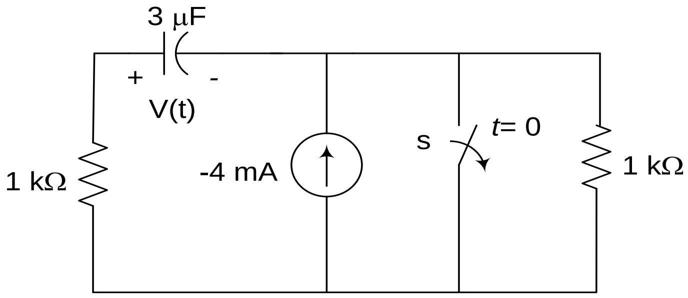
King Saud University
College of Engineering
Dept. of Electrical Engineerin

EE-212: ***Electric Circuits***
Mid-Term Exam (*1*)
Solution

<u>Question #1</u> [10 marks]

In the circuit shown in Fig.1 the switch has been closed for a long time. At t = 0 it is opened. Determine an expression for v(t) for t ≥ 0 and draw a sketch to show its variation with time.

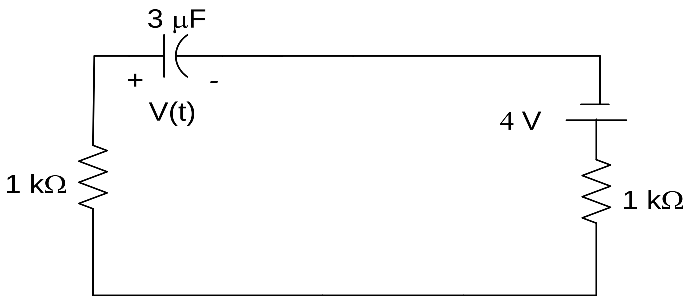


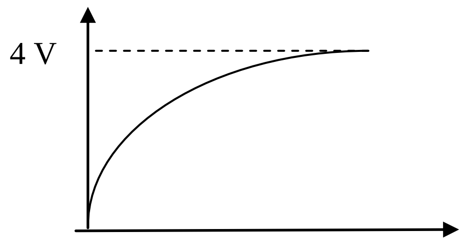
(Fig.1)

<u>Solution:</u>

With switch closed the current source is shorted and hence $V(0^-) = 0$.

With switch S open, converting current source to voltage source V(∞) = 4 V.





$\tau = RC = 6\times10^{-3}$ S.

$V(t) = 4 - 4e^{-166.67t}$

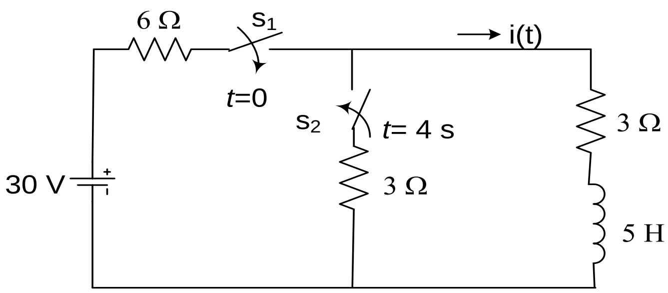
<u>Question #2</u> [15 marks]

In the circuit shown in Fig.2 both switches have been open for a long time. Switch S_1 is closed first at t=0 and then switch S2 is closed at t= 4 seconds later. Both switches remain closed.

a) Determine an expression for i(t) for 0 ≤ t ≤ 4s and for 4s ≤ t ≤ ∞.

b) Find the magnitude of current i at t= 2 s and t = 5s.

c) Draw a sketch to show variation of i(t) with time for 0 ≤ t ≤ ∞.



(Fig.2)

Solution:

a) With switch S1 closed,

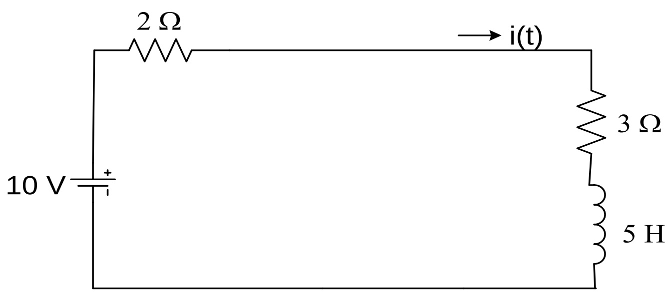
IV =0, FV= 30/(6+3) = 3.33 A Therefore I(t) = 3.33 – 3.33 $e^{-t/\tau}$ where τ = L/R = 5/9 =0.55 S.

Therefore the expression for current is

$$i(t) = 3.33 - 3.33\, e^{-1.8t} \qquad 0 \le t \le 4$$

When switch S_2 closed at t=4 S the value of i(4) is calculated by substituting t= 4S in the expression for i(t) to get i(4) = 3.33 – 3.33e-1.8×4 = 3.33 A, (approximately). This will be the initial value when switch S_2 is closed at t=4S.

By obtaining Thevenin equivalent circuit, the circuit becomes series as shown below



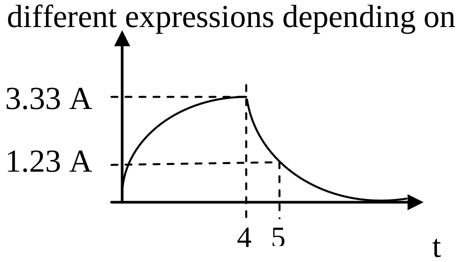
The final value of current is 10/(2+3) =2A, and τ = 5/5 = 1 S. The expression for current is

$$i(t) = 3.33e^{-(t-4)/\tau} = 3.33e^{-(t-4)} \qquad 4 \le t \le \infty$$

Notice since 4s of time is already passed we subtract it from t in the expression.

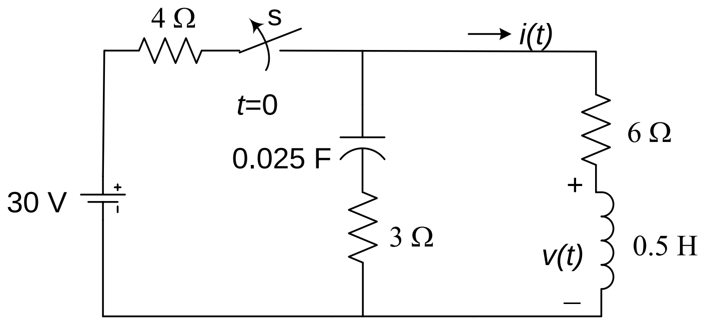
b) At t= 2, i= 3.33 – 3.33 $e^{-1.8\times 2}$ = 3.3 A and At t = 5, i= 3.33 $e^{-(5-4)}$ = 1.23 A

Notice we substitute in different expressions depending on the value of t.



<u>Question # 3</u> [20 marks]

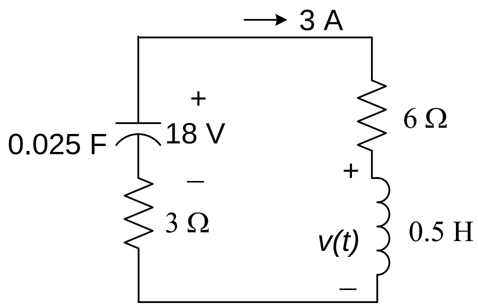
In the circuit shown (**Fig.3**), switch S has been closed for a long time. At t=0, it is opened. Find expression for *i(t)* and *v(t)*.



(Fig.3)

<u>Solution:</u>

With switch S closed for a long time the capacitor is open and fully charged 18 V (6Ω and 4Ω voltage divider). The inductor current will be 30/(4+6) = 3 A. These will be the initial values when switch is open. When switch is open the circuit is series.



Series circuit with initial conditions

The differential equation for this circuit from VR + VL + VC =0 is:

$$\frac{d^2I(t)}{d^2t}+\frac{R}{L}\frac{dI(t)}{dt}+\frac{1}{LC}I(t)=0$$

$$\frac{d^2I(t)}{d^2t}+18\frac{dI(t)}{dt}+180I(t)=0$$

$$m^2+18m+180=0$$

$$(m+10)(m+8)=0$$

$$m=10, m=8$$

Assume solution I(t) = $Ae^{-10t} + Be^{-8t}$

Applying initial conditions,
I(0) = 3= A+B

$$\frac{dI(t)}{dt} = -10Ae^{-10t} - 8Be^{-8t}$$ =-10A - 8B at t=0.

But $\frac{dI(t)}{dt}$ at t=0 is equal to $V_L(0)/L$

From the series circuit $V_L(0^+)$ = 18 - 3×6 -3×3 = -9V. Therefore $\frac{dI(t)}{dt}$ = -9/0.5 = -18

Therefore -10A -8B = -18

But A+ B = 3. Therefore A= -3 and B= 6. Therefore I(t) = $6e^{-8t} - 3e^{-10t}$

V(t) is across the inductor, therefore V(t) = L dI(t)/dt = 0.5× ($-48\ e^{-8t} + 30e^{-10t}$)

V(t) = $15e^{-10t} - 24e^{-8t}$

Check: $V_L(0)$ = 15 – 24 = -9