Equivalence and indifference

▢ Equivalence

Two or more cash flow profiles are equivalent if their time value of money worth at a common point in time are equal.

(time value of money)₁ = (time value of money)₂

$$P_{w1} = P_{w2}$$
$$F_{w1} = F_{w2}$$
$$A_{w1} = A_{w2}$$

▢ Indifference

Potential investor is indifferent between two or more cash flow profiles if they are equivalent.
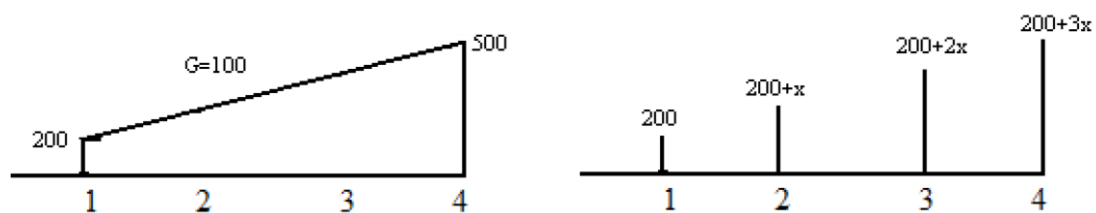

$P_{w1} = P_{w2}$

Ex.5

Determine value of X if two cash flows are equivalent at 10% compounded annually

EOY	Cash Flow A	Cash Flow B
0	0	0
1	200	200
2	300	200+x
3	400	200+2x
4	500	200+3x

Solution

$$Pw = 200(P/A\ 10,4) + 100(P/G10,4)$$
$$= 200(3.16987) + 100(4.37812)$$
$$= \$ 1071.786$$

$$P_W = 200(P/A10,4) + X(P/G10,4)$$
$$= 200(3.16987) + X(4.37812)$$
$$= \$ 633.9746 + 4.37812X$$

$$P_{w1} = P_{w2}$$
$$1071.786 = 633.9746 + 4.37812X \implies X = 100$$

Ex.7 (43/189)

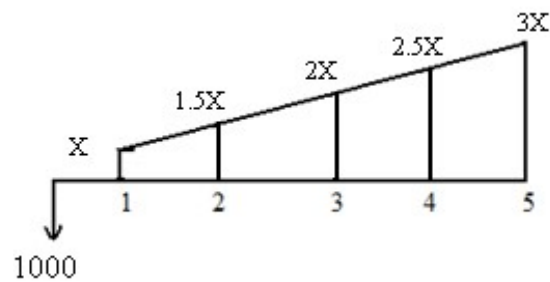Consider the following two cash flow series

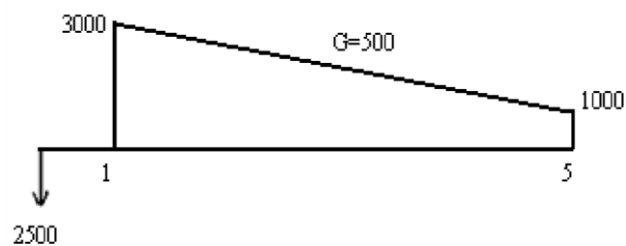| EOY	Cash Flow Series A	Cash Flow Series B
0	-1000	-2500
1	X	3000
2	1.5X	2500
3	2X	2000
4	2.5X	1500
5	3X	1000

Determine the value of X if two cash flows are equivalent at an interest rate of 15 percent per year compounded annually.

Solution

$P_{W A}$ = -1,000+X (P|A 15%,5)+0.5X (P|G 15%,5) =-$1,000+6.239730X

$P_{W B}$ = -2,500+3,000 (P|A 15%,5)- 500 (P|G 15%,5)= $4,668.91

$$P_{WB} = P_{WA}$$

$$\$4,668.91=-\$1,000+6.239730X$$
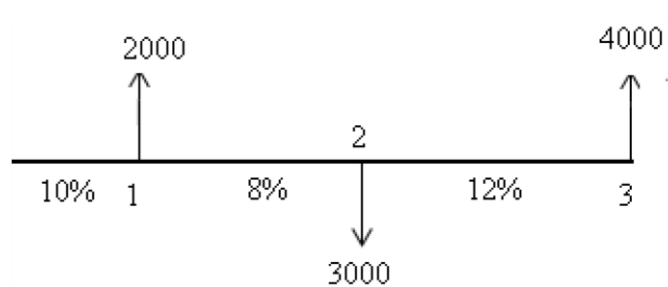
x = \$908.52

Variable interest rate

Ex.6 (60/191)

Consider the following cash flow and interest rates:

EOY	Interest Rate During period	Cash Flow at end of period
0		$0
1	10%	$2000
2	8%	-$3000
3	12%	$4000

a. Determine the present worth of this series of cash flow.
b. Determine the future worth of this series of cash flow.
c. Determine the annual worth of this series of cash flow.

Solution

a)

P_w =2,000 (P|F 10%,1)- 3,000 (P|F 10%,1) (P|F 8%,1)+ 4,000 (P|F 10,1) (P|F 8%,1) (P|F 12%,1)= $ 2,299.19

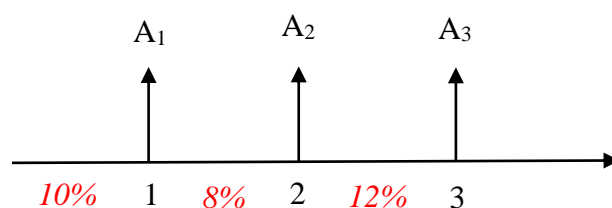P_w =2,000 (0.90909)- 3,000 (0.90909) (0.92593)+ 4,000 (0.90909) (0.92593) (0.89286)= $ 2,299.19

b)

F_W= 2,000 (F|P 8%,1) (F|P 12%,1)- 3,000× (F|P 12%,1)+ 4,000= $3,059.20

F_W= 2,000 (1.08000) (1.12000)- 3,000× (1.12000)+ 4,000= $3,059.20

c)

$P_w = A_1 (P|F\ 10\%,1) + A_2 (P|F\ 10\%,1) (P|F\ 8\%,1) + A_3 (P|F\ 10,1) (P|F\ 8\%,1) (P|F\ 12\%,1)$

$\$\ 2,299.19 = A_1 (0.90909) + A_2 (0.90909) (0.92593) + A_3 (0.90909) (0.92593) (0.89286)$

$A_1 = A_2 = A_3 = A \qquad \Longrightarrow \qquad A = \$\ 918.8$