|_{x-2}^4 = \frac{x+2}{2} \quad ; \quad 4 \leq x < 6 \end{cases}$$

$$E[X|Y] = \int x f(x|y) dx = \int_y^{y+2} \frac{x}{2} dx = \frac{x^2}{4} \Big|_y^{y+2} = \frac{(y+2)^2 - y^2}{4} = \frac{y^2 + 4y + 4 - y^2}{4} = y+1 \quad ; \quad 0 \leq y \leq 4$$