Because learning changes everything.®

# Chapter 6

## Discrete Probability Distributions

# Learning Objectives

LO6-1 Identify the characteristics of a probability distribution.

LO6-2 Distinguish between discrete and continuous random variables.

LO6-3 Compute the mean, variance, and standard deviation of a discrete probability distribution.

LO6-4 Explain the assumptions of the binomial distribution and apply it to calculate probabilities.

LO6-5 Explain the assumptions of the hypergeometric distribution and apply it to calculate probabilities.

LO6-6 Explain the assumptions of the Poisson distribution and apply it to calculate probabilities.

# What is a Probability Distribution? 1

**Probability Distribution** A listing of all the outcomes of an experiment and the probability associated with each outcome.

**Characteristics of a Probability Distribution**

1. The probability of a particular outcome is between 0 and 1 inclusive.
2. The outcomes are mutually exclusive.
3. The list of outcomes is exhaustive. So the sum of the probabilities of the outcomes is equal to 1.

# What is a Probability Distribution? 2

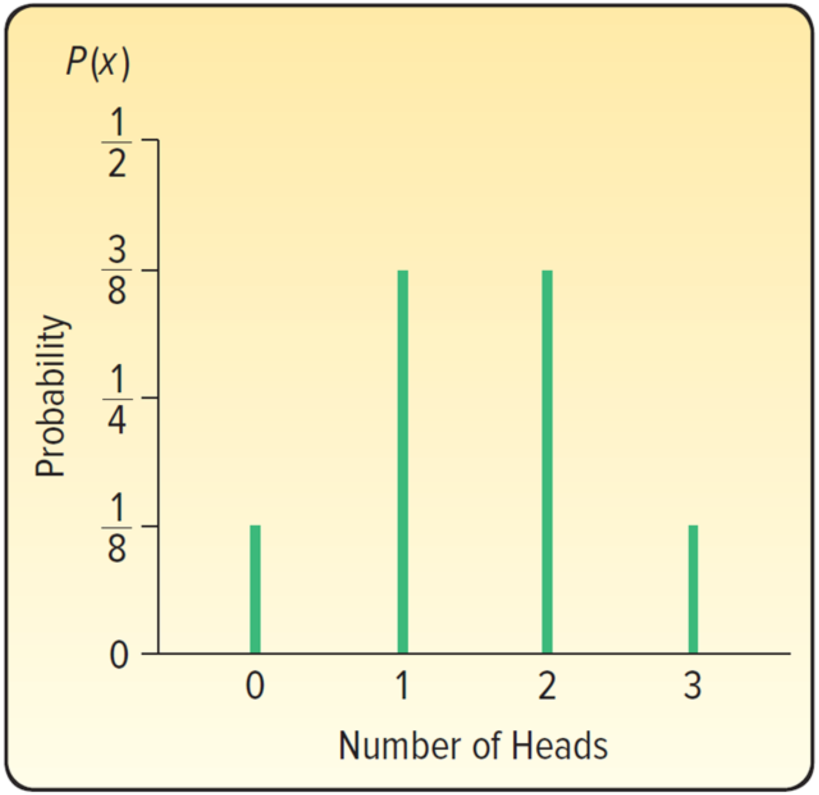
- Example: Suppose we are interested in the number of heads showing face up with 3 tosses of a coin.
- The possible outcomes are 0 heads, 1 head, 2 heads and 3 heads.

| Possible Result | First | Coin Toss Second | Third | Number of Heads |
|---|---|---|---|---|
| 1 | T | T | T | 0 |
| 2 | T | T | H | 1 |
| 3 | T | H | T | 1 |
| 4 | T | H | H | 2 |
| 5 | H | T | T | 1 |
| 6 | H | T | H | 2 |
| 7 | H | H | T | 2 |
| 8 | H | H | H | 3 |

# What is a Probability Distribution? 3

- Example continued.

| Number of Heads,x | Probability of Outcome , P ( x ) |
| --- | --- |
| 0 | $\frac{1}{8} = .125$ |
| 1 | $\frac{3}{8} = .375$ |
| 2 | $\frac{3}{8} = .375$ |
| 3 | $\frac{1}{8} = .125$ |
| **Total** | $\frac{8}{8} = 1.000$ |




Access the text alternative for slide images.

# Random Variables [1]

- In any experiment of chance, the outcomes occur randomly and are called random variables.

**Random Variable** A variable measured or observed as the result of an experiment. By chance, the variable can have different values.

- Example: The grade level (Freshman, Sophomore, Junior, or Senior) of the members of the St. James High School Varsity girls' basketball team. Grade level is the random variable (and notice that it is a qualitative variable).

# Random Variables 2

Discrete variables are usually the result of counting.

The data are summarized with a relative frequency table.

**Discrete Random Variable** A random variable that can assume only certain clearly separated values.

Examples.

- Tossing a coin three times and counting the number of heads.
- A department store offers coupons with discounts of 10%, 15% and 25%.

# Random Variables 3

- Example: The Bank of the Carolinas counts the number of credit cards carried by a group of customers.
- The number of cards carried is the discrete random variable.

| Number of Credit Cards | Relative Frequency |
|---|---|
| 0 | .03 |
| 1 | .10 |
| 2 | .18 |
| 3 | .21 |
| 4 or more | .48 |
| Total | 1.00 |

# Random Variables 4

**Continuous Random Variable** A random variable that may assume an infinite number of values within a given range.

Continuous variables are usually the result of measuring.

Summarized with a probability distribution.

Examples.

- The time between flights between Atlanta and LA are 4.67 hours, 5.13 hours, and so on
- The annual snowfall in Minneapolis, MN measured in inches

# Mean, Variance, and Standard Deviation of a Discrete Probability Distribution 1

The mean is a typical value used to represent the central location of the data.

The long-run average.

Also referred to as the expected value.

$$\mu = \sum [xP(x)]$$

- *x* is a particular value.
- $P(x)$ is the probability of a particular value.

This is a weighted average.

# Mean, Variance, and Standard Deviation of a Discrete Probability Distribution 2

The variance describes the amount of spread (or variation) in the data

$$\sigma^2 = \sum[(x-\mu)^2 P(x)]$$

The computational steps are:

- Subtract the mean from each value, square the difference.
- Multiply each squared difference by its probability.
- Sum the resulting products.

The standard deviation is the positive square root of the variance.

# Mean, Variance, and Standard Deviation of a Discrete Probability Distribution 3

- Example: The number of new cars sold on Saturday has the below probability distribution.

| Number of Cars Sold, *x* | Probability, *P*(*x*) |
|---|---|
| 0 | .1 |
| 1 | .2 |
| 2 | .3 |
| 3 | .3 |
| 4 | .1 |
| | 1.0 |

1. What type of distribution is this?
2. How many cars do you expect to sell?
3. What is the variance?

# Mean, Variance, and Standard Deviation of a Discrete Probability Distribution 4

- Example continued.
- This is a discrete probability distribution.
- The mean is 2.1.

| Number of Cars Sold, x | Probability, P(x) | $x \cdot P(x)$ |
|---|---|---|
| 0 | .1 | 0.0 |
| 1 | .2 | 0.2 |
| 2 | .3 | 0.6 |
| 3 | .3 | 0.9 |
| 4 | .1 | 0.4 |
| | 1.0 | $\mu = 2.1$ |

- The variance is 1.290 so the standard deviation is 1.136.

| Number of Cars Sold, x | Probability, P(x) | $(x - \mu)$ | $(x - \mu)^2$ | $(x - \mu)^2 P(x)$ |
|---|---|---|---|---|
| 0 | .1 | $0 - 2.1$ | 4.41 | 0.441 |
| 1 | .2 | $1 - 2.1$ | 1.21 | 0.242 |
| 2 | .3 | $2 - 2.1$ | 0.01 | 0.003 |
| 3 | .3 | $3 - 2.1$ | 0.81 | 0.243 |
| 4 | .1 | $4 - 2.1$ | 3.61 | 0.361 |
| | 1.0 | | | $\sigma^2 = 1.290$ |

# Binomial Probability Distribution 1

There are four requirements of a binomial probability distribution.

1. There are only two outcomes (success or a failure) that are mutually exclusive.
2. The number of trials is fixed and known.
3. The probability of a success is the same for each trial.
4. Each trial is independent of any other trial.

Example: A young family has two children, both boys.

- The probability of the third birth being a boy is still .50.
- The gender of the third child is independent of the gender of the other two.

# Binomial Probability Distribution 2

To construct a binomial probability, use the number of trials and probability of success

$$P(x) = {}_nC_x \pi^x (1-\pi)^{n-x}$$

- *C* is the combination of the $x$ successes.
- *n* is the number of trials.
- *x* is the number of successes.
- $\pi$ is the probability of success.
- For $x = 0, 1, \cdots, n$
- Do not confuse $\pi$ with 3.1416 . . .

Mean: $n\pi$

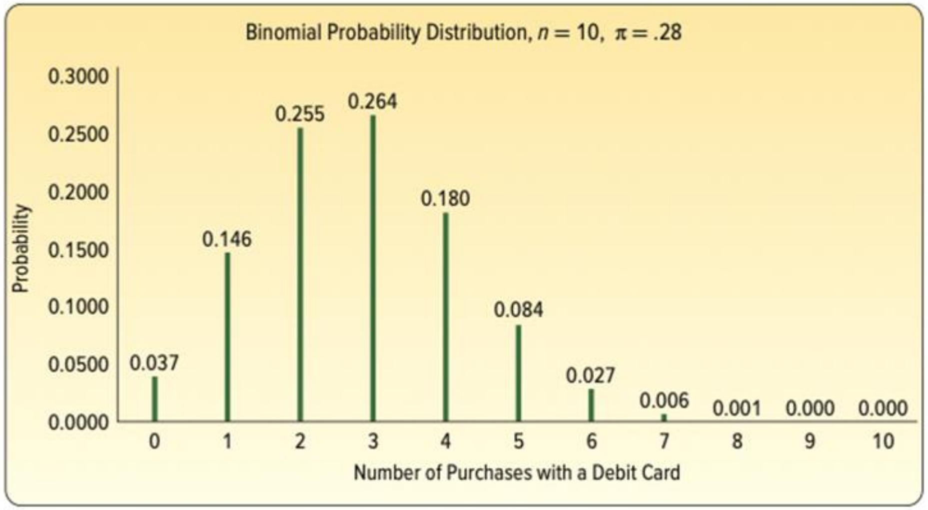
Variance: $n\pi(1-\pi)$

# Binomial Probability Distribution 3

- Example: Recently, www.creditcards.com reported that 28% of purchases at coffee shops were made with a debit card.
- For 10 randomly selected purchases at the Starbucks on the corner of 12th Street and Main, what is the probability that exactly one of the purchases was made with a debit card?
- What is the probability distribution for the number of purchases made with a debit card?
- What is the probability that six or more purchase out of 10 are made with a debit card?
- What is the mean and standard deviation of the number of purchases made with a credit card?

# Binomial Probability Distribution 4

- Example continued.
- $P(1) = {}_{n}C_{x}\pi^{x}(1-\pi)^{n-x} = {}_{10}C_{1}.28^{1}(1-.28)^{10-1} = 0.1456.$




| Number of Debit Card Purchases (*x*) | P(*x*) |
|---|---|
| 0 | 0.037 |
| 1 | 0.146 |
| 2 | 0.255 |
| 3 | 0.264 |
| 4 | 0.180 |
| 5 | 0.084 |
| 6 | 0.027 |
| 7 | 0.006 |
| 8 | 0.001 |
| 9 | 0.000 |
| 10 | 0.000 |

- $P(x \geq 6) = 0.027 + \text{L} + 0.00 = 0.034.$

Access the text alternative for slide images.

# Binomial Probability Distribution 5

- Example continued.
- Mean:
- Variance:

  so the standard deviation is 1.42
- Could find the mean and variance by definition.

| Number of DebitCard Purchases (x) | Probability, P(x) | $x \cdot P(x)$ | $(x-\mu)$ | $(x-\mu)^2$ | $P(x)*(x-\mu)^2$ |
|---|---|---|---|---|---|
| 0 | 0.037 | 0.0000 | $-2.8001$ | 7.841 | 0.2935 |
| 1 | 0.146 | 0.1456 | $-1.8001$ | 3.240 | 0.4718 |
| 2 | 0.255 | 0.5096 | $-0.8001$ | 0.640 | 0.1631 |
| 3 | 0.264 | 0.7927 | 0.1999 | 0.040 | 0.0106 |
| 4 | 0.180 | 0.7193 | 1.1999 | 1.440 | 0.2589 |
| 5 | 0.084 | 0.4196 | 2.1999 | 4.840 | 0.4061 |
| 6 | 0.027 | 0.1632 | 3.1999 | 10.239 | 0.2785 |
| 7 | 0.006 | 0.0423 | 4.1999 | 17.639 | 0.1065 |
| 8 | 0.001 | 0.0071 | 5.1999 | 27.039 | 0.0238 |
| 9 | 0.000 | 0.0007 | 6.1999 | 38.439 | 0.0029 |
| 10 | 0.000 | 0.0000 | 7.1999 | 51.839 | 0.0002 |
| **Total** | | 2.8001 | | | 2.0160 |

# Binomial Probability Distribution [6]

- Tables are available for various $n$ and $\pi$.

*n* = 6 Probability

| $x \setminus \pi$ | **0.5** | .1 | .2 | .3 | .4 | .5 | .6 | .7 | .8 | .9 | .95 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | **.735** | .531 | .262 | .118 | .047 | .016 | .004 | .001 | .000 | .000 | .000 |
| 1 | **.232** | .354 | .393 | .303 | .187 | .094 | .037 | .010 | .002 | .000 | .000 |
| 2 | **.031** | .098 | .246 | .324 | .311 | .234 | .138 | .060 | .015 | .001 | .000 |
| 3 | **.002** | .015 | .082 | .185 | .276 | .313 | .276 | .185 | .082 | .015 | .002 |
| 4 | **.000** | .001 | .015 | .060 | .138 | .234 | .311 | .324 | .246 | .098 | .031 |
| 5 | **.000** | .000 | .002 | .010 | .037 | .094 | .187 | .303 | .393 | .354 | .232 |
| 6 | **.000** | .000 | .000 | .001 | .004 | .016 | .047 | .118 | .262 | .531 | .735 |

# Hypergeometric Probability Distribution [1]

For the binomial distribution, the probability of success must stay the same for each trial.

- Large population.
- Sampling with replacement.

Most sampling is done without replacement.

If the population is small, the probability of success will change for each observation.

Example: The population has 20 items.

- First sample: 1/20.
- Second sample: 1/19.
- Third sample: 1/18.

# Hypergeometric Probability Distribution 2

Hypergeometric criteria.

1. There are only two outcomes (success or a failure) that are mutually exclusive.
2. Fixed number of independent trials.
3. Sample from a finite population without replacement.

- $n/N > 0.05$.
- The probability of success changes for each trial.

$$P(x) = \frac{\left({}_{S}C_{x}\right)\left({}_{N-S}C_{n-x}\right)}{{}_{N}C_{n}}$$

- $N$ is the size of the population, $n$ is size of the sample.
- $S$ is the number of successes in the population.

# Hypergeometric Probability Distribution 3

- Example: Play Time Toys Inc. employs 50 people in the Assembly Dept.
- Forty of the employees belong to a union and 10 do not.
- Five employees are selected at random to form a committee.
- What is the probability that four of the five belong to a union?
- $$P(4)=\frac{\left({}_{40}C_{4}\right)\left({}_{50-40}C_{5-4}\right)}{{}_{50}C_{5}}=\frac{(91,390)}{2,118,760}=0.431$$

| Union Members | Probability |
|---|---|
| 0 | .000 |
| 1 | .004 |
| 2 | .044 |
| 3 | .210 |
| 4 | .431 |
| 5 | .311 |
| | 1.000 |

# Hypergeometric Probability Distribution 4

- Example continued.

| Number of Union Members on Committee | Hypergeometric Probability, P(x) | Binomial Probability ($n = 5$ and $\pi = .80$) |
|---|---|---|
| 0 | .000 | .000 |
| 1 | .004 | .006 |
| 2 | .044 | .051 |
| 3 | .210 | .205 |
| 4 | .210 | .410 |
| 5 | .311 | .328 |
| | 1.000 | 1.000 |

| x | Hypergeometric Probability, P(x) | Binomial Probability ($n = 5$ and $\pi = .80 = (120/150)$) |
|---|---|---|
| 0 | .000 | .000 |
| 1 | .006 | .006 |
| 2 | .049 | .051 |
| 3 | .206 | .205 |
| 4 | .417 | .410 |
| 5 | .322 | .328 |
| | 1.000 | 1.000 |

# Poisson Probability Distribution 1

Poisson describes the number of times some event occurs during a specified interval.

The interval can be time, distance, area or volume.

Assumptions.

- The probability is proportional to the length of the interval.
- The intervals are independent and do not overlap.

Examples.

- The distribution of errors in data entry.
- The number of accidents on I-75 during a three-month period.

# Poisson Probability Distribution 2

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

- $\mu$ is the mean number of occurrences.
- *e* is the constant 2.71828.
- For $x = 0, 1, \cdots$

Mean: $\mu = n\pi.$

Variance: $\sigma^2 = n\pi.$

- *n* is the total number of trials.
- $\pi$ is the probability of success.

# Poisson Probability Distribution 3

- Example: Budget Airlines is a seasonal airline that operates flights from Myrtle Beach, South Carolina, to various cities in the northeast.
- Recently Budget has been concerned about the number of lost bags.
- Ann Poston from the Analytics Department was asked to study the issue.
- She randomly selected a sample of 500 flights and found that a total of twenty bags were lost on the sampled flights.
- What is the mean number of bags lost per flight?
- What is the likelihood that no bags are lost on a flight.

# Poisson Probability Distribution 4

- Example continued.
- The mean is $\mu = 20/500 = .04.$
- The probability of no lost bags is given by.
- $P(0) = \frac{\mu^x e^{-\mu}}{x!} = \frac{.04^0 e^{-.04}}{0!} = 0.9608$

| | A | B |
|---|---|---|
| 1 | Success | Probability |
| 2 | 0 | 0.9608 |
| 3 | 1 | 0.0384 |
| 4 | 2 | 0.0008 |
| 5 | 3 | 0.0000 |
| 6 | 4 | 0.0000 |
| 7 | 5 | 0.0000 |
| 8 | 6 | 0.0000 |
| 9 | 7 | 0.0000 |

Access the text alternative for slide images.

# Poisson Probability Distribution 5

- Like with Binomial, there are tables.

$\mu$

| *x* | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.9048 | 0.8187 | 0.7408 | 0.6703 | 0.6065 | 0.5488 | 0.4966 | 0.4493 | 0.4066 |
| 1 | 0.0905 | 0.1637 | 0.2222 | 0.2681 | 0.3033 | 0.3293 | 0.3476 | 0.3593 | 0.3659 |
| 2 | 0.0045 | 0.0164 | 0.0333 | 0.0536 | 0.0758 | 0.0988 | 0.1217 | 0.1438 | 0.1647 |
| 3 | 0.0002 | 0.0011 | 0.0033 | 0.0072 | 0.0126 | 0.0198 | 0.0284 | 0.0383 | 0.0494 |
| 4 | 0.0000 | 0.0001 | 0.0003 | 0.0007 | 0.0016 | 0.0030 | 0.0050 | 0.0077 | 0.0111 |
| 5 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0002 | 0.0004 | 0.0007 | 0.0012 | 0.0020 |
| 6 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0002 | 0.0003 |
| 7 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

# Chapter 6 Practice Problems

# Question 5

LO6-2,3

The information below is the number of daily emergency service calls made by the volunteer ambulance service of Walterboro, South Carolina, for the last 50 days. To explain, there were 22 days when there were two emergency calls, and 9 days when there were three emergency calls.

| Number of Calls0 | Frequency |
|---|---|
| 0 | 8 |
| 1 | 10 |
| 2 | 22 |
| 3 | 9 |
| 4 | 1 |
| Total | 50 |

a. Convert this information on the number of calls to a probability distribution.

b. Is this an example of a discrete or continuous probability distribution?

c. What is the probability that 3 or more calls are made in a day?

d. What is the mean number of emergency calls per day?

e. What is the standard deviation of the number of calls made daily?

# Question 13

LO6-4

- An American Society of Investors survey found 30% of individual investors have used a discount broker. In a random sample of nine individuals, what is the probability:

a. Exactly two of the sampled individuals have used a discount broker?

b. Exactly four of them have used a discount broker?

c. None of them has used a discount broker?

# Question 21

LO6-4

- In a recent study, 90% of the homes in the United States were found to have large-screen TVs. In a sample of nine homes, what is the probability that:

a. All nine have large-screen TVs?

b. Less than five have large-screen TVs?

c. More than five have large-screen TVs?

d. At least seven homes have large-screen TVs?

# Question 25

LO6-5

- A youth basketball team has 12 players on their roster. Seven of the team members are boys and five are girls. The coach writes each player’s name on a sheet of paper and places the names in a hat. The team captain shuffles the names and the coach selects five slips of paper from the hat to determine the starting lineup.

a. What is the probability the starting lineup consists of three boys and two girls?

b. What is the probability the starting lineup is all boys?

c. What is the probability there is at least one girl in the starting lineup?

# Question 33

LO6-6

- Ms. Bergen is a loan officer at Coast Bank and Trust. From her years of experience, she estimates that the probability is .025 that an applicant will not be able to repay his or her installment loan. Last month she made 40 loans.

a. What is the probability that three loans will be defaulted?

b. What is the probability that at least three loans will be defaulted?

Because learning changes everything.®

www.mheducation.com