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Chapter 5

A Survey of Probability Concepts

Learning Objectives

LO5-1 Define the terms *probability*, *experiment*, *event*, and *outcome.*

LO5-2 Apply the classical approach to assign probabilities.

LO5-3 Determine the number of outcomes using principles of counting.

LO5-4 Apply the empirical approach to assign probabilities.

LO5-5 Apply the subjective approach to assign probabilities.

LO5-6 Calculate probabilities using the rules of addition.

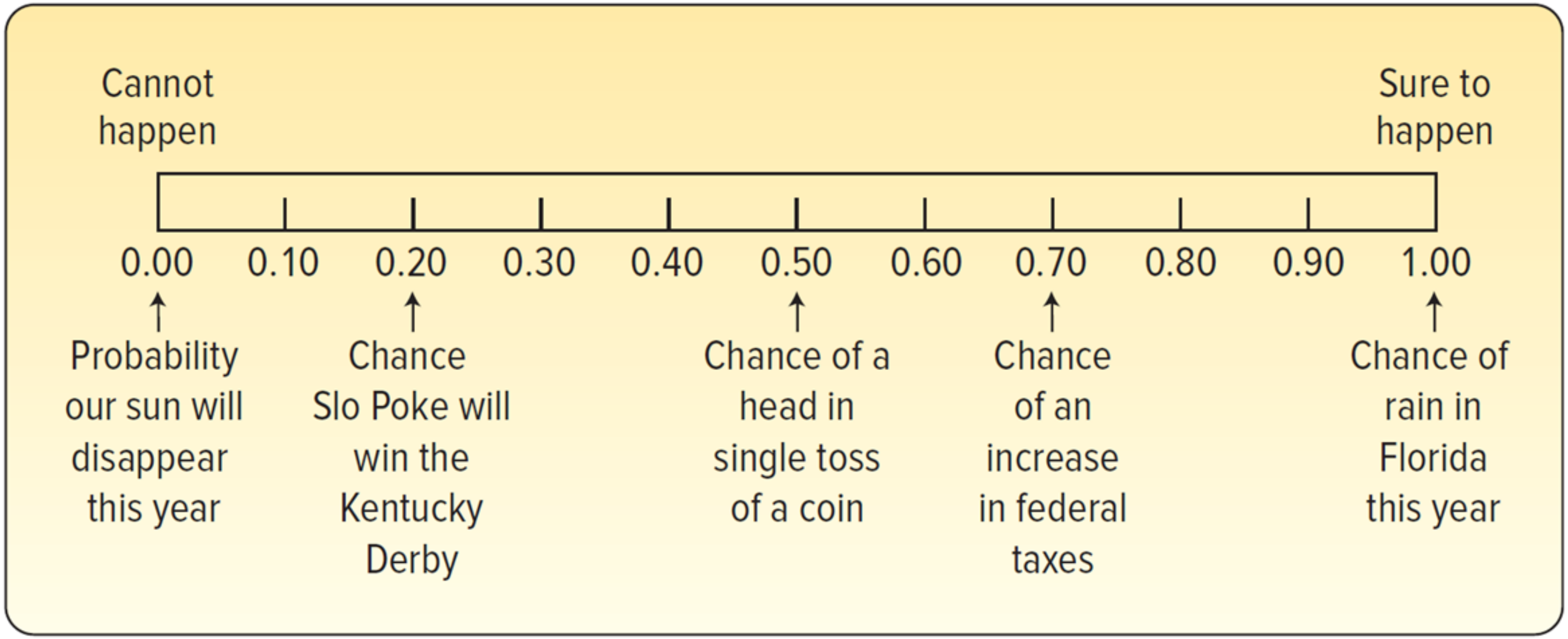
LO5-7 Calculate probabilities using the rules of multiplication.

LO5-8 Compute probabilities using a contingency table.

LO5-9 Calculate probabilities using Bayes’ theorem.

Probability [1]

Probability A value between 0 and 1 inclusive that represents the likelihood a particular event happens.



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Probability 2

Experiment A process that leads to the occurrence of one and only one of several possible results.

Outcome A particular result of an experiment.

Event A collection of one or more outcomes of an experiment.

		
Experiment	Roll a die	Count the number of members of the board of directors for Fortune 500 companies who are over 60 years of age
All possible outcomes	Observe a 1 Observe a 2 Observe a 3 Observe a 4 Observe a 5 Observe a 6	None is over 60 One is over 60 Two are over 60 ... 29 are over 60 ... ... 48 are over 60 ...
Some possible events	Observe an even number Observe a number greater than 4 Observe a number 3 or less	More than 13 are over 60 Fewer than 20 are over 60

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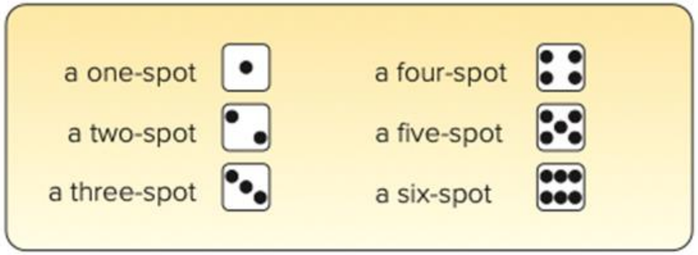
Approaches to Assigning Probabilities 1

- Classical probability is based on the assumption that the outcomes of an experiment are equally likely.

$$\text{Probability of an Event} = \frac{\text{Number of favorable outcomes}}{\text{Number of all possible outcomes}}$$

- Example: Roll a six-sided die once. What is the probability of obtaining even number?



- $\text{Probability of event} = \frac{3}{6} = 0.50$

Approaches to Assigning Probabilities 2

A challenge is determine the values in the numerator and denominator.

A small number of outcomes is easy to count.

If the experiment is complex, there are many possible outcomes.

There are three formulas to help determine the number of outcomes.

- Multiplication formula.
- Permutation formula.
- Combination formula.

Approaches to Assigning Probabilities 3

Suppose there are *m* possible outcomes of an event.

Suppose there are *n* possible outcomes for another event.

The total number of arrangements of the two outcomes is (n)(m).

This can be extended to any number of events.

Example: "Pick 3" lottery where three tumblers each with balls 0 to 9. After tumbled, one ball is selected from each. What is the number of possible outcomes?

- $(10)(10)(10)=1000$

Approaches to Assigning Probabilities 4

- Suppose there is a single group.
- We can determine the number of all possible arrangements of an outcomes from the group.

Permutation Any arrangement of r objects selected from a single group of n possible objects.

$$_nP_r = \frac{n!}{(n-r)!} = \frac{(n-1)(n-2)\cdots(n-(r+1))(n-r)!}{(n-r)!}$$

- This formula is determined by the multiplication formula!

Approaches to Assigning Probabilities 5

- Example: A media company is producing a 1-minute ad video.
- In the production process, eight different video segments were made.
- They can only select three segments.
- How many different ways can the eight video segments be arranged?
- n = 8, r = 3.
- $${}_nP_r = \frac{n!}{(n-r)!} = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{(8)(7)(6)(5!)}{5!} = 336$$

Approaches to Assigning Probabilities 6

If the order of the selected objects is not important, any arrangement is called a combination.

- ABC is the same combination as BAC

The number of combinations is less than the number of permutations.

Combination An event of outcomes when the order of the outcomes does not matter.

The permutation formula overcounts the *r*! permutations, divide out the overcounting.

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

Approaches to Assigning Probabilities 7

- Example: The Grand 16 movie theater uses teams of three employees to work the concession stand each evening.
- There are seven employees available to work.
- How many different teams can be scheduled?
- $${}_nC_r = \frac{n!}{r!(n-r)!} = \frac{7!}{3!(7-3)!} = \frac{7!}{3!4!} = 35$$

Approaches to Assigning Probabilities 8

- Empirical probability is the second type of probability.
- It is based on observation, counting and recording experimental outcomes.

Empirical Probability The probability of an event based on a collection of observations or data.

- An empirical probability is based on relative frequencies.

$$\frac{\text{Number of times the event occurs}}{\text{Total number of observations}}$$

Approaches to Assigning Probabilities 9

- The empirical approach is based on the law of large numbers.

Law of Large Numbers Over a large number of trials, the empirical probability of an event will approach its true probability.

- The key is that more observations provide an accurate estimate of the probability.

Number of Trials	Number of Heads	Relative Frequency of Heads
1	0	.00
10	3	.30
50	26	.52
100	52	.52
500	236	.472
1,000	494	.494
10,000	5,027	.5027

Approaches to Assigning Probabilities 10

- Example: The Standard & Poor's (S&P) 500 stock index was down four time over the course of the previous 20 years.
- What is the probability that the index will be down next year?
- Let A represent the even of a negative yearly return.

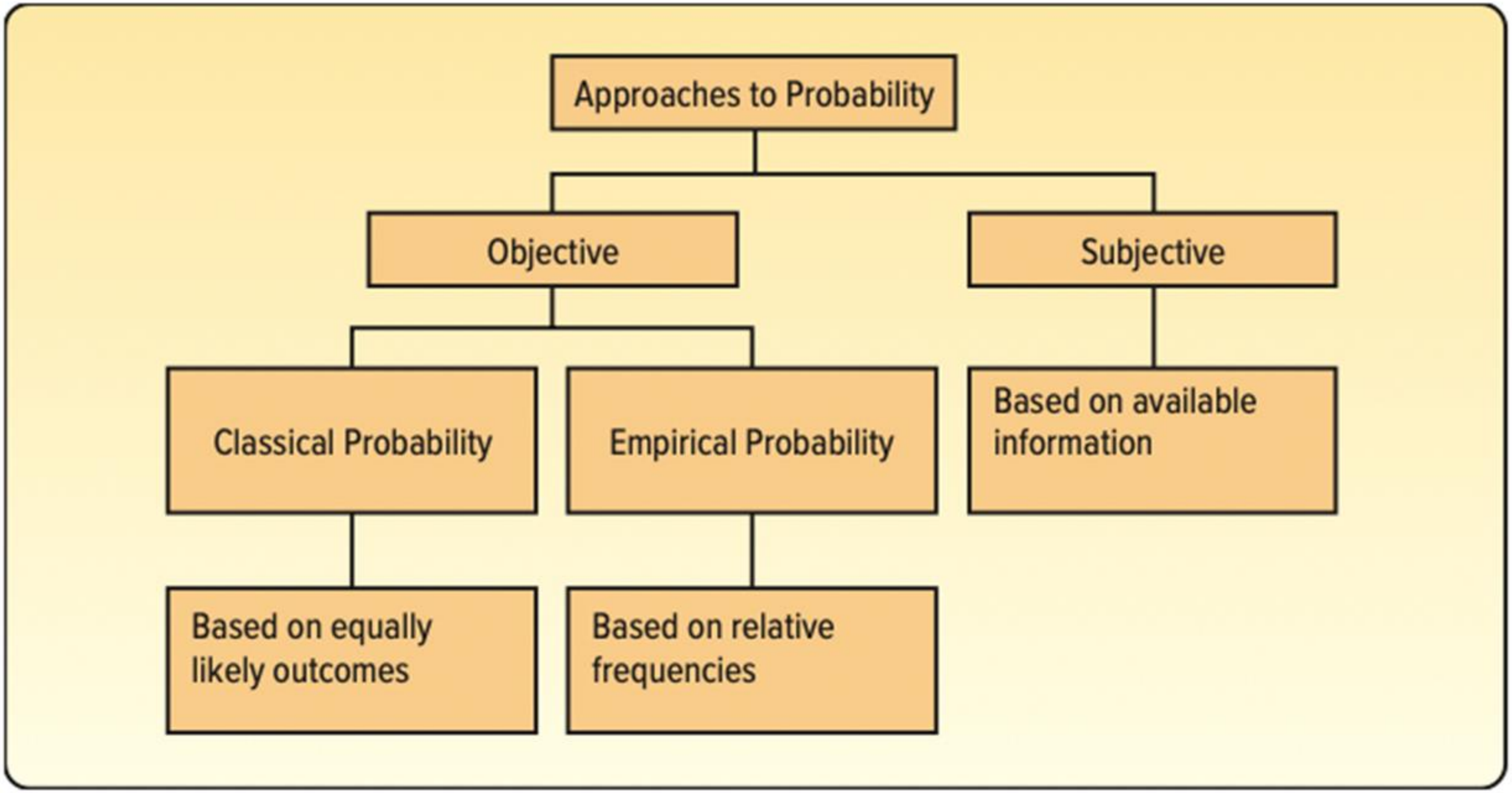
$$P(A) = \frac{\text{Number of times the event occurs}}{\text{Total number of observations}} = \frac{4}{20} = 0.20$$

Approaches to Assigning Probabilities 11

- If there is little or no experience on which to base a probability, it is estimated subjectively.
- An individual evaluates the available opinions and information then assigns a probability.

Subjective Concept of Probabiltiy The likelihood (probability) of a particular event happening that is assigned by an individual based on whatever information is available.

Approaches to Assigning Probabilities 12



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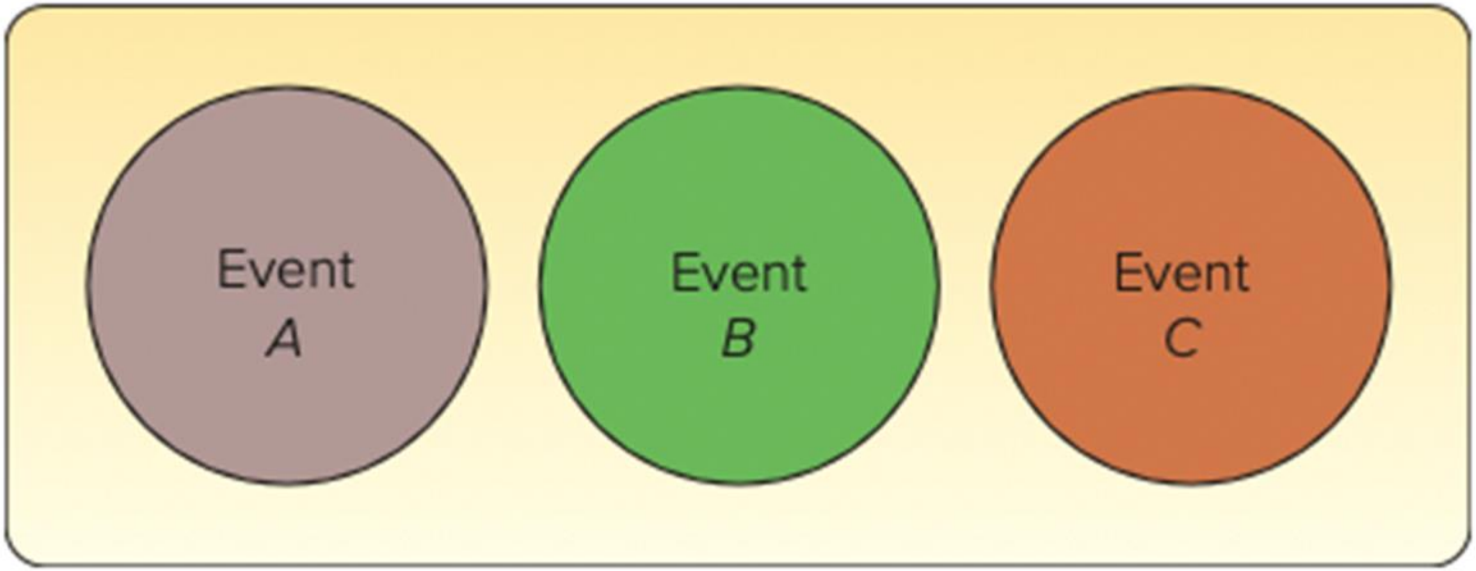
Rules of Addition 1

Mutually Exclusive The occurrence of one event means that none of the other events can occur at the same time.

Collectively Exhaustive At least one of the events must occur when an experiment is conducted.

- If events are mutually exclusive, the special rule of addition is: $P(A \text{ or } B) = P(A) + P(B)$.
- This works for any number of mutually exclusive events.

Rules of Addition 2



Rules of Addition [3]

- Example: A machine fills plastic bags with a mixture of beans, broccoli, and other vegetables.
- Most of the bags contain the correct weight.
- Because of the variation in the size of the beans and other vegetables, a package might be underweight or overweight.

Weight	Event	Number of Packages	Probability of Occurrence
Underweight	*A*	100	.025
Satisfactory	*B*	3,600	.900
Overweight	*C*	300	.075
		4,000	1.000

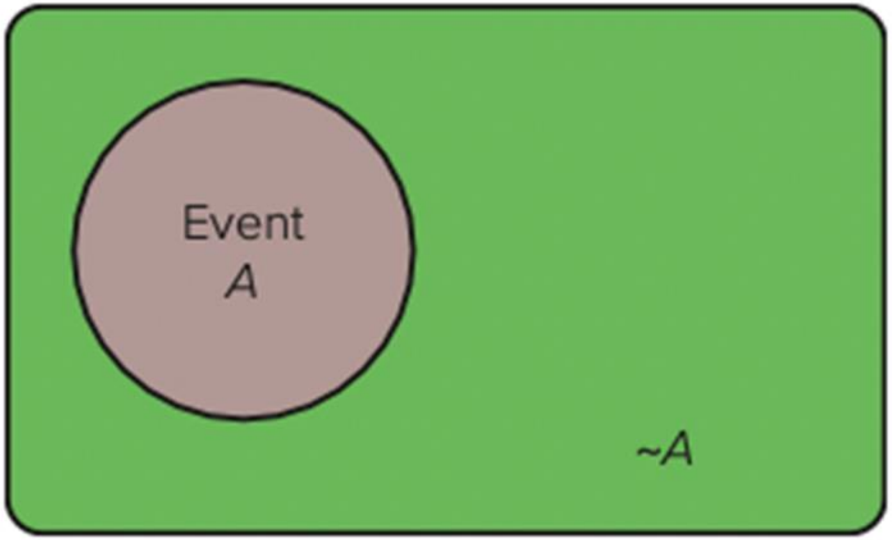
$\leftarrow \frac{100}{4,000}$

- What is the probability that a particular package will be either underweight or overweight?
- $P(A \text{ or } C) = 0.025 + 0.075 = 0.10.$

Rules of Addition 4

- The complement rule is used to determine the probability of an event happening by subtracting the probability of an event not happening from 1.

$$P(A) + P(\sim A) = 1 \text{ so } P(A) = 1 - P(\sim A)$$



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Rules of Addition 5

- Example: Refer to the previous example.

Weight	Event	Number of Packages	Probability of Occurrence
Underweight	*A*	100	.025
Satisfactory	*B*	3,600	.900
Overweight	*C*	300	.075
		4,000	1.000

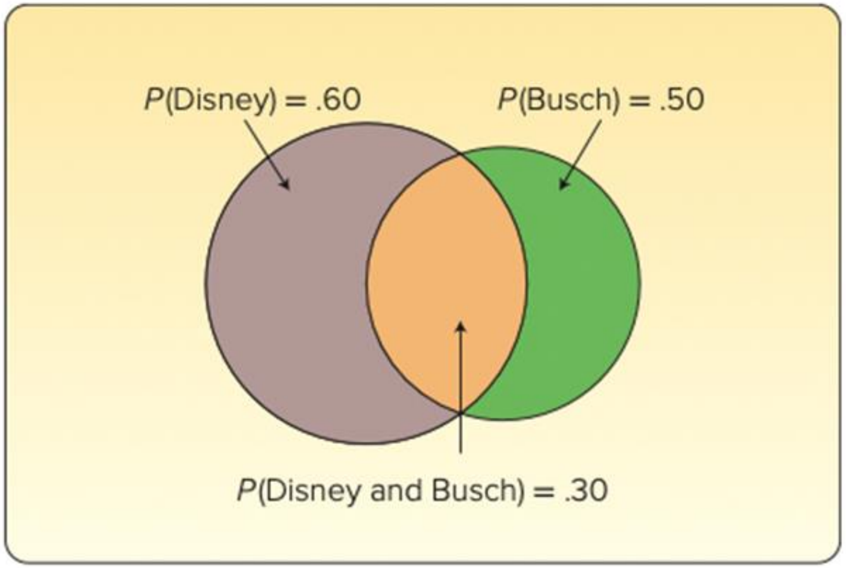
$\leftarrow \frac{100}{4,000}$

- $P(B) = 1 - \left[P(A) + P(C)\right] = 1 - 0.10 = 0.90$

Rules of Addition 6

- The general rule of addition is used when the events are not mutually exclusive.

Joint Probability A probability that measures the likelihood two or more events will happen concurrently.



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Rules of Addition 7

- General Addition Rule.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

- The use of "or" is inclusive: A or B or both.
- Account for the joint probability that is in both A and B.
- Subtract it out so it is not double counted.
- Note $P(A \text{ or } B) = 0$ means events are mutually exclusive.

Rules of Addition [8]

- Example: What is the probability that a card chosen at random from a standard deck of cards will be either a king or a heart?

Card	Probability	Explanation
King	$P(A) = 4/52$	4 kings in a deck of 52 cards
Heart	$P(B) = 13/52$	3 hearts in a deck of 52 cards
King of Hearts	$P(A \text{ and } B) = 1/52$	1 kings of hearts in a deck of 52 cards

- $$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = 0.3077$$

Rules of Multiplication [1]

Independence The occurrence of one event has no effect on the probability of the occurrence of another event.

- A and B occur at different times, do not occur together.
- For two independent events: $P(A \text{ and } B) = P(A)P(B)$.
- This is the special multiplication rule.
- Works for more than two independent events.

Rules of Multiplication 2

- Example: A survey by the American Automobile Association (AA A) revealed 60% of its members made airline reservations last year.
- Two members are selected at random.
- What is the probability both made airline reservations last year?
- $P(R_1 \text{ and } R_2) = P(R_1)P(R_2) = (.60)(.60) = .36.$

Outcomes	Joint Probability	
$R_1 \ R_2$	$(.60)(.60) =$	.36
$R_1 \sim R_2$	$(.60)(.40) =$	.24
$\sim R_1 \ R_2$	$(.40)(.60) =$	.24
$\sim R_1 \sim R_2$	$(.40)(.40) =$	.16
Total		1.00

Rules of Multiplication 3

- If two events are not independent, they are dependent.

Conditional Probability The probability of a particular event occurring, given that another event has occurred.

- The conditional probability is represented a $P(B|A)$.
- Read the “probability of B given A”.
- General multiplication rule: $P(A \text{ and } B) = P(A)P(B|A)$.

Rules of Multiplication 4

- Example: A golfer has 12 golf shirts in his closet.
- Suppose 9 of these shirts are white and the others are blue.
- He gets dressed in the dark, so he just grabs a shirt and puts in on.
- He plays golf two days in a row and does not return the shirts to the closet.
- What is the probability both shirts are white?
- $P(W_1 \text{ and } W_2) = P(W_1)P(W_2 | W_1) = \left(\frac{9}{12}\right)\left(\frac{8}{11}\right) = .55$

Rules of Multiplication 5

Often we tally the results of a survey in a two-way table.

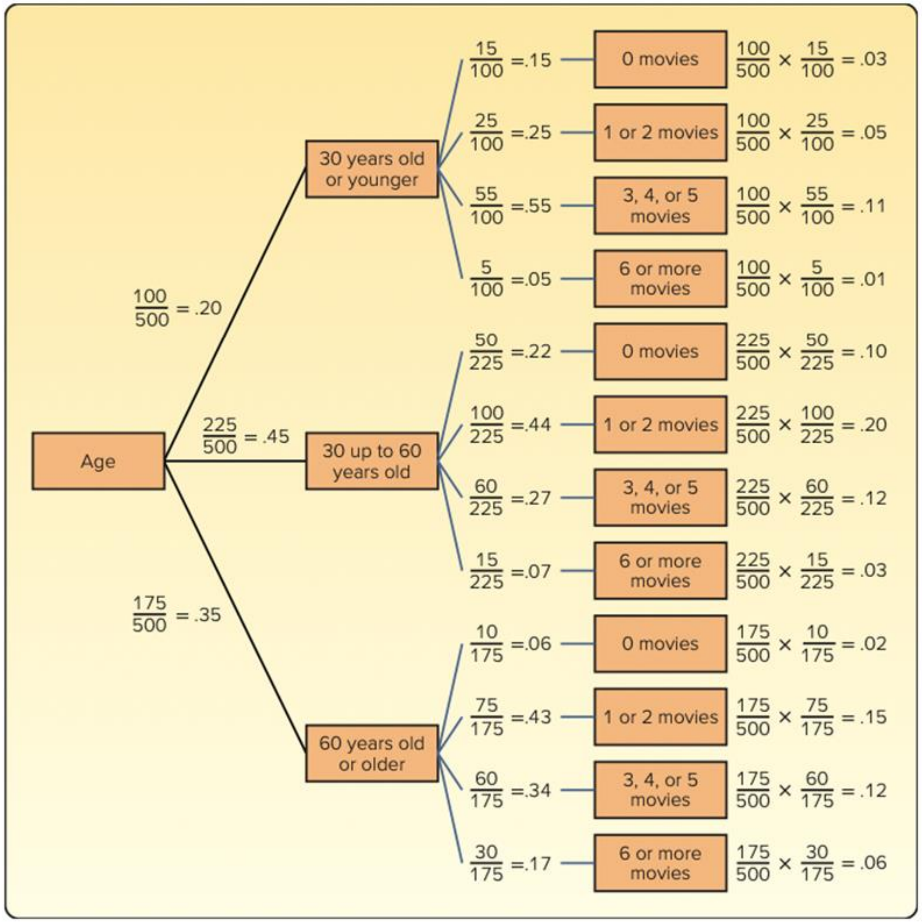
Contingency Table A table used to classify sample observations according to two or more identifiable categories or classes.

- A cross-tabulation that simultaneously summarizes two variables of interest and their relationship.
- Use the results to find probabilities.

Movies per Month	Age Less than 30 B_1	Age 30 up to 60 B_2	Age 60 or Older B_3	Total
0 A_1	15	50	10	75
1 or 2 A_2	25	100	75	200
3, 4, or 5 A_3	55	60	60	175
6 or more A_4	5	15	30	50
Total	100	225	175	500

Tree Diagrams

- A visual that is helpful in organizing and calculating probabilities for problems with several stages.
- Each stage of the problem is represented by a branch of the tree.
- Label the branches with the probabilities.



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Bayes' Theorem 1

- Bayes' Theorem is a method of revising a probability, given that additional information is obtained.

Prior Probability The initial probability based on the present level of information.

Posterior Probability A revised probability based on additional information.

$$P(A_1 | B) = \frac{P(A_1)P(B | A_1)}{P(A_1)P(B | A_1) + P(A_2)P(B | A_2)}$$

Bayes' Theorem 2

- Example: Suppose 5% of the population of Umen have a disease.
- Given someone has the disease, the probability of a positive test result is 0.90.
- Given someone does not have the disease, the probability of a positive test result is 0.15.
- Given someone has a positive test result, what is the probability they have the disease?

Bayes' Theorem 3

- Example continued.
- Let A_1 represent having the disease.
- Let A_2 represent not having the disease.
- Let *B* a positive rest result.

Event, A_i	**PriorProbability,** $P(A_i)$	**Conditional Probability,** $P(B \mid A_i)$	**JointProbability,** $P(A_i \text{ and } B)$	**PosteriorProbability,** $P(A_i \mid B)$
Disease, A_1	.05	.90	.0450	.0450 / .1875 = .24
No disease, A_2	.95	.15	.1425	.1425 / .1875 = .76
			P $(B) = .1875$	1.00

- $$P(A_1 \mid B) = \frac{P(A_1)P(B \mid A_1)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2)} = \frac{(.05)(.90)}{(.05)(.90) + (.95)(.15) = 0.24}$$

Chapter 5 Practice Problems

Question 5

LO5-3

- An overnight express company must include five cities on its route. How many different routes are possible, assuming that it does not matter in which order the cities are included in the routing?

Question 11

LO5-4

A survey of 34 students at the Wall College of Business showed the following majors:

Accounting	10
Finance	5
Economics	3
Management	6
Marketing	10

From the 34 students, suppose you randomly select a student.

- What is the probability he or she is a management major?
- Which concept of probability did you use to make this estimate?

Question 13

LO5-2,4, 5

In each of the following cases, indicate whether classical, empirical, or subjective probability is used.

1. A baseball player gets a hit in 30 out of 100 times at bat. The probability is .3 that he gets a hit in his next at bat.
2. A seven-member committee of students is formed to study environmental issues. What is the likelihood that any one of the seven is randomly chosen as the spokesperson?
3. You purchase a ticket for the Lotto Canada lottery. Over five million tickets were sold. What is the likelihood you will win the $1 million jackpot?
4. The probability of an earthquake in northern California in the next 10 years above 5.0 on the Richter Scale is .80.

Question 21

LO5-6

A study of 200 advertising firms revealed their income after taxes:

Income after Taxes	Number of Firms
Under $1 million	102
$1 million to $20 million	61
$20 million or more	37

1. What is the probability an advertising firm selected at random has under $1 million in income after taxes?
2. What is the probability an advertising firm selected at random has either an income between $1 million and $20 million, or an income of $20 million or more? What rule of probability was applied?

Question 29

LO5-6

The aquarium at Sea Critters Depot contains 140 fish. Eighty of these fish are green swordtails (44 female and 36 male) and 60 are orange swordtails (36 female and 24 males). A fish is randomly captured from the aquarium:

1. What is the probability the selected fish is a green swordtail?
2. What is the probability the selected fish is male?
3. What is the probability the selected fish is a male green swordtail?
4. What is the probability the selected fish is either a male or a green swordtail?

Question 33

LO5-7

- A local bank reports that 80% of its customers maintain a checking account, 60% have a savings account, and 50% have both. If a customer is chosen at random:

1. What is the probability the customer has either a checking or a savings account?
2. What is the probability the customer does not have either a checking or a savings account?

Question 37

LO5-8

- Each salesperson at Puchett, Sheets, and Hogan Insurance Agency is rated either below average, average, or above average with respect to sales ability. Each salesperson also is rated with respect to his or her potential for advancement—either fair, good, or excellent. These traits for the 500 salespeople were cross-classified into the following table.

Sales Ability	Potential for Advancement Fair	Potential for Advancement Good	Potential for Advancement Excellent
Below average	16	12	22
Average	45	60	45
Above average	93	72	135

1. What is this table called?
2. What is the probability a salesperson selected at random will have above average sales ability and excellent potential for advancement?
3. Construct a tree diagram showing all the probabilities, conditional probabilities, and joint probabilities.

Question 43

LO5-9

- The Ludlow Wildcats baseball team, a minor league team in the Cleveland Indians organization, plays 70% of their games at night and 30% during the day. T he team wins 50% of their night games and 90% of their day games. According to today’s newspaper, they won yesterday. What is the probability the game was played at night?

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