

THEOREM 6.4.5 Equivalent Statements

If A is an $n \times n$ matrix, then the following statements are equivalent.

- (a) A is invertible.
- (b) $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.
- (c) The reduced row echelon form of A is I_n .
- (d) A is expressible as a product of elementary matrices.
- (e) $A\mathbf{x} = \mathbf{b}$ is consistent for every $n \times 1$ matrix \mathbf{b} .
- (f) $A\mathbf{x} = \mathbf{b}$ has exactly one solution for every $n \times 1$ matrix \mathbf{b} .
- (g) $\det(A) \neq 0$.
- (h) The column vectors of A are linearly independent.
- (i) The row vectors of A are linearly independent.
- (j) The column vectors of A span \mathbb{R}^n .
- (k) The row vectors of A span \mathbb{R}^n .
- (l) The column vectors of A form a basis for \mathbb{R}^n .
- (m) The row vectors of A form a basis for \mathbb{R}^n .
- (n) A has rank n .
- (o) A has nullity 0.
- (p) The orthogonal complement of the null space of A is \mathbb{R}^n .
- (q) The orthogonal complement of the row space of A is $\{\mathbf{0}\}$.
- (r) The kernel of T_A is $\{\mathbf{0}\}$.
- (s) The range of T_A is \mathbb{R}^n .
- (t) T_A is one-to-one.
- (u) $\lambda = 0$ is not an eigenvalue of A .
- (v) $A^T A$ is invertible.