College of Science
Statistics and Operations Research Department

Chapter 2: Discrete Distributions

Book : PROBABILITY AND STATISTICAL INFERENCE ,Ninth Edition , Robert V. Hogg Elliot A. Tanis Dale L. Zimmerman

Distributions	probability mass function(pmf) P(X=x)	Value of r.v	Mean μ	Variance σ^2 Standard deviation $\sigma = \sqrt{Variance}$	Moment generating $M(t)$	R.V X
Discrete uniform $m > 0$	$\frac{1}{m}$	$x = 1,2, \dots, m$ m: positive integer	$\frac{m+1}{2}$	$\frac{m^2-1}{12}$	$\frac{e^t(1-e^{tm})}{m(1-e^t)}$	Each value of the random variable is equally likely
Bernoulli $0 < p < 1$	$p^x(1-p)^{1-x}$	$x = 0,1$	p	pq	$(q + pe^t)$	Experiment has only two outcomes
Binomial $b(n, p)$	$\binom{n}{x} p^x(1-p)^{n-x}$	$x = 0,1, \dots, n$	np	npq	$(q + pe^t)^n$	X: is the number of successes in a random sample of size n
Hypergeometric	$\frac{\binom{N_1}{x}\binom{N_2}{n-x}}{\binom{N}{n}}$	$x \le n$, $x \le N_1$, $n - x \le N_2$	$n(\frac{N_1}{N})$	$n\left(\frac{N_1}{N}\right)\left(\frac{N_2}{N}\right)\left(\frac{N-n}{N-1}\right)$		$N = N_1 + N_2$ $N_1 > 0, N_2 > 0$ X: number of successes in the sample
Negative binomial 0 < p < 1 r = 1, 2, 3, ...	$\binom{x-1}{r-1} p^r(1-p)^{n-r}$	$x = r, r+1, \dots$	$\frac{r}{p}$	$\frac{rq}{p^2}$	$\frac{(pe^t)^r}{[1-qe^t]^r}$	X: denote the number of trials needed to observe the rth success
Geometric	$p\,(1-p)^{x-1}$	$x = 1,2 \dots$	$\frac{1}{p}$	$\frac{q}{p^2}$	$\frac{pe^t}{[1-qe^t]}$	X : number of the trial on which the first success occurs
Poisson	$\frac{\lambda^x e^{-\lambda}}{x!}$	$x = 0,1, \dots$	λ	λ	$e^{\lambda(e^t-1)}$	X: number of events occurring within a fixed interval of time or space.

2.2 MATHEMATICAL EXPECTATION

2.2-2. Let the random variable X have the pmf

$$f(x) = \frac{(|x| + 1)^2}{9}, x = -1, 0, 1.$$

Compute $E(X), E(X^2), and\ E(3X^2 - 2X + 4)$.

2.2-3. Let the random variable X be the number of days that a certain patient needs to be in the hospital. Suppose X has the pmf

$$f(x) = \frac{5 - x}{10}, x = 1, 2, 3, 4.$$

If the patient is to receive \$200 from an insurance company for each of the first two days in the hospital and \$100 for each day after the first two days, what is the expected payment for the hospitalization?

2.2-4. An insurance company sells an automobile policy with a deductible of one unit. Let X be the amount of the loss having pmf

$$f(x) = \begin{cases} 0.9 & , x = 0. \\ \frac{c}{x} & , x = 1,2,3,4,5,6. \end{cases}$$

where **c** is a constant. Determine **c** and the expected value of the amount the insurance company must pay.

2.3 SPECIAL MATHEMATICAL EXPECTATIONS

2.3-1. Find the mean and variance for the following discrete distributions:

(a) $f(x) = \frac{1}{5}$ $, x = 5, 10, 15, 20, 25.$

(b) $f(x) = 1$ $, x = 5.$

(c) $f(x) = \frac{4-x}{6}$ $, x = 1, 2, 3.$

2.3-2. For each of the following distributions, find $\mu = E(X),\ E[X(X-1)], and\ \sigma^2 = E[X(X-1)] + E(X) - \mu^2$:

(a) $f(x) = \frac{3!}{x!(3-x)!}\left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{3-x}$ $, x = 0, 1, 2, 3.$

(b) $f(x) = \frac{4!}{x!(4-x)!}\left(\frac{1}{2}\right)^4, x = 0, 1, 2, 3, 4.$

2.3-4. Let $\mu\ and\ \sigma^2$ denote the mean and variance of the random variable X. Determine $E[\frac{X-\mu}{\sigma}]\ andE\{\left[\frac{X-\mu}{\sigma}\right]^2\}$.

2.3-5. Consider an experiment that consists of selecting a card at random from an ordinary deck of cards. Let the random variable X equal the value of the selected card, where $Ace = 1$, $Jack = 11, Queen = 12, and\ King = 13$. Thus, the space of X is $S = \{1, 2, 3, \ldots, 13\}$. If the experiment is performed in an unbiased manner, assign probabilities to these 13 outcomes and compute the mean $\boldsymbol{\mu}$ of this probability distribution.

2.3-7. Let X equal an integer selected at random from the first $\boldsymbol{m}$ positive integers, $\{1, 2, \ldots, m\}$. Find the value of $\boldsymbol{m}$ for which $E(X) = Var(X)$. (See Zerger in the references.)

2.3-8. Let X equal the larger outcome when a pair of fair four-sided dice is rolled. The pmf of X is

$$f(x) = \frac{2x-1}{16}, x = 1, 2, 3, 4.$$

Find the mean, variance, and standard deviation of X.

2.3-11. If the moment-generating function of X is

$$M(t) = \frac{2}{5}e^t + \frac{1}{5}e^{2t} + \frac{2}{5}e^{3t},$$

find the mean, variance, and pmf of X.

2.3-19. Given a random permutation of the integers in the set $\{1, 2, 3, 4, 5\}$, let X equal the number of integers that are in their natural position. The moment-generating function of X is

$$M(t) = \frac{44}{120} + \frac{45}{120}e^t + \frac{20}{120}e^{2t} + \frac{10}{120}e^{3t} + \frac{1}{120}e^{5t}$$

(a) Find the mean and variance of X.

2.4 THE BINOMIAL DISTRIBUTION

2.4-1 An urn contains 7 red and 11 white balls. Draw one ball at random from the urn. Let $X = 1$ if a red ball is drawn, and $X = 0$ if a white ball is drawn. Give the pmf, mean, and variance of X .

2.4-2 Suppose that in Exercise 2.4-1, $X = 1$ if a red ball is drawn and $X = -1$ if a white ball is drawn. Give the pmf, mean, and variance of X.

2.4-4 It is claimed that **15%** of the ducks in a particular region have patent schistosome infection. Suppose that **seven ducks** are selected at random. Let X equal the number of ducks that are infected.

(a) Assuming independence, how is X distributed?
(b) Find $(\boldsymbol{i})\ P(X \geq 2), (\boldsymbol{ii})\ P(X = 1), and\ (\boldsymbol{iii})\ P(X \leq 3)$.

2.4-11 A random variable X has a binomial distribution, with $\mathbf{mean\ 6}$ and $\mathbf{variance\ 3.6}$.

Find $P(X = 4)$.

2.4-19 Define the pmf and give the values of $\mu, \sigma^2, and\ \sigma$ when the moment-generating function of X is defined by

(a) $M(t) = \frac{1}{3} + \left(\frac{2}{3}\right)e^t$

(b) $M(t) = (0.25 + 0.75e^t)^{12}$

2.5 THE NEGATIVE BINOMIAL DISTRIBUTION

2.5-2. Show that $63/512$ is the probability that the fifth head is observed on the tenth independent flip of a fair coin.

2.5-4. Suppose an airport metal detector catches a person with metal 99% of the time. That is, it misses detecting a person with metal 1% of the time. Assume independence of people carrying metal. What is the probability that the first metal-carrying person missed (not detected) is among the first 50 metal-carrying persons scanned?

2.6 THE POISSON DISTRIBUTION

2.6-1: Let X have a Poisson distribution with a mean of 4. Find

(a) $P(2 \leq X \leq 5)$.

(b) $P(X \geq 3)$.

(c) $P(X \leq 3)$.

2.6-2: Let X have a Poisson distribution with a variance of 3. Find $P(X = 2)$.

2.6-4: Find $P(X = 4)$ if X has a Poisson distribution such that $3P(X = 1) = P(X = 2)$.

2.6-5: Flaws in a certain type of drapery material appear on the average of one in 150 square feet. If we assume a Poisson distribution, find the probability of at most one flaw appearing in 225 square feet.

2.6-10: The mean of a Poisson random variable X is $\mu = 9$.

Compute $P(\mu - 2\sigma < X < \mu + 2\sigma)$.

Discrete Distributions

Bernoulli $0 < p < 1$	$f(x) = p^x(1-p)^{1-x}, \quad x = 0, 1$ $M(t) = 1 - p + pe^t, \quad -\infty < t < \infty$ $\mu = p, \quad \sigma^2 = p(1-p)$
Binomial $b(n, p)$ $0 < p < 1$	$f(x) = \frac{n!}{x!(n-x)!} p^x(1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n$ $M(t) = (1 - p + pe^t)^n, \quad -\infty < t < \infty$ $\mu = np, \quad \sigma^2 = np(1-p)$
Geometric $0 < p < 1$	$f(x) = (1-p)^{x-1}p, \quad x = 1, 2, 3, \dots$ $M(t) = \frac{pe^t}{1-(1-p)e^t}, \quad t < -\ln(1-p)$ $\mu = \frac{1}{p}, \quad \sigma^2 = \frac{1-p}{p^2}$
Hypergeometric $N_1 > 0, \; N_2 > 0$ $N = N_1 + N_2$	$f(x) = \frac{\binom{N_1}{x}\binom{N_2}{n-x}}{\binom{N}{n}}, \quad x \le n, x \le N_1, n - x \le N_2$ $\mu = n\left(\frac{N_1}{N}\right), \quad \sigma^2 = n\left(\frac{N_1}{N}\right)\left(\frac{N_2}{N}\right)\left(\frac{N-n}{N-1}\right)$
Negative Binomial $0 < p < 1$ $r = 1, 2, 3, \dots$	$f(x) = \binom{x-1}{r-1} p^r(1-p)^{x-r}, \quad x = r, r+1, r+2, \dots$ $M(t) = \frac{(pe^t)^r}{[1-(1-p)e^t]^r}, \quad t < -\ln(1-p)$ $\mu = r\left(\frac{1}{p}\right), \quad \sigma^2 = \frac{r(1-p)}{p^2}$
Poisson $\lambda > 0$	$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \dots$ $M(t) = e^{\lambda(e^t - 1)}, \quad -\infty < t < \infty$ $\mu = \lambda, \quad \sigma^2 = \lambda$
Uniform $m > 0$	$f(x) = \frac{1}{m}, \quad x = 1, 2, \dots, m$ $\mu = \frac{m+1}{2}, \quad \sigma^2 = \frac{m^2-1}{12}$