مع الإجابة الصحيحة

Quiz 1 M.318 first semester 1444H.

Name ______________________________:

University number ______________________:

Question 1(3marks): Solve the following differential equation: $e^x y \frac{dy}{dx} = e^{-y} + e^{-2x-y}$.

Qusetion 2(4marks): Find the solution of the initial value problem:

$$\begin{cases} (x^2 + 2y^2)dx - xydy = 0 \\ y(-1) = 1 \end{cases}, \text{ where } x < 0.$$

Question 3(3marks): Prove that the following differential equation is exact equation and solve it:

$$(x - y)dx + (-x + y + 2)dy = 0.$$

Question ① $e^{x}y\frac{dy}{dx}=e^{-y}+e^{-2x-y}$

① $e^{x}y\frac{dy}{dx}=e^{-y}(1+e^{-2x})$

$e^{x}y\,dy=e^{-y}(1+e^{-2x}) \Rightarrow ye^{y}dy=e^{-x}(1+e^{-2x})dx$

② $\int ye^{y}dy=\int(e^{-x}+e^{-3x})dx$

$\boxed{(ye^{y}-e^{y})=-e^{-x}-\frac{1}{3}e^{-3x}+C}$

Question ② $\begin{cases}(x^2+2y^2)dx-xy\,dy=0, & x<0\\ y(-1)=1\end{cases}$

We have homogeneous D.E, we put $u=\frac{y}{x}$, $y=u.x$,

$dy=u\,dx+x\,du$ ①

$(1+2\frac{y^2}{x^2})dx-\frac{y}{x}dy=0$

$(1+2u^2)dx-u(u\,dx+x\,du)=0$

$(1+2u^2-u^2)dx-ux\,du=0$

① $(1+u^2)dx-ux\,du=0$

$\frac{dx}{x}-\frac{u\,du}{1+u^2}=0 \Rightarrow \ln|x|-\frac{1}{2}\ln(1+u^2)=c$

① $2\ln(-x)-\ln(1+\frac{y^2}{x^2})=c_1 \quad (c_1=2c)$

$\boxed{\ln(x^2)-\ln(1+y^2/x^2)=c_1}$

But $y(-1)=1 \Rightarrow \ln(1)-\ln(1+1)=c_1 \Rightarrow c_1=-\ln 2$ ①

$\boxed{\ln x^2-\ln(1+y^2/x^2)+\ln 2=0}$ is the solution of the IVP

Question (3) $\overbrace{(x-y)}^{M}dx+\overbrace{(-x+y+2)}^{N}dy=0$

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}=-1 \quad (1)$$

Then there exists a function F of x and y s.t.

$$\frac{\partial F}{\partial x}=x-y\,,\quad \frac{\partial F}{\partial y}=-x+y+2$$

$$F(x,y)=\int\frac{\partial F}{\partial x}(x,y)\,dx=\int(x-y)\,dx=\frac{1}{2}x^2-xy+\phi(y)$$

$$\frac{\partial F}{\partial y}=-\cancel{x}+\phi'(y)=-\cancel{x}+y+2$$

$$\phi'(y)=y+2\Rightarrow\phi(y)=\frac{1}{2}y^2+2y+c \quad (1\tfrac{1}{2})$$

Then the solution of the D.E. is

$$\boxed{F(x,y)=\frac{1}{2}x^2+\frac{1}{2}y^2-xy+2y+c=0} \quad (\tfrac{1}{2})$$

م الإجابة الطالب -

Quiz 2, M.318 first semester 1444H (2022).

Name ______________________________:

University number ______________________:

Question1(3): Find the general solution of the differential equation:

$$y^{(4)} + 2y'' + y = 0.$$

Question2(3): Determine a homogenous linear differential equation with constant coefficients having the fundamental set of solutions: $y_1 = 2e^{3x}$, $y_2 = 4\sin(2x)$, $y_3 = \cos(2x)$.

Question3(4): Find the general solution of the differential equation: $x^2y'' - 3xy' + 13y = 0.$, $x > 0$

Question ① $y^{(4)} + 2\ddot{y} + y = 0, \quad y = e^{mx}$

① $m^4 + 2m^2 + 1 = (m^2+1)^2 = 0 \Rightarrow m = \mp i, \quad m = \mp i$

② $\boxed{y = c_1 \cos x + c_2 \sin x + c_3 x \sin x + c_4 x \cos x}$

Question ② $y_1 = 2e^{3x}, \quad y_2 = 4\sin 2x, \quad y_3 = \cos(2x)$

$y_1 \to \quad m - 3 = 0$

$y_2, y_3 \to \quad m = \mp 2i \Rightarrow m^2 + 4 = 0$

② $(m-3)(m^2+4) = 0 \quad m^3 + 4m - 3m^2 - 12 = 0$

Then the D.E is:

① $\boxed{y^{(3)} - 3\ddot{y} + 4y' - 12y = 0}$

Question ③ $x^2\ddot{y} - 3x\dot{y} + 13 = 0 \quad , \quad y = x^m$

$x^m \left(m(m-1) - 3m + 13 \right) = 0$

$\Rightarrow \quad m^2 - 4m + 13 = (m-2)^2 + 9 = 0 \Rightarrow (m-2)^2 = -9 = 9i^2$

② $|m-2| = 3i \Rightarrow \quad m = 2 \mp 3i$

① + ① $\boxed{y = c_1 x^2 \cos(3\ln x) + c_2 x^2 \sin(3\ln x)}$

مع الإجابة العامة

King Sau University	**Mid-Term 1 M.318 First 2022.**
College of Sciences	**Marks 30.**
Mathematical department	**Time 2 hours.**

Question 1(10 marks):

5 **a)** Find the integrating factor of the following differential equation and solve it.

$$y(2x+y+1)dx+(x+y)dy=0, \text{ where } x+y\neq 0.$$

5 **b)** Find the general solution of the following differential equation:

$$(x^2+x)dy=\left(x^5+3xy+3y\right)dx \text{ , where } x>0 \text{ and } x>-1.$$

Question2(12 marks):

6 **a)** Write the following differential equation as he general form of Bernoulli 's equation and solve it.

$$\begin{cases} 5xy^2y'+y^3=6(1+lnx)y^{-2} \\ y(1)=1 \end{cases}$$

6 **b)** A liquid with initial temperature 200 C^0 is surrounded by air at a constant temperature 80 C^0. If the liquid cools to 120 C^0 in 30 minutes what will be the temperature after one hour?

Question3(8 marks):

4 **a)** Find the solution following differential equation:

$$\frac{dy}{dx}=\frac{1-x-y}{2+x+y} \text{ , where } 2+x+y\neq 0.$$

4 **b)** Find the general solution following differential equation:

$(2x+1)y''+4xy'-4y=0$, $x>-\frac{1}{2}$, where $y_1=e^{-2x}$ is the solution of the differential equation.

Question 1 السؤال الأول :

(a)

$$y(2x+y+1)\,dx+(x+y)\,dy=0\ ; \qquad y+x\neq 0$$

$$(\underbrace{2yx+y^2+y}_{M})\,dx+(\underbrace{x+y}_{N})\,dy=0$$

$$\frac{\partial M}{\partial y}=2x+2y+1 \qquad \frac{\partial N}{\partial x}=1\ , \quad f(x)=\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}$$

$$f(x)=\frac{2(x+y)}{x+y}=2\ , \quad \mu(x)=e^{\int 2dx}$$

$$\mu(x)=e^{2x},$$

2

$$(\underbrace{2xe^{2x}y+e^{2x}y^2+ye^{2x}}_{M})\,dx+(\underbrace{xe^{2x}+ye^{2x}}_{N})\,dy=0 \quad \text{---}\ *$$

$$\frac{\partial M}{\partial y}=2xe^{2x}+2ye^{2x}+e^{2x}, \qquad \frac{\partial N}{\partial x}=e^{2x}+2xe^{2x}+2ye^{2x}$$

$\Rightarrow$ The D.E $*$ is exact. $\Rightarrow \exists\ F(x,y)$ s.t

1

$$\frac{\partial F}{\partial x}=M=2xe^{2x}y+e^{2x}y^2+ye^{2x}$$

$$\frac{\partial F}{\partial y}=N=xe^{2x}+ye^{2x}$$

$$F(x,y)=\int(xe^{2x}+ye^{2x})\,dy=xye^{2x}+\tfrac{1}{2}y^2e^{2x}+\phi(x)$$

$$\frac{\partial F}{\partial x}=\cancel{ye^{2x}}+\cancel{2xye^{2x}}+\cancel{y^2e^{2x}}+\phi'(x)=\cancel{2xe^{2x}y}+\cancel{y^2e^{2x}}+\cancel{ye^{2x}}$$

$$\phi'(x)=0 \Rightarrow \phi(x)=c$$

2

The solution of the D.E is

$$\boxed{F(x,y)=xye^{2x}+\tfrac{1}{2}y^2e^{2x}+c=0}$$

(b)

$$(x^2+x)\,dy=(x^5+3xy+3y)\,dx, \qquad x>0\ /\ x>-1$$

$$(x^2+x)\,y'=(x^5+3y(x+1))$$

$$y'-\frac{3(x+1)}{x(x+1)}y=\frac{x^5}{x(x+1)} \Rightarrow y'-\frac{3}{x}\cdot y=\frac{x^5}{x(x+1)}=\frac{x^4}{x+1} \quad \text{is Linear D.E}$$

2

$$\mu(x)=e^{\int\frac{-3dx}{x}}=e^{-3\ln x}=x^{-3}$$

1

$$y\mu(x)=yx^{-3}=\int x^{-3}\frac{x^4}{x+1}\,dx=\int\frac{x+1-1}{x+1}\,dx=\int\left(1-\frac{1}{x+1}\right)dx$$

$$yx^{-3}=x-\ln(x+1)+c$$

$$\boxed{y=x^3(x-\ln(x+1))+cx^3} \quad \text{is the G. Solution.}$$

2

Question 2

ⓐ $5xy^2y' + y^3 = 6(1+\ln x)\,y^{-2}$

$y' + \frac{1}{5x}y = \frac{6(1+\ln x)}{5xy^2}\,y^{-2} = \frac{6(1+\ln x)}{5x}\,y^{-4}$, (2)

$y'y^4 + \frac{1}{5x}y^5 = \frac{6}{5x}(1+\ln x)$ $\qquad y^5 = u,\ 5y^4y' = u'$

$\qquad y'y^4 = \frac{u'}{5}$ (2)

$\frac{u'}{5} + \frac{1}{5x}u = \frac{6}{5x}(1+\ln x)$

$u' + \frac{1}{x}u = 6\left(\frac{1}{x} + \frac{\ln x}{x}\right)$, $\mu(x) = e^{\int \frac{1}{x}dx} = x$

(2) $ux = 6\int x\left(\frac{1}{x} + \frac{\ln x}{x}\right)dx = 6\int (1+\ln x)\,dx$

$ux = 6(x + x\ln x - x) + c$

$\boxed{y^5 x = 6x\ln x + c}$

(7) ⓑ $\frac{dT}{T-T_s} = k\,dt$. $\quad \ln|T-T_s| = kt + c$

$T = T_s + c_1 e^{kt}$ (2)

$T(0) = 200\ C^\circ$, $T_s = 80$, $T(30) = 120\ c^\circ$,

1 hour = 60 minutes $\quad T(60) = ?$

$T(t) = 80 + ce^{kt}$

$T(0) = 200 = 80 + c \Rightarrow c = 120$

$T(t) = 80 + 120e^{kt}$ (2)

$T(30) = 80 + 120e^{30k} = 120 \Rightarrow 40 = 120e^{30k}$

$\frac{40}{120} = \frac{1}{3} = e^{30k} \Rightarrow -\ln(3) = 30k$

(1) $k = \frac{-\ln 3}{30} \cong -0.037$

$T(t) = 80 + 120e^{-0.037t}$

$T(60) = 80 + 120e^{(-0.037)(60)}$

$= 80 + 120e^{-2.22}$ (2)

$= 80 + 120(0.109)$

$T(60) = 93.033 \cong 93\ C^\circ$ $\qquad = 80 + 13.033 = 93.033$

(2)

Question 3

4 (a) $y' = \frac{1-(x+y)}{2+(x+y)}$, $2+x+y=0$, $x+y=u \Rightarrow 1+y'=u'$

$y' = u'-1$ ①

① $u'-1 = \frac{1-u}{2+u} \Rightarrow u' = 1 + \frac{1-u}{2+u} = \frac{2+u+1-u}{2+u} = \frac{3}{u+2} = \frac{du}{dx}$

② $3dx = (u+2)du$

$3x = (u+2)^2 \frac{1}{2} + C$

$\boxed{6x = (x+y+2)^2 + c_1}$ o.k

4 (b) $(3x+1)y'' - (9x+6)y' + 9y = 0$, $x > \frac{-1}{3}$, $y = e^{3x}$ is a solution of the D.E

$y'' - \frac{3(3x+2)}{3x+1}y' + \frac{9}{3x+1}y = 0$ $\quad P = -3\,\frac{3x+1+1}{3x+1} = -3\left(1 + \frac{1}{3x+1}\right)$

$e^{-\int P(x)dx} = e^{\int \left(3 + \frac{3}{3x+1}\right)dx} = e^{3x + \ln(1+3x)} = e^{3x}(1+3x)$

$y_2 = y_1 \int \frac{e^{-\int P(x)dx}}{(y_1)^2} dx$

$= e^{3x} \int \frac{e^{3x}(1+3x)}{e^{6x}} dx = e^{3x} \int e^{-3x}(1+3x)\,dx$

$-\frac{1}{3}(1+3x)e^{-3x} + \int \frac{1}{3} e^{-3x} 3dx$

$e^{3x}\left[-\frac{1}{3}(1+3x)e^{-3x} - \frac{1}{3}e^{-3x}\right] = \frac{-1}{3}(1+3x) - \frac{1}{3}$

$y_2 = -\frac{1}{3} - x - \frac{1}{3} = \frac{-2}{3} - x$

$\boxed{y = c_1 y_1 + c_2 y_2 = c_1 e^{3x} + c_2\left(-x - \frac{2}{3}\right)}$

Question 3

4 (b) $(2x+1)\ddot{y} + 4xy' - 4y = 0$, $x > -1/2$, $y = e^{-2x}$ is a particular solution of the D.E

$$\ddot{y} + 2 \cdot \frac{2x}{2x+1} y' - \frac{4}{2x+1} y = 0, \quad P = \frac{4x}{2x+1}$$ (1)

$$e^{-\int P(x)dx} = e^{-2\int \frac{2x+1-1}{2x+1}dx} = e^{-2\int (1 - \frac{1}{2x+1})dx}$$

$$= e^{-2x + \ln(1+2x)} = e^{-2x}(1+2x)$$ (1)

(1) $$y_2 = y_1 \int \frac{\int e^{-P(x)dx}}{(y_1)^2} dx = e^{-2x} \int \frac{e^{-2x}(2x+1)}{e^{-4x}} dx$$

$$= e^{-2x} \int e^{2x}(1+2x)dx$$

$$= e^{-2x}\left[\tfrac{1}{2} e^{2x}(1+2x) - \tfrac{1}{2}\int e^{2x} 2 dx\right]$$

$$= e^{-2x}\left[\tfrac{1}{2} e^{2x}(1+2x) - \tfrac{1}{2} e^{2x}\right] = x$$ (1)

$y = c_1 e^{-2x} + c_2(x)$ is the G. solution of the D.E