# GE 403 <br> Engineering Economy 

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## Equivalence \& Indifference

- Two cash flow streams are said to be equivalent at $k \%$ interest if and only if their present worth are equal at $\mathrm{k} \%$ interest.
- OR Two cash flow profiles are equivalent if their time value of money worth at a common point in time are equal.
- In engineering economic analyses, indifference means "to have no preference."
- Specifically, a potential investor is indifferent between two (or more) cash flow profiles if they are equivalent.

Ex. Determine the value of X if the two cash flows are equivalent at $10 \%$ compounded annually .

| EOY | Cash Flow A | Cash Flow B |
| :---: | :---: | :---: |
| 0 | $\$ 0$ | $\$ 0$ |
| 1 | $\$ 200$ | $\$ 200$ |
| 2 | $\$ 300$ | $\$ 200+\mathrm{X}$ |
| 3 | $\$ 400$ | $\$ 200+2 \mathrm{X}$ |
| 4 | $\$ 500$ | $\$ 200+3 \mathrm{X}$ |

## Solution


$\mathrm{Pw}_{\mathrm{A}}=200(\mathrm{P} / \mathrm{A} 10 \%, 4)+100(\mathrm{P} / \mathrm{G} 10 \%, 4)$
$\mathrm{Pw}_{\mathrm{A}}=200(3.16987)+100(4.37812)$
$\mathrm{Pw}_{\mathrm{B}}=200(\mathrm{P} / \mathrm{A} 10 \%, 4)+\mathrm{X}(\mathrm{P} / \mathrm{G} 10 \%, 4)$
$\mathrm{Pw}_{\mathrm{A}}=\$ 1071.786$
$\mathrm{Pw}_{\mathrm{B}}=200(3.16987)+\mathrm{X}(4.37812)$
$\mathrm{Pw}_{\mathrm{B}}=\$ 633.974+4.37812 \mathrm{X}$

$$
\begin{gathered}
\mathrm{Pw}_{\mathrm{A}}=\mathrm{Pw}_{\mathrm{B}} \\
1071.786=633.974+4.37812 \mathrm{X} \quad \longrightarrow \mathrm{X}=\$ 100
\end{gathered}
$$

Ex. Consider the following two cash flow series

| EOY | Cash Flow A | Cash Flow B |
| :---: | :---: | :---: |
| 0 | $-\$ 1000$ | $-\$ 2500$ |
| 1 | X | $\$ 3000$ |
| 2 | 1.5 X | $\$ 2500$ |
| 3 | 2 X | $\$ 2000$ |
| 4 | 2.5 X | $\$ 1500$ |
| 5 | 3 X | $\$ 1000$ |

Determine the value of X if two cash flows are equivalent at an interest rate of 15 percent per year compounded annually.

## Solution



$$
\begin{array}{ll}
\mathrm{Pw}_{\mathrm{A}}=-1,000+\mathrm{X}(\mathrm{P} \mid \mathrm{A} 15 \%, 5)+0.5 \mathrm{X}(\mathrm{P} \mid \mathrm{G} 15 \%, 5) & \mathrm{Pw}_{\mathrm{B}}=-2,500+3,000(\mathrm{P} \mid \mathrm{A} 15 \%, 5)-500(\mathrm{P} \mid \mathrm{G} 15 \%, 5) \\
\mathrm{Pw}_{\mathrm{A}}=-\$ 1,000+6.239730 \mathrm{X} & \mathrm{Pw}_{\mathrm{B}}=\$ 4,668.91
\end{array}
$$

$$
\mathrm{Pw}_{\mathrm{A}}=\mathrm{Pw}_{\mathrm{B}}
$$

$$
-\$ 1,000+6.239730 \mathrm{X}=\$ 4,668.91 \quad \mathrm{X}=\$ 908.5
$$

## Variable Interest Rates

Consider the following cash flows and interest rates:

| End of Year | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cash Flow at End of Period | $\$ 0$ | $\$ 2000$ | $-\$ 3000$ | $\$ 0$ | $\$ 0$ | $\$ 0$ | $\$ 4000$ |
| Interest Rate during Period |  | $10 \%$ | $8 \%$ | $12 \%$ | $12 \%$ | $12 \%$ | $12 \%$ |

- Determine the present worth of this series of cash flows.
- Determine the future worth of this series of cash flows.
- Determine a 6-year uniform annual series that is equivalent to the original series.


## Solution

Pw


Pw =2,000 (P|F 10\%,1)- 3,000 (P|F 10\%,1) (P|F 8\%,1)+ 4,000 (P|F 10\%,1) (P|F $8 \%, 1$ ) (P|F 12\%,4)
$\mathrm{Pw}=2,000 \quad(0.90909)-\quad 3,000 \quad(0.90909) \quad(0.92593)+\quad 4,000 \quad$ (0.90909) $(0.92593)(0.63552)=\$ 1,432.72$

## Solution

Fw

$\mathrm{FW}=2,000(\mathrm{~F} \mid \mathrm{P} 8 \%, 1)(\mathrm{F} \mid \mathrm{P} \mathrm{12} \mathrm{\%,4)-3,000} \mathrm{\times(F\mid P12} \mathrm{\%,4)+4,000}$ $\mathrm{FW}=2,000$ (1.08000) (1.57352)-3,000×(1.57352)+4,000=\$2,678.24 OR
$\mathrm{FW}=\mathrm{Pw}(\mathrm{F} \mid \mathrm{P} 10 \%, 1)(\mathrm{F} \mid \mathrm{P} 8 \%, 1)(\mathrm{F} \mid \mathrm{P} 12 \%, 4)=\$ 2,678.24$

## Solution

Aw

$\mathrm{Pw}=\mathrm{A}_{1}(\mathrm{P} \mid \mathrm{F} \quad 10 \%, 1)+\mathrm{A}_{2}(\mathrm{P} \mid \mathrm{F} 10 \%, 1)(\mathrm{P} \mid \mathrm{F} 8 \%, 1)+\mathrm{A}(\mathrm{P} \mid \mathrm{A} 12 \%, 4)(\mathrm{P} \mid \mathrm{F} 10 \%, 1)$ (P|F 8\%,1)
$\$ 1,432.72=\mathrm{A}(0.90909)+\mathrm{A}(0.90909)(0.92593)+\mathrm{A}(3.03735)(0.90909)(0.92593)$
$\mathrm{A}=\$ 332.6$

