

#### THEOREM 6.4.5 Equivalent Statements

If  $A$  is an  $n \times n$  matrix, then the following statements are equivalent.

- (a)  $A$  is invertible.
- (b)  $A\mathbf{x} = \mathbf{0}$  has only the trivial solution.
- (c) The reduced row echelon form of  $A$  is  $I_n$ .
- (d)  $A$  is expressible as a product of elementary matrices.
- (e)  $A\mathbf{x} = \mathbf{b}$  is consistent for every  $n \times 1$  matrix  $\mathbf{b}$ .
- (f)  $A\mathbf{x} = \mathbf{b}$  has exactly one solution for every  $n \times 1$  matrix  $\mathbf{b}$ .
- (g)  $\det(A) \neq 0$ .
- (h) The column vectors of  $A$  are linearly independent.
- (i) The row vectors of  $A$  are linearly independent.
- (j) The column vectors of  $A$  span  $\mathbb{R}^n$ .
- (k) The row vectors of  $A$  span  $\mathbb{R}^n$ .
- (l) The column vectors of  $A$  form a basis for  $\mathbb{R}^n$ .
- (m) The row vectors of  $A$  form a basis for  $\mathbb{R}^n$ .
- (n)  $A$  has rank  $n$ .
- (o)  $A$  has nullity 0.
- (p) The orthogonal complement of the null space of  $A$  is  $\mathbb{R}^n$ .
- (q) The orthogonal complement of the row space of  $A$  is  $\{\mathbf{0}\}$ .
- (r) The kernel of  $T_A$  is  $\{\mathbf{0}\}$ .
- (s) The range of  $T_A$  is  $\mathbb{R}^n$ .
- (t)  $T_A$  is one-to-one.
- (u)  $\lambda = 0$  is not an eigenvalue of  $A$ .
- (v)  $A^T A$  is invertible.