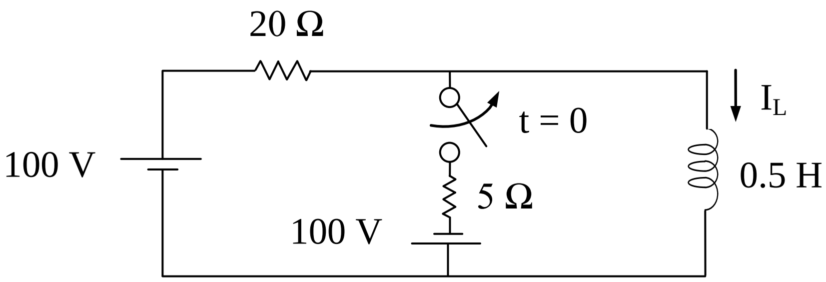
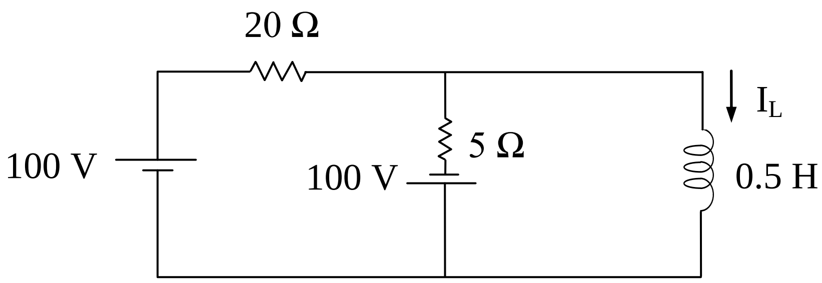
Question #1



a) t < 0 (switch closed)



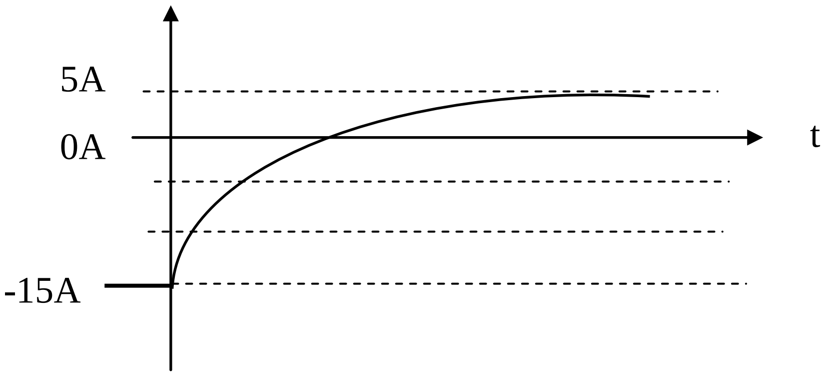
$$I_L = \frac{100}{20} - \frac{100}{5} = 5 - 20 = -15A \qquad \therefore I_L(0^-) = -15A$$

b) $I_L(0^+) = I_L(0^-) = -15A$

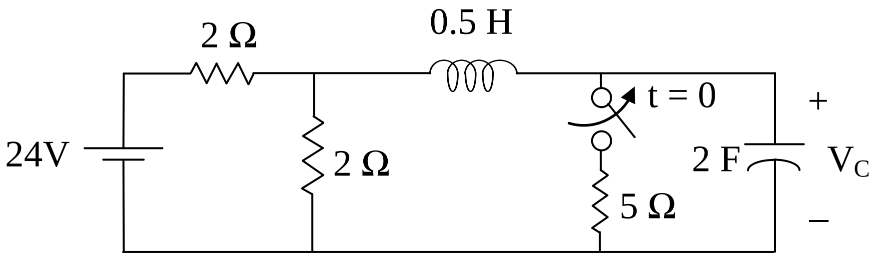
c) $I_L(\infty) = \frac{100}{20} = 5A$

d) $\tau = \frac{L}{R} = \frac{0.5}{20} = 0.025s\,, \quad \frac{1}{\tau} = 40$

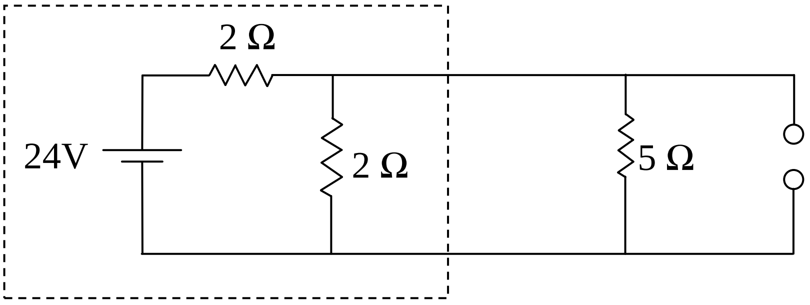
$$I_L(t) = I_l(\infty) + \left[I_L(0) - I_L(\infty)\right]e^{-40t} \quad = 5 + \left[-15 - 5\right]e^{-40t} \quad = 5 - 20\,e^{-40t}$$

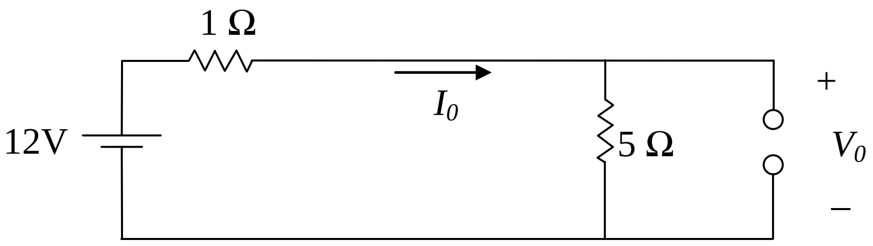


Question #2



t < 0 (switch closed), *L*= short, *C*= open.





$$I_0 = \frac{12}{1+5} = 2\ A. \qquad V_0 = 2\times 5 = 10\,V.$$

t > 0 (switch opened):

$$V_C(\infty) = 12\,V. \qquad I_L(0^+) = I_L(0^-) = 2\ A. = I_C(0^+)$$

$$\alpha = \frac{R}{2L} = 1,\ \omega^2{}_0 = \frac{1}{LC} = 1 \qquad \therefore$$ <u>Critical damping step response</u>

$$V_C(t) = e^{-t}\left[A_1 + A_2 t\right] + V_C(\infty)$$

At t=0 $V_C(0^+) = 10 \qquad \therefore 10 = A_1 + 0 + 12 \qquad A_1 = -2$

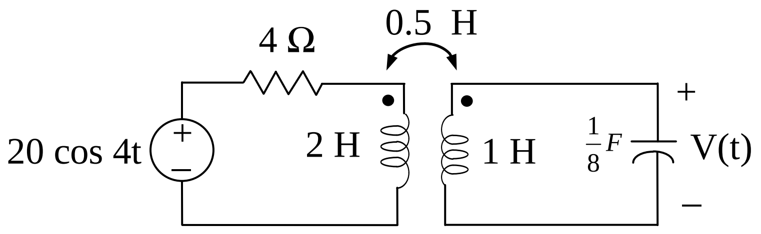
$$\frac{dV_C}{dt} = \frac{I_C(0^+)}{C} = \frac{2}{2} = 1$$

$$\frac{dV_C}{dt} = -A_1 e^{-t} + A_2 e^{-t} + A_2 t\,(-e^{-t}) + 0 = -A_1 + A_2 = 2 + A_2 = 1$$ at t = 0. $\therefore A_2 = -1$

$$\therefore V_C(t) = 12 - 2e^{-t} - t\,e^{-t}$$

Question #3

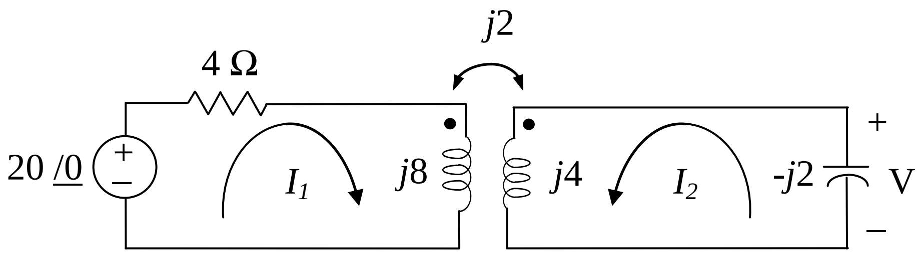
A)



$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.5}{\sqrt{2 \times 1}} = 0.3535$$

$\omega = 4$ rad/s.

$j\omega L_1 = j8\,\Omega$, $j\omega L_2 = j4\,\Omega$, $j\omega M = j2\,\Omega$, and $1/j\omega C = -j2\,\Omega$



Loop1: $20\underline{/0} = (4 + j8)I_1 + j2I_2$ (1)

Loop2: $0 = (j4 - j2)I_2 + j2I_1$ (2)

From Equ.(2) $I_1 = -I_2$. Substitute in Equ. 1 $\therefore$ $20\underline{/0} = -(4 + j8)I_2 + j2I_2 = -(4 + j6)I_2$

$20\underline{/180} = (4 + j6)I_2$, $I_2 = 2.774\ \underline{/123.7^o}$ A.

$V = -j2(-I_2) = j2I_2 = 5.547\underline{/213.7^o}$ V.

$V(t) = 5.547\cos(4t + 213.7^o)$ V.

B) For the following Laplace transform functions $F(s)$ find the corresponding $f(t)$

i) $$F(s) = \frac{s+3}{(s+1)(s+2)} = \frac{k_1}{s+1} + \frac{k_2}{s+2}$$

$k_1 = \frac{s+3}{s+2}\Big|s = -1$ $\quad k_1 = \frac{-1+3}{-1+2} = \frac{2}{1} = 2$

$k_2 = \frac{s+3}{s+1}\Big|s = -2$ $\quad k_2 = \frac{-2+3}{-2+1} = \frac{1}{-1} = -1$

$f(t) = (2e^{-t} - e^{-2t})\,u(t)$

ii) $$F(s)=\frac{4}{(s+2)(s+1)^2}=\frac{k_1}{s+2}+\frac{k_2}{(s+1)^2}+\frac{k_3}{s+1}$$

$$k_1=\frac{4}{(s+1)^2}\Big|s=-2 \qquad k_1=\frac{4}{(-2+1)^2}=\frac{4}{(-1)^2}=4$$

$$k_2=\frac{4}{(s+2)}\Big|s=-1 \qquad k_2=\frac{4}{(-1+2)}=4$$

$$k_3=\frac{d}{ds}\left(\frac{4}{s+2}\right)\Bigg|s=-1 \qquad k_3=\frac{4\times-1}{(s+2)^2}\Bigg|s=-1 \qquad k_3=\frac{-4}{(-1+2)^2}=-4$$

$$f(t)=(4e^{-2t}+4te^{-t}-4e^{-t})u(t)$$

iii) $$F(s)=\frac{10}{(s+3)(s^2+6s+10)}=\frac{k_1}{s+3}+\frac{k_2}{s+3-j}+\frac{k_3}{s+3+j}$$

$$k_1=\frac{10}{s^2+6s+10}\Big|s=-3 \qquad k_1=\frac{10}{9-18+10}=10$$

$$k_2=\frac{10}{(s+3)(s+3+j)}\Bigg|s=-3+j \quad k_2=\frac{10}{(-3+j+3)(-3+j+3+j)}=\frac{10}{(j)(2j)}=\frac{10}{-2}=-5$$

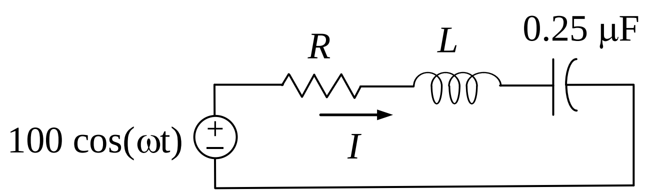
$$k_3=\frac{10}{(s+3)(s+3-j)}\Bigg|s=-3-j \quad k_3=\frac{10}{(-3-j+3)(-3-j+3-j)}=\frac{10}{(-j)(-2j)}=\frac{10}{-2}=-5$$

$$f(t)=(10e^{-3t}-5e^{-(3+j)t}-5e^{-(3-j)t})u(t)$$

$$f(t)=(10e^{-3t}-10e^{-3t}\cos t)u(t)$$

Question #4

A)



At $\omega = \omega_0$ (Resonance) , $I = 1.0\cos(\omega_0 t)$ $Z_T = 100\underline{/0}$ $\therefore R = 100\ \Omega$

If $\omega = 5000$ rad/s I = $(1/\sqrt{2})\cos(5000 - 45^o)$ A. $\therefore Z_T = 100\sqrt{2}\underline{/45^o}\ \Omega$

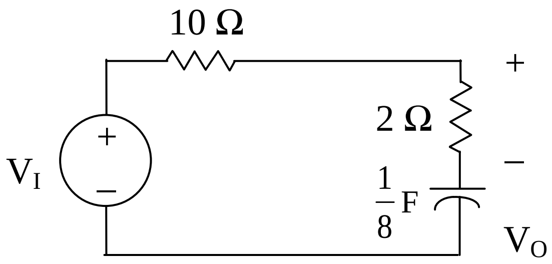
$Z_T = 100 + j100 \therefore \omega_{c2} = 5000$ rad/s. $X_C = -\dfrac{10^6}{5000\times 0.25} = -800\ \Omega \therefore X_L = 900\ \Omega$

$X_T = 900 - 800 = 100\ \Omega$ $\qquad L = \dfrac{900}{5000} = 0.18\ \text{H}$

$\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{0.18\times 0.25\times 10^{-6}}} = 4714.$ rad/s $= \sqrt{\omega_{c1}\omega_{c2}}$ $\qquad \omega_{c1} = \dfrac{(4714)^2}{5000} = 4444$ rad/s.

$B = \omega_{c2} - \omega_{c1} = 5000 - 444 = 556$ rad/s. $\qquad Q = \dfrac{\omega_0}{B} = \dfrac{4714}{556} = 8.5$

B)



$$H(s)=\frac{V_O(s)}{V_I(s)}=\frac{2}{12+\frac{8}{s}}=\frac{2s}{12s+8}=\frac{\frac{1}{6}s}{s+\frac{2}{3}}$$

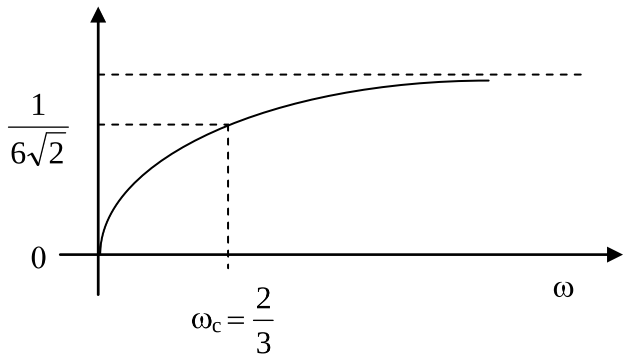
$$H(j\omega)=\frac{\frac{1}{6}j\omega}{j\omega+\frac{2}{3}}$$

At $\omega=0$ (low frequency) $H(j0)=0$. At $\omega=\infty$ (high frequency) $H(j\infty)=\frac{1}{6}$

$\therefore$ It is a high pass (HP) filter.

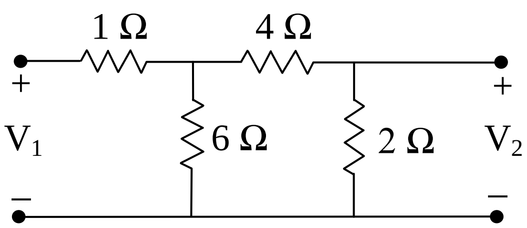
At cut-off frequency $|H(j\omega_c)|=\frac{1}{6\sqrt{2}}$ $\qquad \therefore \left|\frac{j\omega_c}{j\omega_c+\frac{2}{3}}\right|=\frac{1}{\sqrt{2}}$

$\therefore \frac{\omega_c}{\sqrt{\omega_c^2+\frac{4}{9}}}=\frac{1}{\sqrt{2}}$ $\qquad \therefore \omega_c=\frac{2}{3}$ rad/s.



Question #5

A)



$Z_{11} = \frac{V_1}{I_1}\Big| I_2 = 0 \qquad Z_{11} = 1 + 3 = 4\,\Omega \qquad Z_{22} = \frac{V_2}{I_2}\Big| I_1 = 0 \qquad Z_{22} = \frac{2\times 10}{2+10} = \frac{5}{3}\,\Omega$

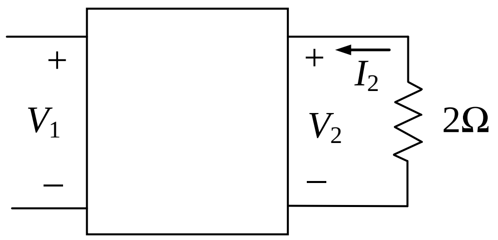
$Z_{12} = \frac{V_2}{I_1}\Big| I_2 = 0 \qquad Z_{12} = 1\,\Omega \qquad Z_{21} = \frac{V_1}{I_2}\Big| I_1 = 0 \qquad Z_{21} = 1\,\Omega$

B)

$Y_{11} = 0.5$ S $\qquad Y_{12} = Y_{21} =$ -0.4 S $\qquad Y_{22} = 0.6$ S

$I_1 = Y_{11}V_1 + Y_{12}V_2 \ = 0.5\,V_1 - 0.4\,V_2 \qquad (1)$

$I_2 = Y_{21}V_1 + Y_{22}V_2 \ = -0.4\,V_1 + 0.6\,V_2 \qquad (2)$



For this termination, $I_2 = -0.5\,V_2$, substituting in (2),

$-0.5\,V_2 = -0.4\,V_1 + 0.6\,V_2$, dividing by V_1,

$-0.5\,\frac{V_2}{V_1} = -0.4 + 0.6\,\frac{V_2}{V_1} \qquad \therefore \quad \frac{V_2}{V_1} = \frac{4}{11}$, substituting in (1),

$I_1 = 0.5\,V_1 - 0.4\,V_2 = 0.5\,(\frac{11}{4}V_2) - 0.4\,V_2$, $V_2 = -2I_2$

$I_1 = -\frac{11}{4}I_2 + 0.8\,I_2$, divide by I_1,

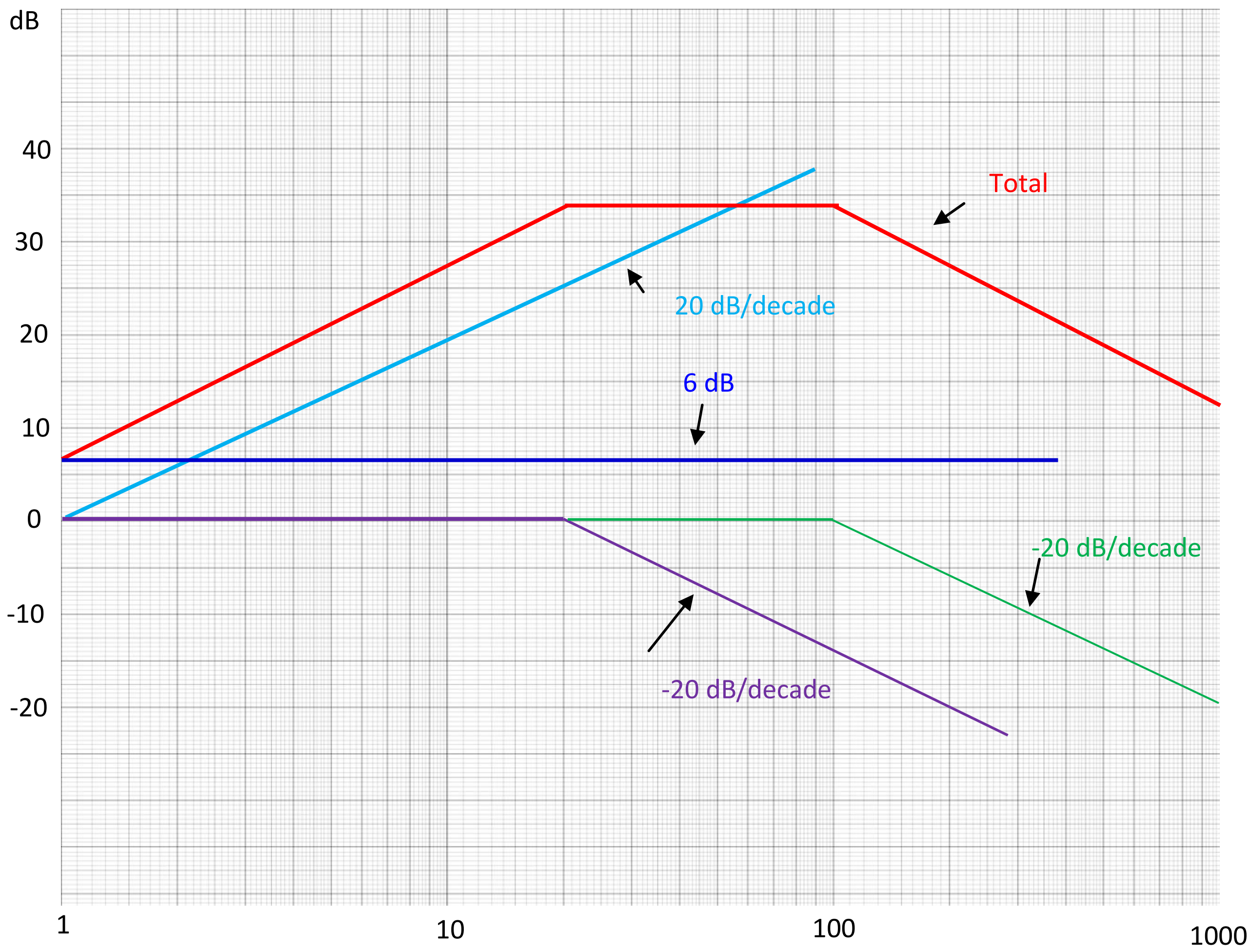
$1 = -\frac{11}{4}\frac{I_2}{I_1} + 0.8\,\frac{I_2}{I_1} \qquad \therefore \quad \frac{I_2}{I_1} = -\frac{20}{39}$

C)

$$H(s) = \frac{4000(s+1)}{(s+20)(s+100)} = \frac{4000(1+s)}{20(1+\frac{s}{20})100(1+\frac{s}{100})} = \frac{2(1+s)}{(1+\frac{s}{20})(1+\frac{s}{100})}$$

$|H(j\omega)|$ in dB $= 20\log(2) + 20\log|1 + j\omega| - 20\log\left|1+\frac{j\omega}{20}\right| - 20\log\left|1+\frac{j\omega}{100}\right|$

$$= 6 + 20\log|1 + j\omega| - 20\log\left|1+\frac{j\omega}{20}\right| - 20\log\left|1+\frac{j\omega}{100}\right|$$


dB
40
30
20
10
0
-10
-20
Total
20 dB/decade
6 dB
-20 dB/decade
-20 dB/decade
1
10
100
1000