Chapter 6: Electromagnetic Induction

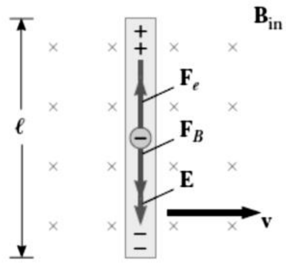
6.1 Motional electromotive force

As discussed in chapter five, when a conducting wire is moving with constant velocity in a direction perpendicular to the direction of a uniform magnetic field, as illustrated in the following figure, the charges in the wire experience a magnetic force ($\mathbf{F}_{mag} = q\mathbf{v}\times\mathbf{B}$) and this will create an electric field E inside the wire, and at equilibrium:

$$qE = qvB$$

$$E = vB$$

$$\Rightarrow \mathbf{E} = \mathbf{v}\times\mathbf{B}$$

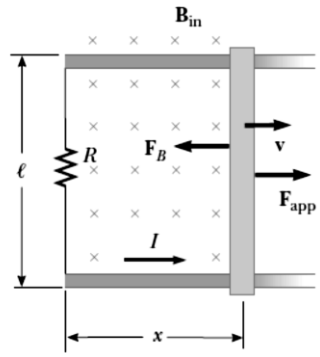


And the potential difference ΔV across the ends of the conductor is:

$$\Delta \mathbf{V} = \mathbf{E}\boldsymbol{l} = \mathbf{vB}\boldsymbol{l}$$

What will happen if the moving conductor is part of a closed conducting path?

Consider the following circuit which consists of a conducting bar of length l sliding along two fixed parallel conducting rails and a uniform magnetic field is applied perpendicular to the plane of the circuit. Assume the bar has zero resistance.



When an applied force F_{app} moves the bar to the right with a velocity v, the free charges in the wire experience a magnetic force directed along the length of the bar. This force creates motional electromotive force that causes an induced a counterclockwise current I in the loop.

The induced motional emf across the moving bar are proportional to the change in area of the circuit (the rate of change of magnetic flux through the circuit).

Also due to the induced current in the loop, there will be a magnetic force on the bar carrying this current and this magnetic force opposes its motion:
Thus, work is needed to keep the bar moving to the right

$$dW = Fdx = Fvdt = ILBvdt$$

$$dW = LBvdq$$

$$\Rightarrow \varepsilon = dW/dq = BLv$$

which is same as the potential difference ΔV across the ends of the conductor that we discussed above

In general

$$\varepsilon = \int (v \times B) \cdot dL$$

6.2 Faraday's Law

From the previous section if we move the wire to the right with dx, the change in the magnetic flux $d\Phi_B$ will be:

$$d\Phi_B = Bda = Bldx$$

$$\frac{d\Phi_B}{dt} = Bl\frac{dx}{dt}$$

$$\frac{d\Phi_B}{dt} = Blv$$

From the previous section

$$\varepsilon = BLv$$

This gives us

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

This is the flux rule for motional emf which is called the **Faraday induction law**

It shows that when the magnetic flux through the loop changes with time, an electromotive force is induced.

And the minus sign comes from **Lenz's law** which states that the induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop.

Remember **the magnetic flux** can be changed with time by:

- Changing the magnitude of B with time.
- Changing the area enclosed by the loop with time.
- Changing the angle θ between B and A (area : the normal to the loop) with time.

From Faraday's law: changing magnetic field induces an electric field

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}$$

then E is related to the change in B by

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{a}$$

We have used the partial derivative of B because we now require the rate of change of B with time at a fixed point.

This is Faraday's law, in integral form. We can convert it to differential form by applying Stokes' theorem:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

This is one of the four Maxwell equations.

Faraday's law reduces to the old rule:

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0$$

(or, in differential form, $\nabla \times E = 0$) in the static case (constant B)

Example 6.1

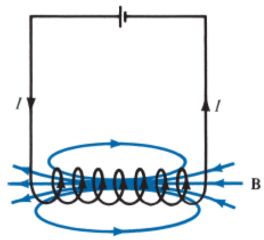
A coil consists of 200 turns of wire. Each turn is a square of side 18 cm, and a uniform magnetic field is directed perpendicular to the plane of the coil. If the field changes linearly from 0 to 0.5 T in 0.8 s, what is the magnitude of the induced emf in the coil while the field is changing?

$$\mathcal{E} = -N\frac{d\Phi}{dt} \Rightarrow |\mathcal{E}| = N\frac{\Delta\Phi}{\Delta t} = NA\frac{\Delta B}{\Delta t}$$

$$\mathcal{E} = 200\,(18\times10^{-2})^2\,\frac{(0.5-0)}{0.8} = 4\ \text{V}$$

6.3 Inductors and Inductances

From the previous chapter we found that a circuit (or closed conducting path) carrying current *I* produces a magnetic field that causes a magnetic flux Φ_B to pass through each turn of the circuit as shown in the figure below.



The magnetic flux is proportional to B, which in turn is proportional to the current in the circuit.

$$\Phi_B \propto B$$
$$B \propto I$$
$$\Rightarrow \Phi_B = k\, I$$

where k is the proportionality constant

Therefore, from Faraday's law when there are N turns a self-induced emf is always proportional to the time rate of change of the current.

$$\varepsilon_L = -N\frac{d\Phi_B}{dt} = -Nk\frac{dI}{dt}$$

If L=kN

$$\varepsilon_L = -L\frac{dI}{dt}$$

where L is a constant of proportionality called the **inductance** of the circuit (commonly referred to as **self-inductance**)

The inductance L of an inductor is the ratio of the magnetic flux to the current *I* through the inductor.

$$L = \frac{N\Phi_B}{I}$$

And L is a property of the physical arrangement of the circuit.

A circuit or part of a circuit that has inductance is called an **inductor**.

The unit of inductance is the henry (H), which is the same as webers per ampere

or $1\text{H} = 1\,\frac{\text{V.s}}{\text{A}}$

Example 6.2

Consider a uniformly wound solenoid having N turns and length l. Assume l is much longer than the radius of the windings and the core of the solenoid is air.

a) Find the inductance of the solenoid.
b) Calculate the inductance of the solenoid if it contains 300 turns, its length is 25 cm, and its cross-sectional area is 4cm^2.
c) Calculate the self-induced emf in the solenoid if the current it carries decreases at the rate of 50 A/s.

a) From chapter 5

$$B = \mu_0 \frac{N}{l} I \Rightarrow \Phi_B = \frac{\mu_0 N I A}{l}$$

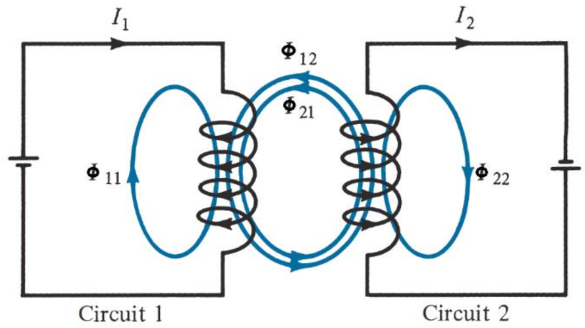
$$L = \frac{N \Phi_B}{I} \Rightarrow L = \mu_0 \frac{N^2}{l} A$$

b) $$L = \frac{4\pi \times 10^{-7} (300)^2 (4 \times 10^{-4})}{25 \times 10^{-2}} = 181\ \mu H$$

c) $$\varepsilon = -L \frac{dI}{dt} = -(181 \times 10^{-6})(-50) = 9\ mV$$

6.4 Mutual Inductance

If instead of having a single circuit, we have two circuits carrying current I_1 and I_2 as shown in the figure below, a magnetic interaction exists between the circuits. Four component fluxes Φ_{11}, Φ_{12}, Φ_{21}, Φ_{22} are produced.



Let us take Φ_{12} which is the flux passing through circuit 1 due to current I_2 in circuit 2.
the mutual inductance M_{12} is:

$$M_{12} = \frac{N_1 \Phi_{12}}{I_2}$$

Similarly

$$M_{21} = \frac{N_2 \Phi_{21}}{I_1}$$