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Chapter 7

Continuous Probability Distributions

Learning Objectives

LO7-1 Describe the uniform probability distribution and use it to calculate probabilities.

LO7-2 Describe the characteristics of a normal probability distribution.

LO7-3 Describe the standard normal probability distribution and use it to calculate probabilities.

LO7-4 Describe the exponential probability distribution and use it to calculate probabilities.

Introduction

A continuous probability distribution usually results from measuring something.

A continuous random variable has an infinite number of values within a particular range.

Probability is for a range of values.

The probability for a specific value is 0.

Properties of continuous distributions.

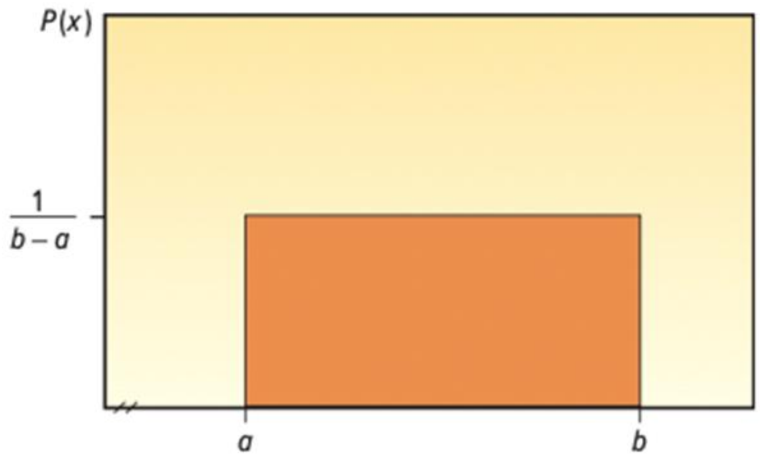
- Area under the curve is 1.
- $P(x) \geq 0$ for any x.

Uniform Distribution 1

- Why use a uniform distribution?
- We do not have any information about the shape of a random variable's distribution.
- No information that any event is more likely than any other.
- Each event is equally likely.
- "Information-less" distribution.
- We only need to know or estimate the min and max.

Uniform Distribution 2

- Defined between the min (a) and max (b).
- $P(x) = \frac{1}{b-a}$ if $a \le x \le b$ and 0 elsewhere.



- $P(x) \geq 0$.
- $\text{Area} = (\text{height})(\text{base}) = \frac{1}{(b-a)}(b-a) = 1$

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Uniform Distribution 3

The mean is the midpoint of the interval.

- $\mu = \dfrac{a+b}{2}$
- The mean is also the median.

The standard deviation is related to the min and max.

- $\sigma = \sqrt{\dfrac{(b-a)^2}{12}}$
- The standard deviation describes the dispersion of the distribution.

Uniform Distribution 4

Example: Southwest Arizona State University provides bus service to students while they are on campus.

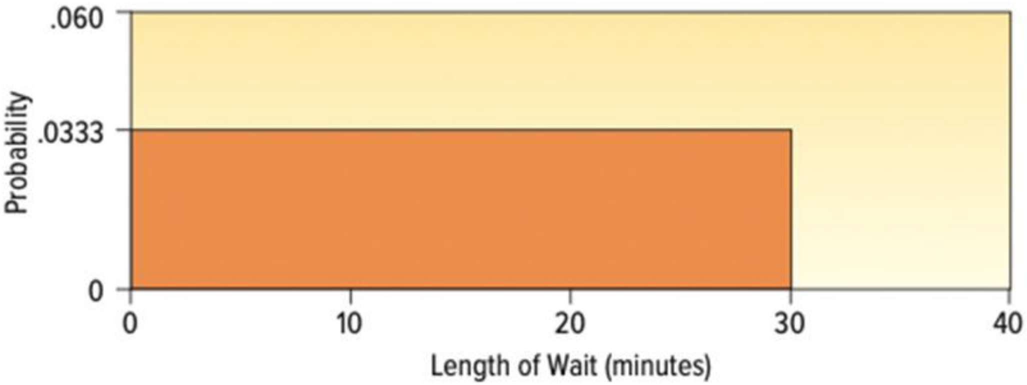
Students arrive at the bus stop at random times. The time that a student waits is uniformly distributed from 0 to 30 minutes.

- Draw a graph of this distribution.
- Show that the probability of any value between 0 and 30 is equal to 1.0.
- What is the mean of the distribution?
- What is the standard deviation of the distribution?
- What is the probability a student will wait more than 25 minutes?

Uniform Distribution 5

Example continued.

Draw a graph of this distribution.



Show that the probability of any value between 0 and 30 is equal to 1.0.

- Probability $= (\text{base})(\text{height}) = \dfrac{1}{(30-0)}(30-0) = 1.00$

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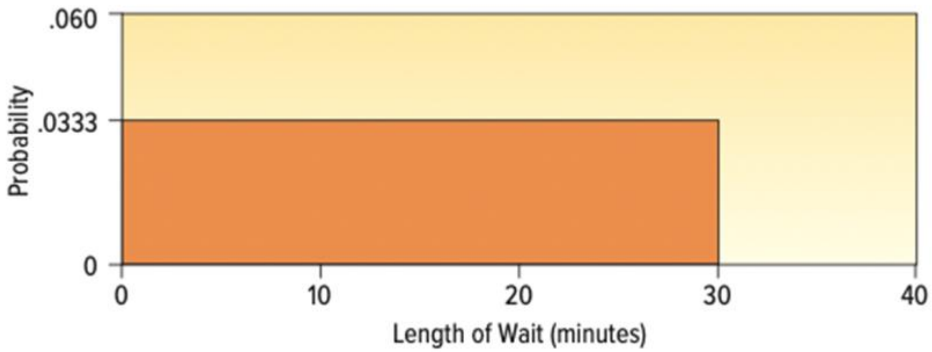
Uniform Distribution 6

Example continued.



What is the mean of the distribution?

- $\mu = \dfrac{a+b}{2} = \dfrac{0+30}{2} = 15 \text{ minutes}$

What is the standard deviation of the distribution?

- $\sigma = \sqrt{\dfrac{(b-a)^2}{12}} = \sqrt{\dfrac{(30-0)^2}{12}} = 8.66 \text{ minutes}$

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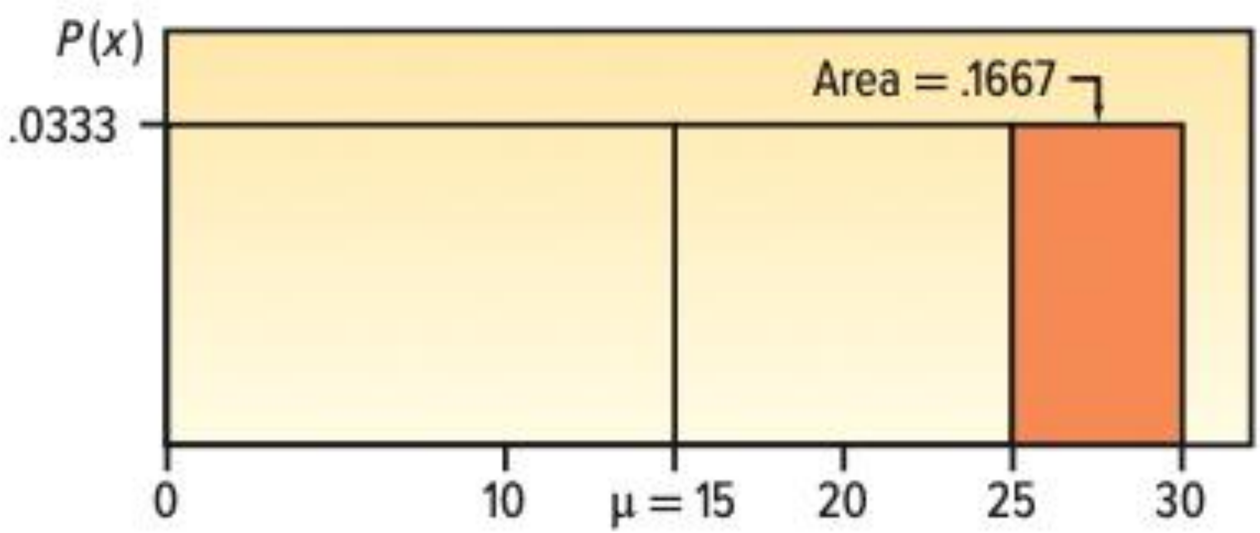
Uniform Distribution [7]

Example continued.

What is the probability a student will wait more than 25 minutes?

- $P(25 < \text{wait time} < 30) = (\text{height})(\text{base}) = \dfrac{1}{(30-5)}(5) = .1667$



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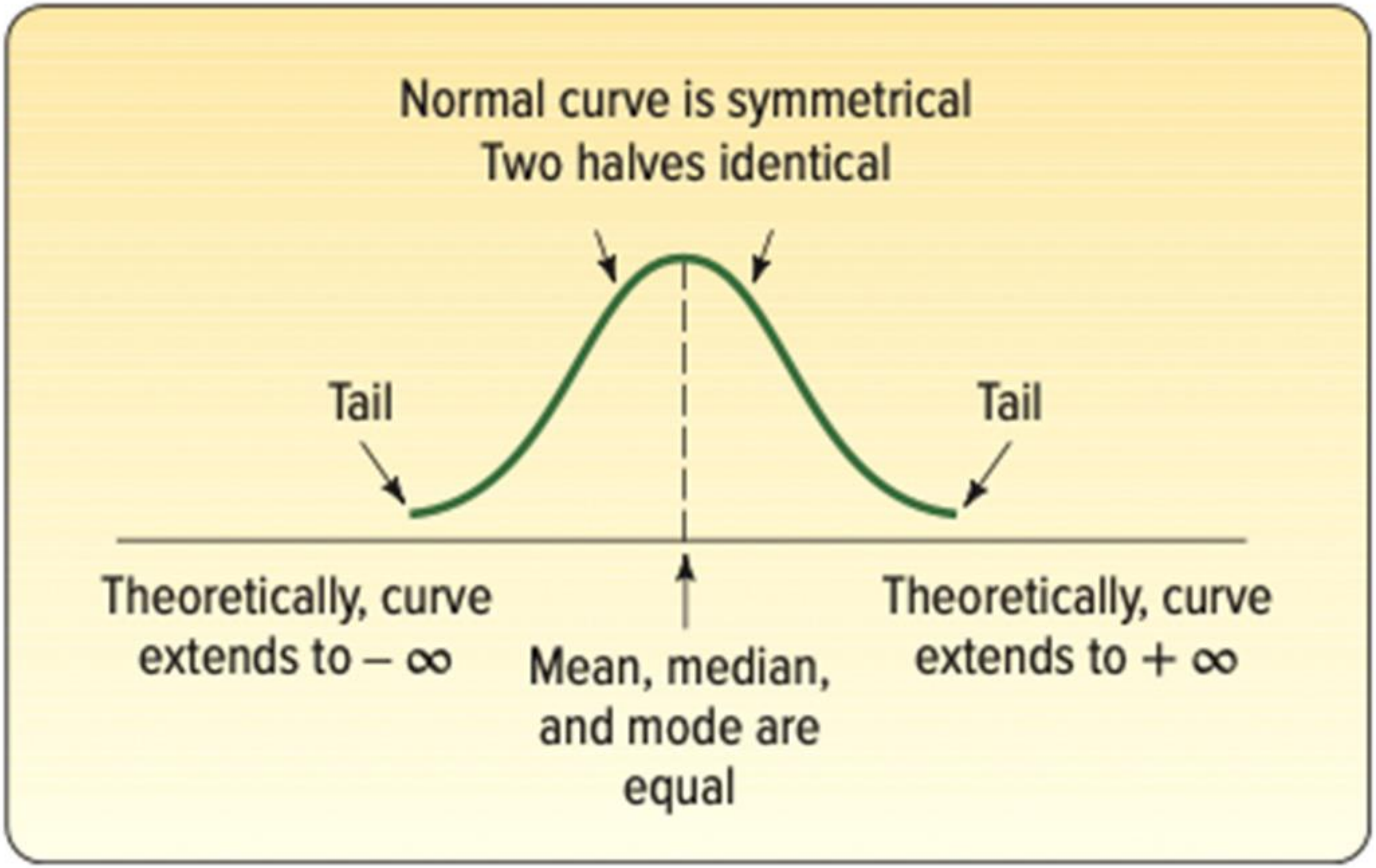
The Family of Normal Probability Distribution 1

The normal probability distribution is a continuous distribution with the following characteristics:

- Bell-shaped.
- A single peak at the center of the distribution.
- Symmetric.
- Asymptotic: The curve approaches but never touches the x-axis.
- Completely described by its mean and standard deviation.

There is a family of normal probability distributions for each combination of the mean and standard deviation.

The Family of Normal Probability Distribution 2



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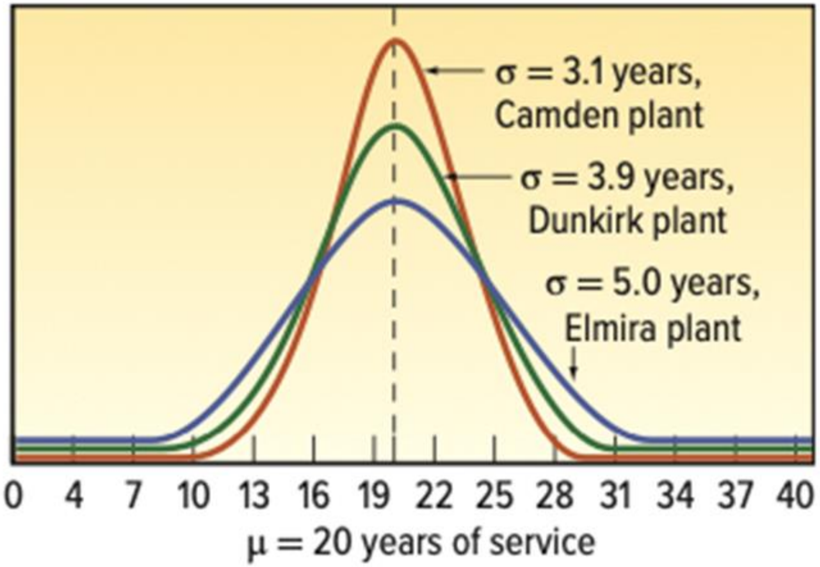
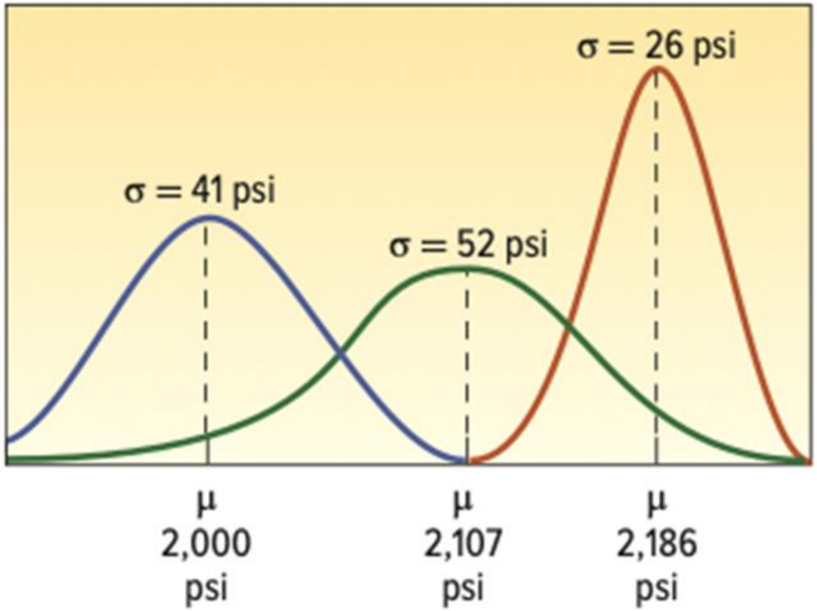
The Family of Normal Probability Distribution [3]





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The Standard Normal Probability Distribution [1]

Standard normal distribution.

- Mean of 0.
- Standard deviation of 1.

z Value The signed distance between a selected value, designated x, and the mean, μ, divided by the standard deviation, σ.

Any normal distribution can be converted to the standard normal distribution.

$$z = \frac{x - \mu}{\sigma}$$

The Standard Normal Probability Distribution 2

- Areas under the standard normal distribution
- The area between 0 and the value of z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	...	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	...	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	...	0.0753
•	•	•	•	•	•	•	•	...	•
•	•	•	•	•	•	•	•	...	•
•	•	•	•	•	•	•	•	...	•
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	...	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	...	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	...	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	...	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	...	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	...	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	...	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	...	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	...	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	...	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	...	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	...	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	...	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	...	0.4890
•	•	•	•	•	•	•	•	...	•
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The Standard Normal Probability Distribution 3

Example: Rideshare services are available internationally. A customer uses a smartphone app to request a ride.

Suppose the weekly income of rideshare drivers follows the normal probability distribution with a mean of $1,000 and a standard deviation of $100.

What is the *z* value of the income for a driver who earns $1,100 per week?

- $z = \dfrac{x - \mu}{\sigma} = \dfrac{\$1{,}100 - \$1{,}000}{\$100} = 1.00$
- This indicates that $1,100 is 1 standard deviation from the mean of $1,000.

The Standard Normal Probability Distribution 4

Example continued.

What is the likelihood that a randomly selected driver earns between $1,000 and $1,100 per week?

z-value for $1,100 is 1; z-value for $1000 is 0.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	•••	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	•••	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	•••	0.0753
•	•	•	•	•	•	•	•	•••	•
•	•	•	•	•	•	•	•	•••	•
•	•	•	•	•	•	•	•	•••	•
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	•••	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	•••	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	•••	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	•••	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	•••	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	•••	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	•••	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	•••	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	•••	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	•••	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	•••	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	•••	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	•••	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	•••	0.4890
•	•	•	•	•	•	•	•	•••	•
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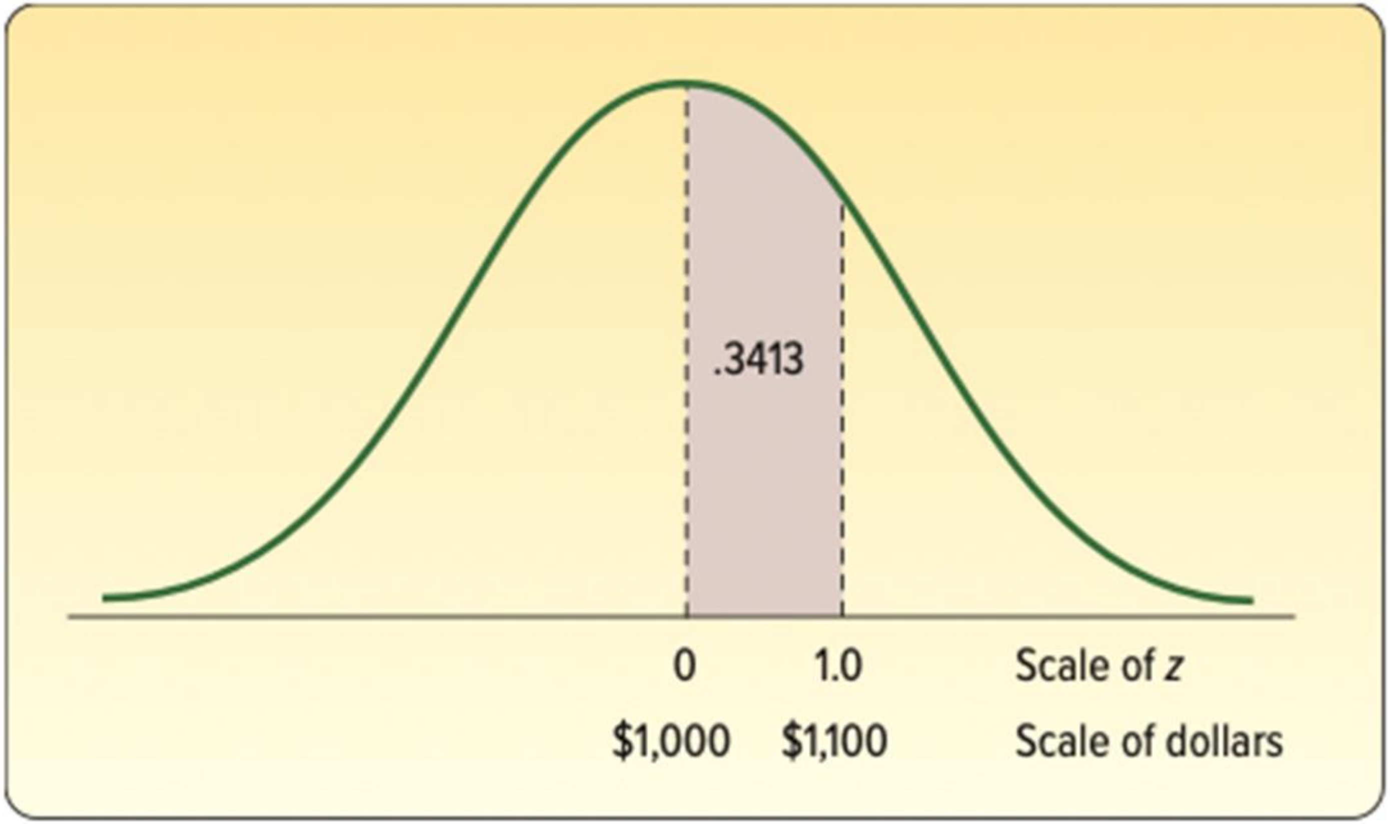
The Standard Normal Probability Distribution [5]

- Example continued.



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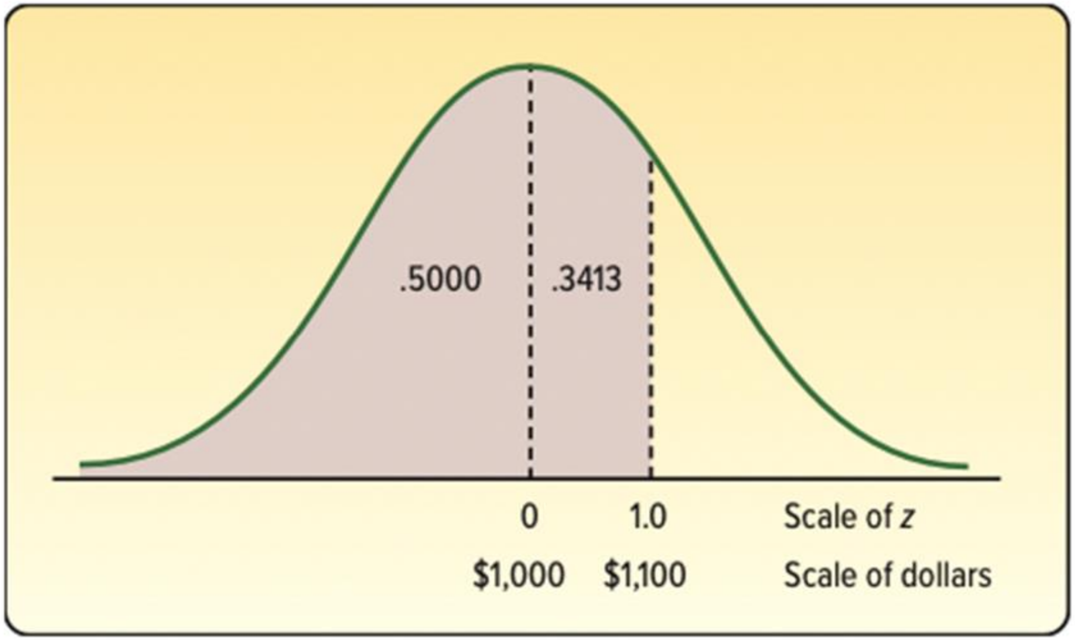
The Standard Normal Probability Distribution 6

- Example continued.
- What is the probability a randomly selected driver earns less than $1,100 per week?
- Half of the area is less than the mean of $1,000.



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The Standard Normal Probability Distribution 7

- Example continued.
- What is the probability a randomly selected driver earns between \$790 and \$1,000?
- $$z = \frac{x - \mu}{\sigma} = \frac{\$790 - \$1,000}{\$100} = -2.10$$
- The normal curve is symmetric.
- The area from -2.10 to 0 is the same as 0 to 2.10.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	•••	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	•••	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	•••	0.0753
•	•	•	•	•	•	•	•	•••	•
•	•	•	•	•	•	•	•	•••	•
•	•	•	•	•	•	•	•	•••	•
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	•••	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	•••	0.3621
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1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	•••	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	•••	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	•••	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	•••	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	•••	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	•••	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	•••	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	•••	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	•••	0.4890
•	•	•	•	•	•	•	•	•••	•
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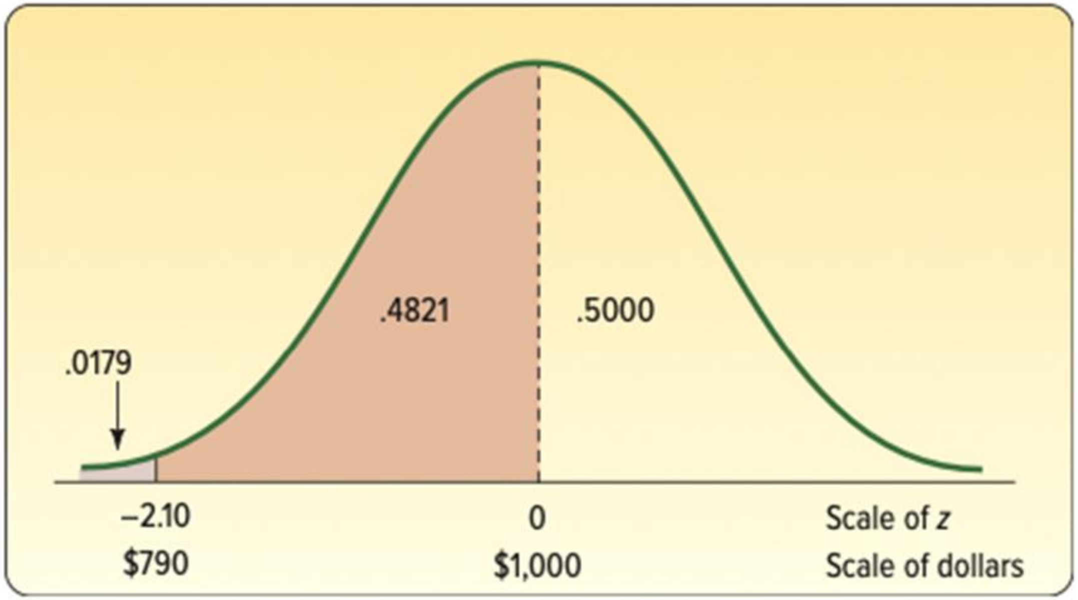
The Standard Normal Probability Distribution 8

- Example continued.
- What is the probability a randomly selected driver earns less than $790 per week?
- $z = -2.10$.



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The Standard Normal Probability Distribution 9

- Example continued.
- What is the probability a randomly selected driver earns between \$845 and \$1,200 per week?
- $z = \dfrac{\$845 - \$1,000}{\$100} = -1.55$
- $z = \dfrac{\$1,200 - \$1,000}{\$100} = 2.00$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	•••	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	•••	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	•••	0.0753
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•	•	•	•	•	•	•	•	•••	•
•	•	•	•	•	•	•	•	•••	•
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	•••	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	•••	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	•••	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	•••	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	•••	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	•••	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	•••	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	•••	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	•••	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	•••	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	•••	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	•••	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	•••	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	•••	0.4890
•	•	•	•	•	•	•	•	•••	•
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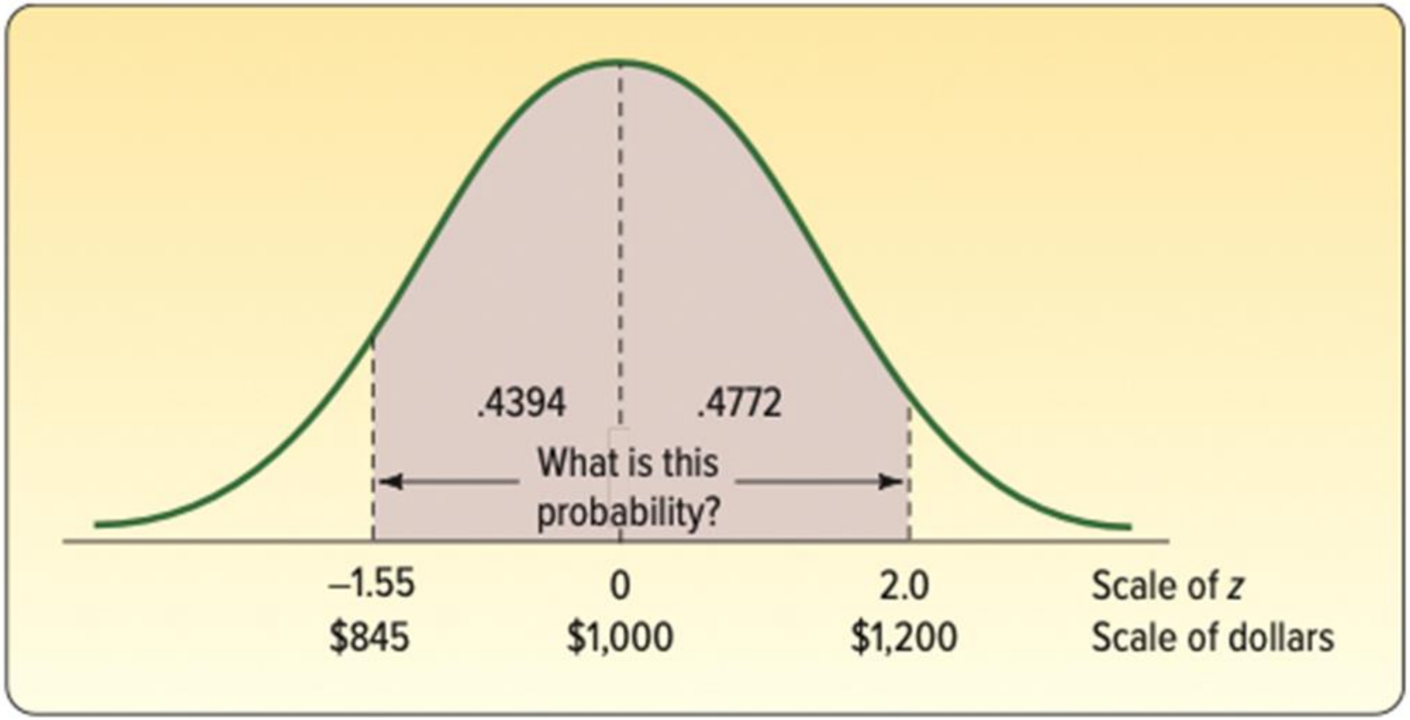
The Standard Normal Probability Distribution 10

- Example continued.



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The Standard Normal Probability Distribution 11

- Example continued
- What is the probability a randomly selected driver earns between \$1,150 and \$1,215 per week?
- $z = \dfrac{\$1,150 - \$1,000}{\$100} = 1.5$
- $z = \dfrac{\$1,215 - \$1,000}{\$100} = 2.15$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	···	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	···	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	···	0.0753
·	·	·	·	·	·	·	·	···	·
·	·	·	·	·	·	·	·	···	·
·	·	·	·	·	·	·	·	···	·
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	···	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	···	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	···	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	···	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	···	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	···	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	···	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	···	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	···	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	···	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	···	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	···	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	···	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	···	0.4890
·	·	·	·	·	·	·	·	···	·
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·	·	·	·	·	·	·	·	···	·

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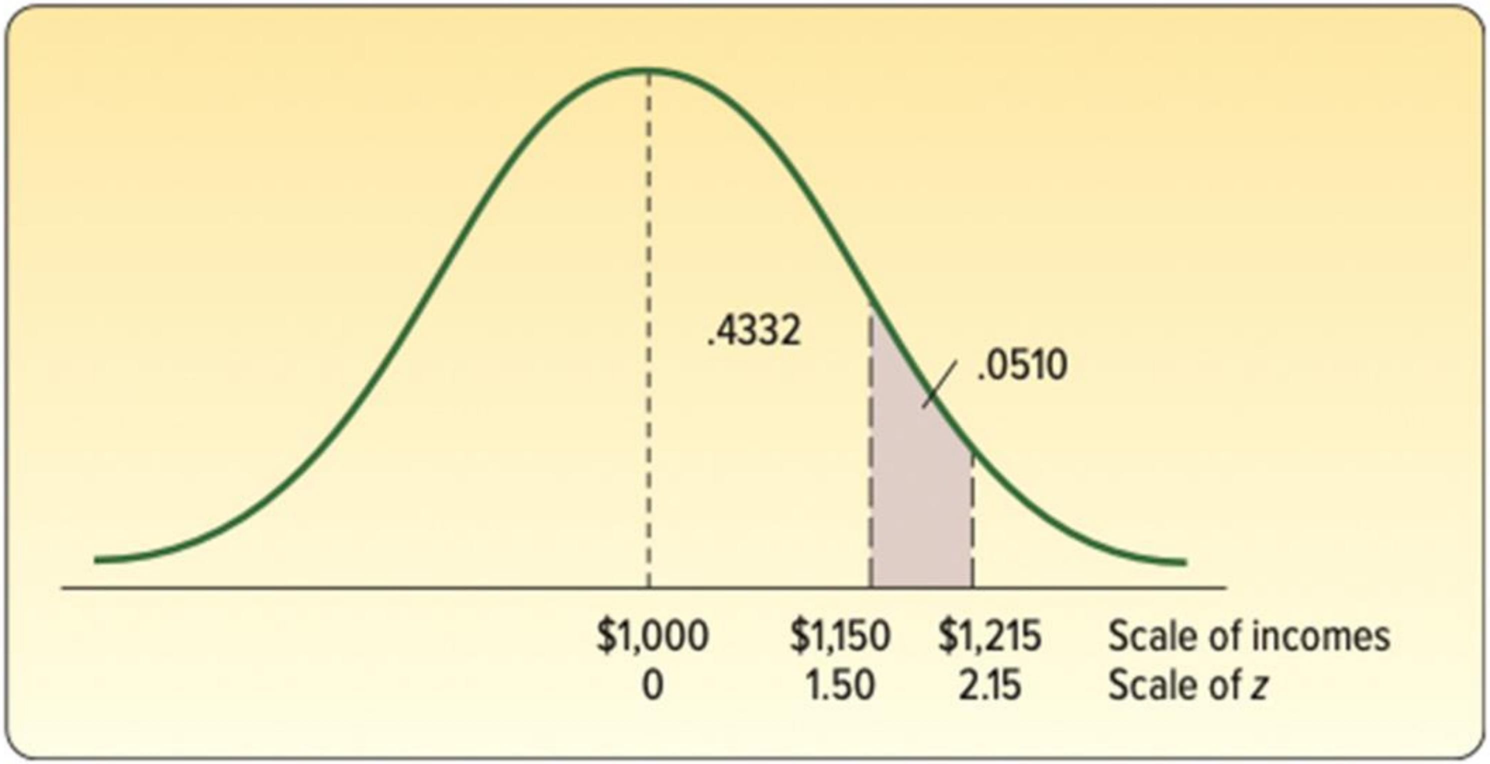
The Standard Normal Probability Distribution 12

- Example continued.



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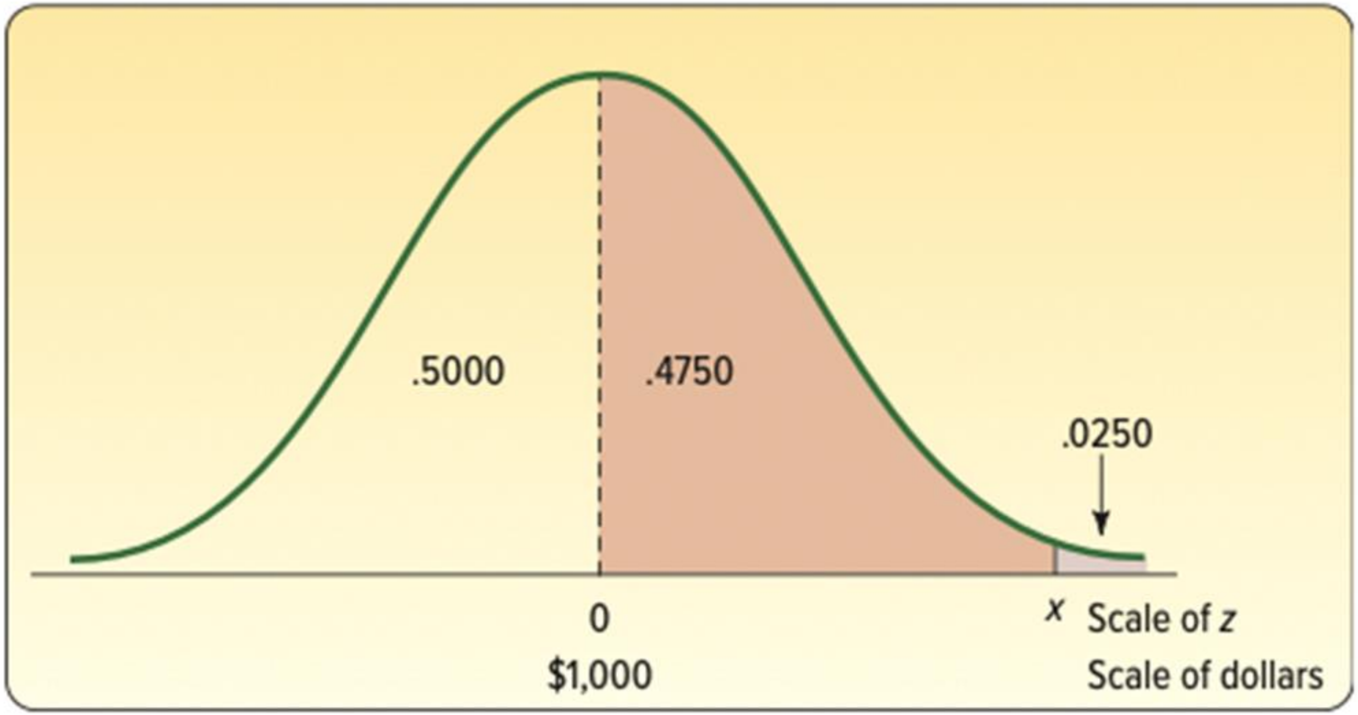
The Standard Normal Probability Distribution 13

- Example continued.
- What is the 97.5 percentile of the weekly earnings?



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The Standard Normal Probability Distribution 14

- Example continued.

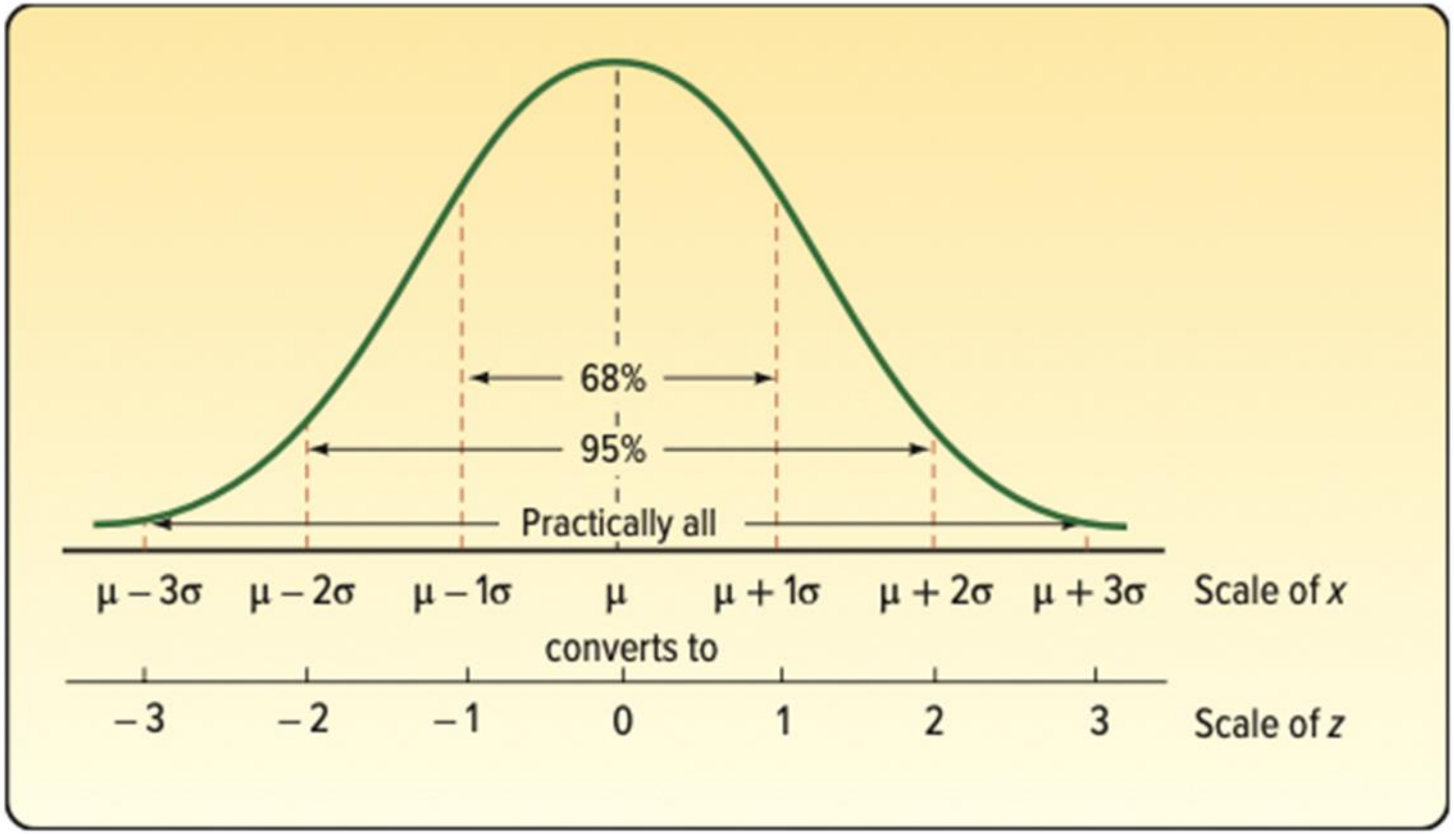
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	•••	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	•••	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	•••	0.0753
•	•	•	•	•	•	•	•	•••	•
•	•	•	•	•	•	•	•	•••	•
•	•	•	•	•	•	•	•	•••	•
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	•••	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	•••	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	•••	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	•••	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	•••	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	•••	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	•••	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	•••	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	•••	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	•••	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	•••	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	•••	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	•••	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	•••	0.4890
•	•	•	•	•	•	•	•	•••	•
•	•	•	•	•	•	•	•	•••	•
•	•	•	•	•	•	•	•	•••	•

- $x = \mu + z\sigma = 1{,}000 + 1.96(100) = \$1{,}196$

Access the text alternative for slide images.

The Standard Normal Probability Distribution 15

- Recall the empirical rule.



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The Family of Exponential Distributions 1

Describes times between events in a sequence.

Events are independent at a constant rate.

They are non-negative, always positive.

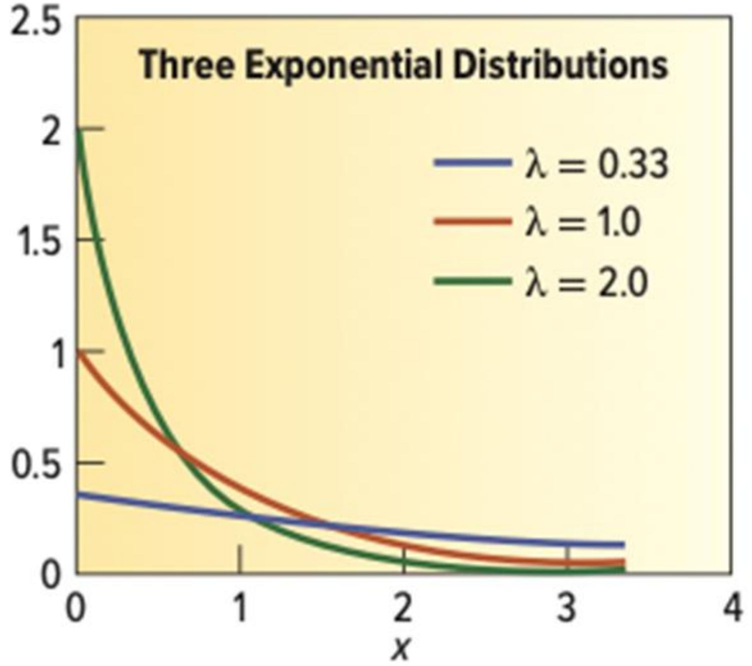
Positively skewed.

Examples of situations using the exponential distributions.

- The service time for customers at the information desk at Dallas Public Library.
- The time until the next phone call arrives in a customer service center.

The Family of Exponential Distributions 2

- Distribution is described by a single parameter.
- λ (lambda): the rate parameter.
- Decrease λ shape is "less skewed".



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The Family of Exponential Distributions 3

- Related to the Poisson distribution for the number of occurrences.
- Exponential distribution: $P(x) = \lambda e^{-\lambda x}$
- Area under the curve: $P(\text{Arrival time} < x) = 1 - e^{-\lambda x}$
- Mean: $\mu = \dfrac{1}{\lambda}$
- Standard deviation: $\sigma = \dfrac{1}{\lambda}$

The Family of Exponential Distributions 4

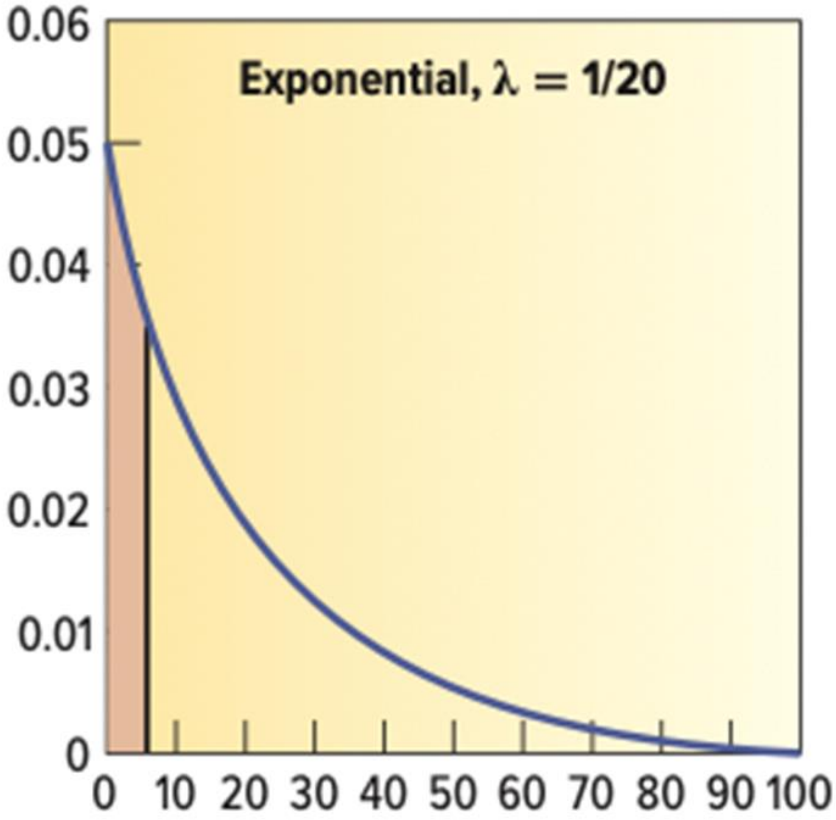
- Example: Orders for prescriptions arrive at a pharmacy website according to an exponential probability distribution at a mean of one every 20 seconds.
- Find the probability the next order arrives in less than 5 seconds.
- $\lambda = \frac{1}{20}$
- $P(Arrival\,time < 5) = 1 - e^{-\frac{1}{20}5}$

$$= 1 - .7788$$

$$= .2212$$

The Family of Exponential Distributions 5

- Example continued.



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Chapter 7 Practice Problems

Question 1

LO-1

A uniform distribution is defined over the interval from 6 to 10.

a. What are the values for *a* and *b*?

b. What is the mean of this uniform distribution?

c. What is the standard deviation?

d. Show that the probability of any value between 6 and 10 is equal to 1.0.

e. What is the probability that the random variable is more than 7?

f. What is the probability that the random variable is between 7 and 9?

g. What is the probability that the random variable is equal to 7.91?

Question 7

LO7-3

A normal population has a mean of 20.0 and a standard deviation of 4.0.

a. Compute the z value associated with 25.0.

b. What proportion of the population is between 20.0 and 25.0?

c. What proportion of the population is less than 18.0?

Question 10

LO7-3

The mean of a normal probability distribution is 400; the standard deviation is 10.

a. About 68% of the observations lie between what two values?

b. About 95% of the observations lie between what two values?

c. Practically all of the observations lie between what two values?

Question 13

LO7-3

The Internal Revenue Service reported the average refund in 2022 was $3,401 with a standard deviation of $82.5. Assume the amount refunded is normally distributed.

a. What percent of the refunds are more than $3,500?

b. What percent of the refunds are more than $3,500 but less than $3,579?

c. What percent of the refunds are more than $3,325 but less than $3,579?

Question 21

LO7-3

According to media research, the typical American listened to 195 hours of music in the last year. This is down from 290 hours 4 years earlier. Dick Trythall is a big country and western music fan. He listens to music while working around the house, reading, and riding in his truck. Assume the number of hours spent listening to music follows a normal probability distribution with a standard deviation of 8.5 hours.

a. If Dick is in the top 1% in terms of listening time, how many hours did he listen last year?

b. Assume that the distribution of times 4 years earlier also follows the normal probability distribution with a standard deviation of 8.5 hours. How many hours did the 1% who listen to the *least* music actually listen?

Question 31

LO7-4

Waiting times to receive food after placing an order at the local Subway sandwich shop follow an exponential distribution with a mean of 60 seconds. Calculate the probability a customer waits:

a. Less than 30 seconds.

b. More than 120 seconds.

c. Between 45 and 75 seconds.

d. Fifty percent of the patrons wait less than how many seconds? What is the median?

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