Ways of Expressing Concentrations of Solutions
Mass Percentage

Mass % of A = \frac{\text{mass of A in solution}}{\text{total mass of solution}} \times 100
Parts per Million and Parts per Billion

Parts per Million (ppm)

$$\text{ppm} = \frac{\text{mass of A in solution}}{\text{total mass of solution}} \times 10^6$$

Parts per Billion (ppb)

$$\text{ppb} = \frac{\text{mass of A in solution}}{\text{total mass of solution}} \times 10^9$$
A solution whose solute concentration is 1 ppm contains 1 g of solute for each million \((10^6)\) grams of solution or, equivalently, 1 mg of solute per kilogram of solution. Because the density of water is 1 g/mL, 1 kg of a dilute aqueous solution will have a volume very close to 1 L. Thus, 1 ppm also corresponds to 1 mg of solute per liter of aqueous solution.

**SAMPLE EXERCISE 13.4 | Calculation of Mass-Related Concentrations**

(a) A solution is made by dissolving 13.5 g of glucose \((C_6H_{12}O_6)\) in 0.100 kg of water. What is the mass percentage of solute in this solution? (b) A 2.5-g sample of groundwater was found to contain 5.4 \(\mu\)g of Zn\(^{2+}\). What is the concentration of Zn\(^{2+}\) in parts per million?

**SOLUTION**

(a) **Analyze:** We are given the number of grams of solute (13.5 g) and the number of grams of solvent (0.100 kg = 100 g). From this we must calculate the mass percentage of solute.

**Plan:** We can calculate the mass percentage by using Equation 13.5. The mass of the solution is the sum of the mass of solute (glucose) and the mass of solvent (water).

**Solve:** 

\[
\text{Mass \% of glucose} = \frac{\text{mass glucose}}{\text{mass soln}} \times 100 = \frac{13.5 \text{ g}}{13.5 \text{ g} + 100 \text{ g}} \times 100 = 11.9\%
\]

**Comment:** The mass percentage of water in this solution is \((100 - 11.9)\% = 88.1\%\).

(b) **Analyze:** In this case we are given the number of micrograms of solute. Because 1 \(\mu\)g is \(1 \times 10^{-6}\) g, 5.4 \(\mu\)g = \(5.4 \times 10^{-6}\) g.

**Plan:** We calculate the parts per million using Equation 13.6.

**Solve:** 

\[
\text{ppm} = \frac{\text{mass of solute}}{\text{mass of soln}} \times 10^6 = \frac{5.4 \times 10^{-6} \text{ g}}{2.5 \text{ g}} \times 10^6 = 2.2 \text{ ppm}
\]
Mole Fraction \((X)\)

\[
X_A = \frac{\text{moles of } A}{\text{total moles in solution}}
\]

- In some applications, one needs the mole fraction of *solvent*, not solute — make sure you find the quantity you need!
Sample Exercise 13.6 Calculation of Mole Fraction and Molality

An aqueous solution of hydrochloric acid contains 36% HCl by mass. (a) Calculate the mole fraction of HCl in the solution. (b) Calculate the molality of HCl in the solution.

Solution

Analyze: We are asked to calculate the concentration of the solute, HCl, in two related concentration units, given only the percentage by mass of the solute in the solution.

Plan: In converting concentration units based on the mass or moles of solute and solvent (mass percentage, mole fraction, and molality), it is useful to assume a certain total mass of solution. Let’s assume that there is exactly 100 g of solution. Because the solution is 36% HCl, it contains 36 g of HCl and (100 – 36) g = 64 g of H₂O. We must convert grams of solute (HCl) to moles to calculate either mole fraction or molality. We must convert grams of solvent (H₂O) to moles to calculate mole fractions, and to kilograms to calculate molality.

Solve: (a) To calculate the mole fraction of HCl, we convert the masses of HCl and H₂O to moles and then use Equation 13.7:

\[
\text{Moles HCl} = \left(36 \text{ g HCl}\right) \left(\frac{1 \text{ mol HCl}}{36.5 \text{ g HCl}}\right) = 0.99 \text{ mol HCl}
\]

\[
\text{Moles H₂O} = \left(64 \text{ g H₂O}\right) \left(\frac{1 \text{ mol H₂O}}{18 \text{ g H₂O}}\right) = 3.6 \text{ mol H₂O}
\]

\[
X_{\text{HCl}} = \frac{\text{moles HCl}}{\text{moles H₂O} + \text{moles HCl}} = \frac{0.99}{3.6 + 0.99} = \frac{0.99}{4.6} = 0.22
\]

(b) To calculate the molality of HCl in the solution, we use Equation 13.9. We calculated the number of moles of HCl in part (a), and the mass of solvent is 64 g = 0.064 kg:

\[
\text{Molality of HCl} = \frac{0.99 \text{ mol HCl}}{0.064 \text{ kg H₂O}} = 15 m
\]
Sample Exercise 13.6 Calculation of Mole Fraction and Molality

Practice Exercise

A commercial bleach solution contains 3.62 mass % NaOCl in water. Calculate (a) the mole fraction and (b) the molality of NaOCl in the solution.

*Answer:* (a) $9.00 \times 10^{-3}$, (b) 0.505 $m$. 
Molarity

- Two solutions can contain the same compounds but be quite different because the proportions of those compounds are different.
- Molarity is one way to measure the concentration of a solution.

\[
M \text{olarity }(M) = \frac{\text{moles of solute}}{\text{volume of solution in liters}}
\]
Sample Exercise 4.11 Calculating Molarity

Calculate the molarity of a solution made by dissolving 23.4 g of sodium sulfate (Na₂SO₄) in enough water to form 125 mL of solution.

Solution

Analyze: We are given the number of grams of solute (23.4 g), its chemical formula (Na₂SO₄), and the volume of the solution (125 ml). We are asked to calculate the molarity of the solution.

Plan: We can calculate molarity using Equation 4.33. To do so, we must convert the number of grams of solute to moles and the volume of the solution from milliliters to liters.

Solve: The number of moles of Na₂SO₄ is obtained by using its molar mass:

\[ \text{Moles Na}_2\text{SO}_4 = \left( \frac{23.4 \text{ g Na}_2\text{SO}_4}{142 \text{ g Na}_2\text{SO}_4} \right) \left( \frac{1 \text{ mol Na}_2\text{SO}_4}{142 \text{ g Na}_2\text{SO}_4} \right) = 0.165 \text{ mol Na}_2\text{SO}_4 \]

Converting the volume of the solution to liters:

\[ \text{Liters soln} = \left( \frac{125 \text{ mL}}{1000 \text{ mL}} \right) = 0.125 \text{ L} \]

Thus, the molarity is

\[ \text{Molarity} = \frac{0.165 \text{ mol Na}_2\text{SO}_4}{0.125 \text{ L soln}} = 1.32 \frac{\text{mol Na}_2\text{SO}_4}{\text{L soln}} = 1.32 \text{ M} \]

Check: Because the numerator is only slightly larger than the denominator, it is reasonable for the answer to be a little over 1 M. The units (mol/L) are appropriate for molarity, and three significant figures are appropriate for the answer because each of the initial pieces of data had three significant figures.
Sample Exercise 4.11 Calculating Molarity

**Practice Exercise**

Calculate the molarity of a solution made by dissolving 5.00 g of glucose (C\textsubscript{6}H\textsubscript{12}O\textsubscript{6}) in sufficient water to form exactly 100 mL of solution.

*Answer: 0.278 M*
Sample Exercise 4.13 Using Molarity to Calculate Grams of Solute

How many grams of Na$_2$SO$_4$ are required to make 0.350 L of 0.500 M Na$_2$SO$_4$?

Solution

Analyze: We are given the volume of the solution (0.350 L), its concentration (0.500 M), and the identity of the solute Na$_2$SO$_4$ and asked to calculate the number of grams of the solute in the solution.

Plan: We can use the definition of molarity (Equation 4.33) to determine the number of moles of solute, and then convert moles to grams using the molar mass of the solute.

\[
M_{\text{Na}_2\text{SO}_4} = \frac{\text{moles Na}_2\text{SO}_4}{\text{liters soln}}
\]

Solve: Calculating the moles of Na$_2$SO$_4$ using the molarity and volume of solution gives

\[
M_{\text{Na}_2\text{SO}_4} = \frac{\text{moles Na}_2\text{SO}_4}{\text{liters soln}} = \text{liters soln} \times M_{\text{Na}_2\text{SO}_4} = (0.350 \text{ L-soln})(0.500 \text{ mol Na}_2\text{SO}_4/1 \text{ L-soln}) = 0.175 \text{ mol Na}_2\text{SO}_4
\]

Because each mole of Na$_2$SO$_4$ weighs 142 g, the required number of grams of Na$_2$SO$_4$ is

\[
\text{grams Na}_2\text{SO}_4 = (0.175 \text{ mol Na}_2\text{SO}_4)(142 \text{ g Na}_2\text{SO}_4/1 \text{ mol Na}_2\text{SO}_4) = 24.9 \text{ g Na}_2\text{SO}_4
\]

Check: The magnitude of the answer, the units, and the number of significant figures are all appropriate.
Sample Exercise 4.13 Using Molarity to Calculate Grams of Solute

Practice Exercise

(a) How many grams of Na$_2$SO$_4$ are there in 15 mL of 0.50 $M$ Na$_2$SO$_4$? (b) How many milliliters of 0.50 $M$ Na$_2$SO$_4$ solution are needed to provide 0.038 mol of this salt?

Answers: (a) 1.1 g, (b) 76 mL
Sample Exercise 4.12 Calculating Molar Concentrations of Ions

What are the molar concentrations of each of the ions present in a 0.025 M aqueous solution of calcium nitrate?

Solution

Analyze: We are given the concentration of the ionic compound used to make the solution and asked to determine the concentrations of the ions in the solution.

Plan: We can use the subscripts in the chemical formula of the compound to determine the relative concentrations of the ions.

Solve: Calcium nitrate is composed of calcium (Ca$^{2+}$) ions and nitrate ions NO$_3^-$, so its chemical formula is Ca(NO$_3$)$_2$. Because there are two NO$_3^-$ ions for each Ca$^{2+}$ ion in the compound, each mole of Ca(NO$_3$)$_2$ that dissolves dissociates into 1 mol of Ca$^{2+}$ and 2 mol of NO$_3^-$. Thus, a solution that is 0.025 M in Ca(NO$_3$)$_2$ is 0.025 M in Ca$^{2+}$ and 2 × 0.025 M = 0.050 M in NO$_3^-$:

$$\frac{\text{mol NO}_3^-}{\text{L}} = \left(\frac{0.025 \text{ mol Ca(NO}_3)_2}{\text{L}}\right)\left(\frac{2 \text{ mol NO}_3^-}{1 \text{ mol Ca(NO}_3)_2}\right) = 0.050 \text{ M}$$

Check: The concentration of NO$_3^-$ ions is twice that of Ca$^{2+}$ ions, as the subscript 2 after the NO$_3^-$ in the chemical formula Ca(NO$_3$)$_2$ suggests it should be.

Practice Exercise

What is the molar concentration of K$^+$ ions in a 0.015 M solution of potassium carbonate?

Answer: 0.030 M K$^+$
Mixing a Solution

• To create a solution of a known molarity, one weighs out a known mass (and, therefore, number of moles) of the solute.

• The solute is added to a volumetric flask, and solvent is added to the line on the neck of the flask.
Dilution

• One can also dilute a more concentrated solution by
  – Using a pipet to deliver a volume of the solution to a new volumetric flask, and
  – Adding solvent to the line on the neck of the new flask.
Dilution

The molarity of the new solution can be determined from the equation

\[ M_c \times V_c = M_d \times V_d, \]

where \( M_c \) and \( M_d \) are the molarity of the concentrated and dilute solutions, respectively, and \( V_c \) and \( V_d \) are the volumes of the two solutions.
Sample Exercise 4.14 Preparing A solution by Dilution

How many milliliters of 3.0 \( M \) \( \text{H}_2\text{SO}_4 \) are needed to make 450 mL of 0.10 \( M \) \( \text{H}_2\text{SO}_4 \)?

Solution

Analyze: We need to dilute a concentrated solution. We are given the molarity of a more concentrated solution (3.0 \( M \)) and the volume and molarity of a more dilute one containing the same solute (450 mL of 0.10 \( M \) solution). We must calculate the volume of the concentrated solution needed to prepare the dilute solution.

Plan: We can calculate the number of moles of solute, \( \text{H}_2\text{SO}_4 \), in the dilute solution and then calculate the volume of the concentrated solution needed to supply this amount of solute. Alternatively, we can directly apply Equation 4.35. Let’s compare the two methods.

Solve: Calculating the moles of \( \text{H}_2\text{SO}_4 \) in the dilute solution:

\[
\text{moles } \text{H}_2\text{SO}_4 \text{ in dilute solution} = (0.450 \text{ L soln}) \left( \frac{0.10 \text{ mol } \text{H}_2\text{SO}_4}{1 \text{ L soln}} \right) = 0.045 \text{ mol } \text{H}_2\text{SO}_4
\]

Calculating the volume of the concentrated solution that contains 0.045 mol \( \text{H}_2\text{SO}_4 \):

\[
\text{L conc soln} = (0.045 \text{ mol } \text{H}_2\text{SO}_4) \left( \frac{1 \text{ L soln}}{3.0 \text{ mol } \text{H}_2\text{SO}_4} \right) = 0.015 \text{ L soln}
\]

Converting liters to milliliters gives 15 mL.

If we apply Equation 4.35, we get the same result:

\[
(3.0 \text{ M})(V_{\text{conc}}) = (0.10 \text{ M})(450 \text{ mL})
\]

\[
V_{\text{conc}} = \frac{(0.10 \text{ M})(450 \text{ mL})}{3.0 \text{ M}} = 15 \text{ mL}
\]
Sample Exercise 4.14 Preparing A solution by Dilution

Solution (continued)

Either way, we see that if we start with 15 mL of 3.0 $M$ $H_2SO_4$ and dilute it to a total volume of 450 mL, the desired 0.10 $M$ solution will be obtained.

Check: The calculated volume seems reasonable because a small volume of concentrated solution is used to prepare a large volume of dilute solution.

Practice Exercise

(a) What volume of 2.50 $M$ lead(II) nitrate solution contains 0.0500 mol of Pb$^{2+}$?
(b) How many milliliters of 5.0 $M$ $K_2Cr_2O_7$ solution must be diluted to prepare 250 mL of 0.10 $M$ solution?
(c) If 10.0 mL of a 10.0 $M$ stock solution of NaOH is diluted to 250 mL, what is the concentration of the resulting stock solution?

Answers: (a) 0.0200 L = 20.0 mL, (b) 5.0 mL, (c) 0.40 $M$
Molality (m)

\[
m = \frac{\text{mol of solute}}{\text{kg of solvent}}
\]

Since both moles and mass do not change with temperature, molality (unlike molarity) is *not* temperature-dependent.
Sample Exercise 13.5 Calculation of Molality

A solution is made by dissolving 4.35 g glucose (C₆H₁₂O₆) in 25.0 mL of water at 25 °C. Calculate the molality of glucose in the solution. Water has a density of 1.00 g/mL.

Solution

Analyze: We are asked to calculate a molality. To do this, we must determine the number of moles of solute (glucose) and the number of kilograms of solvent (water).

Plan: We use the molar mass of C₆H₁₂O₆ to convert grams to moles. We use the density of water to convert milliliters to kilograms. The molality equals the number of moles of solute divided by the number of kilograms of solvent (Equation 13.9).

Solve: Use the molar mass of glucose, 180.2 g/mL, to convert grams to moles:

\[
\text{Mol C}_6\text{H}_{12}\text{O}_6 = \frac{4.35 \text{ g C}_6\text{H}_{12}\text{O}_6}{180.2 \text{ g C}_6\text{H}_{12}\text{O}_6} = 0.0241 \text{ mol C}_6\text{H}_{12}\text{O}_6
\]

Because water has a density of 1.00 g/mL, the mass of the solvent is

\[
(25.0 \text{ mL})(1.00 \text{ g/mL}) = 25.0 \text{ g} = 0.0250 \text{ kg}
\]

Finally, use Equation 13.9 to obtain the molality:

\[
\text{Molality of C}_6\text{H}_{12}\text{O}_6 = \frac{0.0241 \text{ mol C}_6\text{H}_{12}\text{O}_6}{0.0250 \text{ kg H}_2\text{O}} = 0.964 \text{ m}
\]

Practice Exercise

What is the molality of a solution made by dissolving 36.5 g of naphthalene (C₁₀H₈) in 425 g of toluene (C₇H₈)?

Answer: 0.670 m
Sample Exercise 13.7 Calculation of Molality Using the Density of a Solution

A solution with a density of 0.876 g/mL contains 5.0 g of toluene (C₇H₈) and 225 g of benzene. Calculate the molarity of the solution.

Solution

Analyze: Our goal is to calculate the molarity of a solution, given the masses of solute (5.0 g) and solvent (225 g) and the density of the solution (0.876 g/mL).

Plan: The molarity of a solution is the number of moles of solute divided by the number of liters of solution (Equation 13.8). The number of moles of solute (C₇H₈) is calculated from the number of grams of solute and its molar mass. The volume of the solution is obtained from the mass of the solution (mass of solute + mass of solvent = 5.0 g + 225 g = 230 g) and its density.

Solve: The number of moles of solute is

\[
\text{Moles C}_7\text{H}_8 = (5.0 \text{ g C}_7\text{H}_8) \left( \frac{1 \text{ mol C}_7\text{H}_8}{92 \text{ g C}_7\text{H}_8} \right) = 0.054 \text{ mol}
\]

The density of the solution is used to convert the mass of the solution to its volume:

\[
\text{Milliliters soln} = (230 \text{ g}) \left( \frac{1 \text{ mL}}{0.876 \text{ g}} \right) = 263 \text{ mL}
\]

Molarity is moles of solute per liter of solution:

\[
\text{Molarity} = \left( \frac{\text{moles C}_7\text{H}_8}{\text{liter soln}} \right) = \left( \frac{0.054 \text{ mol C}_7\text{H}_8}{263 \text{ mL soln}} \right) \left( \frac{1000 \text{ mL soln}}{1 \text{ L soln}} \right) = 0.21 \text{ M}
\]

Check: The magnitude of our answer is reasonable. Rounding moles to 0.05 and liters to 0.25 gives a molarity of

\[
\frac{0.05 \text{ mol}}{0.25 \text{ L}} = 0.2 \text{ M}
\]

The units for our answer (mol/L) are correct, and the answer, 0.21 M, has two significant figures, corresponding to the number of significant figures in the mass of solute (2).
Sample Exercise 13.7 Calculation of Molality Using the Density of a Solution

Solution (continued)

Comment: Because the mass of the solvent (0.225 kg) and the volume of the solution (0.263 L) are similar in magnitude, the molarity and molality are also similar in magnitude:

\[
\frac{(0.054 \text{ mol } C_7H_8O_3)}{0.225 \text{ kg solvent}} = 0.24 \text{ m}
\]

Practice Exercise

A solution containing equal masses of glycerol (C\(_3\)H\(_8\)O\(_3\)) and water has a density of 1.10 g/mL. Calculate (a) the molality of glycerol, (b) the mole fraction of glycerol, (c) the molarity of glycerol in the solution.

Answer: (a) 10.9 m, (b) \(X_{C_3H_8O_3} = 0.163\), (c) 5.97 M
If we know the density of the solution, we can calculate the molality from the molarity and vice versa.