College of Science
Statistics and Operations Research Department

Chapter 2: Discrete Distributions

Book : PROBABILITY AND STATISTICAL INFERENCE ,Ninth Edition , Robert V. Hogg Elliot A. Tanis Dale L. Zimmerman

Distributions	probability mass function(pmf) P(X=x)	Value of r.v	Mean μ	Variance σ^2 Standard deviation $\sigma = \sqrt{Variance}$	Moment generating $M(t)$	R.V X
Discrete uniform $m > 0$	$\frac{1}{m}$	$x = 1,2,\dots,m$ m: positive integer	$\frac{m+1}{2}$	$\frac{m^2-1}{12}$	$\frac{e^t(1-e^{tm})}{m(1-e^t)}$	Each value of the random variable is equally likely
Bernoulli $0 < p < 1$	$p^x(1-p)^{1-x}$	$x = 0,1$	p	pq	$(q + pe^t)$	Experiment has only two outcomes
Binomial b(n, p)	$\binom{n}{x} p^x(1-p)^{n-x}$	$x = 0,1,\dots,n$	np	npq	$(q + pe^t)^n$	X: is the number of successes in a random sample of size n
Hypergeometric	$\frac{\binom{N_1}{x}\binom{N_2}{n-x}}{\binom{N}{n}}$	$x \leq n$, $x \leq N_1$, $n - x \leq N_2$	$n(\frac{N_1}{N})$	$n\left(\frac{N_1}{N}\right)\left(\frac{N_2}{N}\right)\left(\frac{N-n}{N-1}\right)$		$N = N_1 + N_2$ $N_1 > 0, N_2 > 0$ X: number of successes in the sample
Negative binomial 0 < p < 1 r = 1, 2, 3, …	$\binom{x-1}{r-1} p^r(1-p)^{x-r}$	$x = r, r+1, \dots$.	$\frac{r}{p}$	$\frac{rq}{p^2}$	$\frac{(pe^t)^r}{[1-qe^t]^r}$	X: denote the number of trials needed to observe the rth success
Geometric	$p\,(1-p)^{x-1}$	$x = 1,2\dots$	$\frac{1}{p}$	$\frac{q}{p^2}$	$\frac{pe^t}{[1-qe^t]}$	X : number of the trial on which the first success occurs
Poisson	$\frac{\lambda^x e^{-\lambda}}{x!}$	$x = 0,1,\dots$.	λ	λ	$e^{\lambda(e^t-1)}$	X: number of events occurring within a fixed interval of time or space.

2.2 MATHEMATICAL EXPECTATION

2.2-2. Let the random variable X have the pmf

$$f(x) = \frac{(|x| + 1)^2}{9}, x = -1, 0, 1.$$

Compute $E(X), E(X^2), and\ E(3X^2 - 2X + 4)$.

2.2-3. Let the random variable X be the number of days that a certain patient needs to be in the hospital. Suppose X has the pmf

$$f(x) = \frac{5 - x}{10}, x = 1, 2, 3, 4.$$

If the patient is to receive \$200 from an insurance company for each of the first two days in the hospital and \$100 for each day after the first two days, what is the expected payment for the hospitalization?

2.2-4. An insurance company sells an automobile policy with a deductible of one unit. Let X be the amount of the loss having pmf

$$f(x) = \begin{cases} 0.9 & , x = 0. \\ \frac{c}{x} & , x = 1,2,3,4,5,6. \end{cases}$$

where **c** is a constant. Determine **c** and the expected value of the amount the insurance company must pay.

2.3 SPECIAL MATHEMATICAL EXPECTATIONS

2.3-1. Find the mean and variance for the following discrete distributions:

(a) $f(x) = \frac{1}{5}$ $, x = 5, 10, 15, 20, 25.$

(b) $f(x) = 1$ $, x = 5.$

(c) $f(x) = \frac{4-x}{6}$ $, x = 1, 2, 3.$

2.3-2. For each of the following distributions, find $\mu = E(X),\ E[X(X - 1)], and\ \sigma^2 = E[X(X - 1)] + E(X) - \mu^2$:

(a) $f(x) = \frac{3!}{x!(3-x)!}\left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{3-x}$ $, x = 0, 1, 2, 3.$

(b) $f(x) = \frac{4!}{x!(4-x)!}\left(\frac{1}{2}\right)^4$ $, x = 0, 1, 2, 3, 4.$

2.3-4. Let $\mu\ and\ \sigma^2$ denote the mean and variance of the random variable X. Determine $E[\frac{X - \mu}{\sigma}]\ and E\{\left[\frac{X - \mu}{\sigma}\right]^2\}$.

2.3-5. Consider an experiment that consists of selecting a card at random from an ordinary deck of cards. Let the random variable X equal the value of the selected card, where $Ace = 1$, $Jack = 11, Queen = 12, and\ King = 13$. Thus, the space of X is $S = \{1, 2, 3, \ldots, 13\}$. If the experiment is performed in an unbiased manner, assign probabilities to these 13 outcomes and compute the mean $\boldsymbol{\mu}$ of this probability distribution.

2.3-7. Let X equal an integer selected at random from the first $\boldsymbol{m}$ positive integers, $\{1, 2, \ldots, m\}$. Find the value of $\boldsymbol{m}$ for which $E(X) = Var(X)$. (See Zerger in the references.)

2.3-8. Let X equal the larger outcome when a pair of fair four-sided dice is rolled. The pmf of X is

$$f(x) = \frac{2x - 1}{16}, x = 1, 2, 3, 4.$$

Find the mean, variance, and standard deviation of X.

2.3-11. If the moment-generating function of X is

$$M(t) = \frac{2}{5} e^{t} + \frac{1}{5} e^{2t} + \frac{2}{5} e^{3t},$$

find the mean, variance, and pmf of X.

2.3-19. Given a random permutation of the integers in the set $\{1, 2, 3, 4, 5\}$, let X equal the number of integers that are in their natural position. The moment-generating function of X is

$$M(t) = \frac{44}{120} + \frac{45}{120} e^{t} + \frac{20}{120} e^{2t} + \frac{10}{120} e^{3t} + \frac{1}{120} e^{5t}$$

(a) Find the mean and variance of X.

2.4 THE BINOMIAL DISTRIBUTION

2.4-1 An urn contains 7 red and 11 white balls. Draw one ball at random from the urn. Let $X = 1$ if a red ball is drawn, and $X = 0$ if a white ball is drawn. Give the pmf, mean, and variance of X.

2.4-2 Suppose that in Exercise 2.4-1, $X = 1$ if a red ball is drawn and $X = -1$ if a white ball is drawn. Give the pmf, mean, and variance of X.

2.4-4 It is claimed that **15%** of the ducks in a particular region have patent schistosome infection. Suppose that **seven ducks** are selected at random. Let X equal the number of ducks that are infected.

(a) Assuming independence, how is X distributed?
(b) Find $(\boldsymbol{i})\ P(X \geq 2), (\boldsymbol{ii})\ P(X = 1), and\ (\boldsymbol{iii})\ P(X \leq 3).$

2.4-11 A random variable X has a binomial distribution, with $\mathbf{mean\ 6}$ and $\mathbf{variance\ 3.6}$.

Find $P(X = 4)$.

2.4-19 Define the pmf and give the values of $\mu, \sigma^2, and\ \sigma$ when the moment-generating function of X is defined by

(a) $M(t) = \frac{1}{3} + \left(\frac{2}{3}\right) e^{t}$

(b) $M(t) = (0.25 + 0.75e^{t})^{12}$

2.5 THE NEGATIVE BINOMIAL DISTRIBUTION

2.5-2. Show that $63/512$ is the probability that the fifth head is observed on the tenth independent flip of a fair coin.

2.5-4. Suppose an airport metal detector catches a person with metal 99% of the time. That is, it misses detecting a person with metal 1% of the time. Assume independence of people carrying metal. What is the probability that the first metal-carrying person missed (not detected) is among the first 50 metal-carrying persons scanned?

2.6 THE POISSON DISTRIBUTION

2.6-1: Let X have a Poisson distribution with a mean of 4. Find

(a) $P(2 \le X \le 5)$.

(b) $P(X \ge 3)$.

(c) $P(X \le 3)$.

2.6-2: Let X have a Poisson distribution with a variance of 3. Find $P(X = 2)$.

2.6-4: Find $P(X = 4)$ if X has a Poisson distribution such that $3P(X = 1) = P(X = 2)$.

2.6-5: Flaws in a certain type of drapery material appear on the average of one in 150 square feet. If we assume a Poisson distribution, find the probability of at most one flaw appearing in 225 square feet.

2.6-10: The mean of a Poisson random variable X is $\mu = 9$.

Compute $P(\mu - 2\sigma < X < \mu + 2\sigma)$.

Discrete Distributions

Bernoulli $0 < p < 1$	$f(x) = p^x(1-p)^{1-x}, \quad x = 0, 1$ $M(t) = 1 - p + pe^t, \quad -\infty < t < \infty$ $\mu = p, \quad \sigma^2 = p(1-p)$
Binomial $b(n, p)$ $0 < p < 1$	$f(x) = \dfrac{n!}{x!(n-x)!} p^x(1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n$ $M(t) = (1 - p + pe^t)^n, \quad -\infty < t < \infty$ $\mu = np, \quad \sigma^2 = np(1-p)$
Geometric $0 < p < 1$	$f(x) = (1-p)^{x-1}p, \quad x = 1, 2, 3, \ldots$ $M(t) = \dfrac{pe^t}{1-(1-p)e^t}, \quad t < -\ln(1-p)$ $\mu = \dfrac{1}{p}, \quad \sigma^2 = \dfrac{1-p}{p^2}$
Hypergeometric $N_1 > 0, \ N_2 > 0$ $N = N_1 + N_2$	$f(x) = \dfrac{\dbinom{N_1}{x}\dbinom{N_2}{n-x}}{\dbinom{N}{n}}, \quad x \le n, x \le N_1, n - x \le N_2$ $\mu = n\left(\dfrac{N_1}{N}\right), \quad \sigma^2 = n\left(\dfrac{N_1}{N}\right)\left(\dfrac{N_2}{N}\right)\left(\dfrac{N-n}{N-1}\right)$
Negative Binomial $0 < p < 1$ $r = 1, 2, 3, \ldots$	$f(x) = \dbinom{x-1}{r-1} p^r(1-p)^{x-r}, \quad x = r, r+1, r+2, \ldots$ $M(t) = \dfrac{(pe^t)^r}{[1-(1-p)e^t]^r}, \quad t < -\ln(1-p)$ $\mu = r\left(\dfrac{1}{p}\right), \quad \sigma^2 = \dfrac{r(1-p)}{p^2}$
Poisson $\lambda > 0$	$f(x) = \dfrac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \ldots$ $M(t) = e^{\lambda(e^t-1)}, \quad -\infty < t < \infty$ $\mu = \lambda, \quad \sigma^2 = \lambda$
Uniform $m > 0$	$f(x) = \dfrac{1}{m}, \quad x = 1, 2, \ldots, m$ $\mu = \dfrac{m+1}{2}, \quad \sigma^2 = \dfrac{m^2-1}{12}$

$\boxed{2.2-2}$

$$f(x) = \frac{(|x|+1)^2}{9}, \quad x = -1, 0, 1$$

$$|x| = \begin{cases} x & ; x \geq 0 \\ -x & ; x < 0 \end{cases}$$

$$f(x) = \begin{cases} \frac{(x+1)^2}{9} & x = 0, 1 \\ \frac{(-x+1)^2}{9} & x = -1 \end{cases}$$

x	-1	0	1	Total
$f(x) = P(X=x)$	$\frac{4}{9}$	$\frac{1}{9}$	$\frac{4}{9}$	1

- mean $= E[x] = \sum x \; p(x=x)$

$$= \left(-1 \times \frac{4}{9}\right) + \left(0 \times \frac{1}{9}\right) + \left(1 \times \frac{4}{9}\right) = 0$$

- $E[x^2] = \sum x^2 \; p(X=x) = \frac{8}{9}$
- $E[3x^2 - 2x + 4] = 3E[x^2] - 2E[x] + 4$

$$= 3\left(\frac{8}{9}\right) - 2(0) + 4 = \frac{20}{3} = 6.667$$

$\boxed{2.2-3}$

$$f(x) = \frac{5-x}{10}, \quad x = 1, 2, 3, 4$$

x	1	2	3	4	Total
$f(x)$	$\frac{4}{10}$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{1}{10}$	1
Payment	200	400	500	600	

Expected payment $= \left(200 \times \frac{4}{10}\right) + \left(400 \times \frac{3}{10}\right) + \left(500 \times \frac{2}{10}\right) + \left(600 \times \frac{1}{10}\right)$

$= 360 \$$

$\boxed{2.2-4}$

$$f(x) = \begin{cases} 0.9 & x = 0 \\ \frac{c}{x} & x = 1, 2, 3, 4, 5, 6 \end{cases}$$

x	0	1	2	3	4	5	6
$f(x)$	0.9	$\frac{c}{1}$	$\frac{c}{2}$	$\frac{c}{3}$	$\frac{c}{4}$	$\frac{c}{5}$	$\frac{c}{6}$

$\Rightarrow \quad \sum f(x) = 1$

$$0.9 + c\left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}\right] = 1$$

$$\frac{49}{20} c = \underbrace{1 - 0.9}_{0.1 = \frac{1}{10}} \Rightarrow * \boxed{c = \frac{2}{49}} = 0.0408$$

$* \; E[x] = \sum_{x=0}^{6} x \; p(x) = \frac{12}{49} = 0.244$

$=$

Expected payment by Insurance Company:

$* \; E[\text{payment}] = \left(\overset{2-1}{1} \cdot \frac{c}{2}\right) + \left(\overset{3-1}{2} \cdot \frac{c}{3}\right) + \left(\overset{4-1}{3} \cdot \frac{c}{4}\right) + \left(\overset{5-1}{4} \cdot \frac{c}{5}\right) + \left(\overset{6-1}{5} \cdot \frac{c}{6}\right)$

$= \frac{71}{490} = 0.14489$ Unite

2.3-1 (a) $f(x) = \frac{1}{5}$, $x = 5, 10, 15, 20, 25$

x	5	10	15	20	25
$f(x)$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

mean : $E[x] = \sum x \, p(x)$

$= 15$

$E[x^2] = \sum x^2 \, p(x) = 275$

$\sigma^2 = E[x^2] - (E[x])^2$

$= 50$

(b) $f(x) = 1$, $x = 5$

mean : $E[x] = \sum x \, p(x) = 5 \times 1 = 5$

$E[x^2] = x^2 p(x) = 25 \times 1 = 25$

Variance : $\sigma^2 = E[x^2] - (E[x])^2 = 25 - 5^2 = 0$

(c) $f(x) = \frac{4-x}{6}$, $x = 1, 2, 3$

x	1	2	3
$f(x)$	$\frac{3}{6}$	$\frac{2}{6}$	$\frac{1}{6}$

mean : $E[x] = \sum x \, p(x) = \frac{10}{6} = \frac{5}{3}$

$E[x^2] = \sum x^2 p(x) = \frac{10}{3}$

Variance $\sigma^2 = E[x^2] - (E[x])^2 = \frac{5}{9}$

2.3-2 (a) $f(x) = \frac{3!}{x!\,(3-x)!} \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{3-x}$; $x = 0, 1, 2, 3$ $\Rightarrow f(x) = \binom{3}{x} \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{3-x}$

$x \sim Bin\left(n = 3, P = \frac{1}{4}\right)$

x	0	1	2	3
$f(x)$	0.421875	~~0.4222~~ 0.421875	0.140625	0.015625

$E[x] = np = \frac{3}{4}$

Variance $= \sigma^2 = npq = \frac{9}{16}$

$\sigma^2 = E[x^2] - (E[x])^2$

$\Rightarrow E[x^2] = \sigma^2 + (E(x))^2 = \frac{9}{8}$

$E[x] = \sum x \, p(x) = 0.75$

$E[x^2] = 1.125 = \frac{9}{8}$

$\sigma^2 = \frac{9}{16} =$

(b) H.W

2.3-4

$$E\left[\frac{X-\mu}{\sigma}\right] = E\left[\frac{X}{\sigma} - \frac{\mu}{\sigma}\right] = \frac{1}{\sigma}E[X] - \frac{1}{\sigma}\mu = \frac{\mu}{\sigma} - \frac{\mu}{\sigma} = 0$$

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^2\right] = E\left[\left(\frac{X}{\sigma} - \frac{\mu}{\sigma}\right)^2\right] = E\left[\left(\frac{X}{\sigma}\right)^2\right] - 2\frac{\mu}{\sigma}E\left[\frac{X}{\sigma}\right] + \frac{\mu^2}{\sigma^2}$$

$$= \frac{1}{\sigma^2}E[X^2] - \frac{2\mu^2}{\sigma^2} + \frac{\mu^2}{\sigma^2}$$

$$= \frac{1}{\sigma^2}E[X^2] - \frac{\mu^2}{\sigma^2}$$

$$= \frac{1}{\sigma^2}(\sigma^2 + \mu^2) - \frac{\mu^2}{\sigma^2}$$

$$= \frac{\sigma^2}{\sigma^2} + \cancel{\frac{\mu^2}{\sigma^2}} - \cancel{\frac{\mu^2}{\sigma^2}} = 1$$

2.3-7

X is an integer Selected at random from the first m positive integer.

* Discrete Uniform distrbution

$E[X] = \frac{1+m}{2}$ $\quad\quad$ $Var(X) = \frac{m^2-1}{12}$

$\Rightarrow E[X] = Var(X) \Rightarrow \frac{1+m}{2} = \frac{m^2-1}{12}$

$\Rightarrow 12m + 12 = 2m^2 - 2$

$2m^2 - 12m - 14 = 0$

$\Rightarrow \boxed{m=7}$ or $\boxed{m=-1}$ $\quad$ Since m represent the numbe of positive integer

$\Rightarrow \boxed{m=7}$

2.3-8 $f(x) = \frac{2x-1}{16}$, $x = 1, 2, 3, 4$

x	1	2	3	4
$f(x)$				

Complet H.W

mean: $E[x] = \sum x\, p(x)$

$E[x^2] = \sum x^2\, p(x)$

Variance: $\sigma^2 = E[x^2] - (E[x])^2$

S.D: $\sigma = \sqrt{\sigma^2} =$

2.3.11 $M(t) = \frac{2}{5} e^{t} + \frac{1}{5} e^{2t} + \frac{2}{5} e^{3t}$

$M'(t=0) = \frac{2}{5} e^{t} + \frac{2}{5} e^{2t} + \frac{2\times3}{5} e^{3t} \Big|_{t=0} = 2 \Rightarrow E[x] = 2$

$M''(t=0) = \frac{2}{5} e^{t} + \frac{2\times2}{5} e^{2t} + \frac{6\times3}{5} e^{3t} \Big|_{t=0} = \frac{24}{5} \Rightarrow E[x^2] = \frac{24}{5}$

$Var(x) = E[x^2] - (E[x])^2$

$= \frac{24}{5} - (2)^2 = \frac{4}{5} = 0.8$

2.3.19 $M(t) = \frac{44}{120} + \frac{45}{120} e^{t} + \frac{20}{120} e^{2t} + \frac{10}{120} e^{3t} + \frac{1}{120} e^{5t}$

$E[x] = M'(t=0) = 1$

$Var(x) = M''(t=0) - [M'(t=0)]^2$

$= 2 - 1^2$

$= 1$

$b = \frac{10}{30}$

2.4-1

$$\underbrace{\begin{matrix} 7 & 11 \\ R & W \end{matrix}}_{N=18}$$

The Experiment has only two outcoms $\left\langle \begin{matrix} R \\ W \end{matrix} \right.$

$X=1$ if a red ball drawn

$X=0$ if a White ball drawn

$X=1 \Rightarrow R$ = Successe $\rightarrow P = \frac{7}{18}$

$X=0 \Rightarrow W$ = falulr

pmf

$$P(X=x) = \begin{cases} p^x (1-p)^{1-x} & x=0,1 \\ 0 & o\,w \end{cases}$$

or

$$P(X=x) = \begin{cases} \frac{7}{18} & \text{if } x=1 \\ \frac{11}{18} & x=0 \end{cases}$$

mean : $E[x] = P = \frac{7}{18}$

Variance:

$Var(x) = pq = \frac{7}{18} \cdot \frac{11}{18} = \frac{77}{324} = 0.2376$

2.4-2

$X = 1$ if ball R

$X = -1$ if ball W

$$P(X=x) = \begin{cases} \frac{7}{18} & x=1 \\ \frac{11}{18} & x=-1 \end{cases}$$

mean: $E[x] = \sum x\, p(x)$

$= \frac{7}{18} - \frac{11}{18} = \frac{-2}{9}$

$E[x^2] = \sum x^2\, p(x)$

$= \frac{7}{18} + \frac{11}{18} = 1$

Vaiance:

$Var(x) = E[x^2] - (E[x])^2$

$= 1 - \left(\frac{-2}{9}\right)^2 = \frac{77}{81} = 0.951$

2.4-4 $X \sim Bin\,(n = 7\ ,\ P = 0.15)$

$P(X=x) = \binom{n}{x} p^x q^{n-x}$, $x = 0, 1, \ldots, n$

$P(X \geq 2) = 0.2834$ $\quad$ $P(X=1) = 0.3960$ $\quad$ $P(X \leq 3) = 0.9879$

2.4-11 $X \sim Bin(n, p)$ $\quad P(X=x) = \binom{n}{x} p^x q^{n-x} \; ; \; x = 0, 1, \dots n$

mean: $E[X] = np = 6$ $\quad$ Variance: $Var(X) = npq = 3.6$

$$6q = 3.6$$

$$q = 0.6$$

$\Rightarrow p = 1 - q = 0.4$ $\quad (p + q = 1)$

$\Rightarrow n = 15$ $\quad \begin{cases} np = 6 \\ n(0.4) = 6 \\ n = 15. \end{cases}$

$$P(X = 4) = \binom{15}{4} (0.4)^4 (0.6)^{15-4} = 0.12678$$

2.4-19

a) $M(t) = \frac{1}{3} + \frac{2}{3} e^t$ $\quad \Rightarrow$ Bernoulli $\Rightarrow M(t) = q + pe^t$

$$p = \frac{2}{3}$$

pmf: $P(X=x) = \begin{cases} \left(\frac{2}{3}\right)\left(1 - \frac{2}{3}\right)^{1-x} & ; \; x = 0, 1 \\ 0 & \text{ow} \end{cases}$

mean $= p = \frac{2}{3}$ $\quad$ Variance: $pq = \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9}$

S.D: $\sqrt{pq} = \sqrt{\frac{2}{9}}$

b) $M(t) = (0.25 + 0.75 e^t)^{12}$ $\Rightarrow$ Binomial $(n = 12, p = 0.7)$

$$M(t) = (q + pe^t)^n$$

$P(X=x) = \binom{12}{x} 0.75^x (0.25)^{12-x} \; ; \; x = 0, 1, \dots 12$

mean: np $\quad$ Variance (σ^2): npq $\quad$ S.D (σ): $\sqrt{npq}$

2.5-2

"Negative Binomial distribution"

$$P(X=x) = \binom{x-1}{r-1} p^r q^{x-r}$$

P: Probability of Heads $= \frac{1}{2}$

X = # of trial on which the r^{th} success occurs. $\Rightarrow$ $x = 10$

$r = 5$

$$P(X=7) = \binom{10-1}{5-1} \left(\tfrac{1}{2}\right)^5 \left(\tfrac{1}{2}\right)^{10-5} = \frac{63}{512}$$ #

2.5-4

"Geometric distribution"

X = # of trial on which the first success occurs. (first miss occurs)

$P = 0.01$ Probability of missing a metal-carrying person

$$P(X \leq 50) = \sum_{x=1}^{50} p\, q^{x-1} = 0.39499$$ $p(1-p)^{x-1}$

$\simeq 0.395$

$= 39.5\%$

2.6-1

$X \sim \text{Poisson}(\lambda = 4)$

$f(x) = P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$; $x = 0, 1, \ldots$

$$P(2 \leq X \leq 5) = \sum_{x=2}^{5} \left(\frac{4^x e^{-4}}{x!} \right) = 0.69355$$

$$P(X \geq 3) = 1 - P(X \leq 2) = 0.76189$$

$$P(X \leq 3) = \sum_{x=0}^{3} \left(\frac{4^x e^{-4}}{x!} \right) = 0.43347$$

2.6-2

$X \sim \text{Poisson}(\lambda = 3)$

$$P(X=2) = \frac{3^2 e^{-3}}{2!} =$$

$\boxed{2.6\text{-}4}$ $3\,p(X=1) = p(X=2)$; $X \sim \text{Poisson}(\lambda)$

$$3\,\frac{\cancel{\lambda^1}\,\cancel{e^{-\lambda}}}{1!} = \frac{\lambda^2\,\cancel{e^{-\lambda}}}{2!}$$

$$3 = \frac{\lambda}{2}$$

$$\boxed{6 = \lambda}$$

Find : $p(X=4) = \frac{6^4 e^{-6}}{4!} = 0.13385$

$\boxed{2.6\text{-}5}$

$$P(X \leqslant 1) = p(X=0) + P(X=1)$$

$$= \sum_{x=0}^{1} \frac{(\lambda^*)^x e^{-\lambda^*}}{x!}$$

$$= \sum_{x=0}^{1} \frac{(1.5)^x e^{-1.5}}{x!}$$

$$= 0.5578$$

$X \sim \text{Poisson}(\lambda)$

$\lambda =$ avegge $=$ mean

$= 1$ in 150 Squae feet (unite)

$\times 1.5$

$\lambda^* = 1.5$ in 225 Squae feet

$\boxed{2.6\text{-}10}$ $X \sim \text{Poisson}(\lambda)$

mean $= \mu = 9 = \lambda$ Variance $= \sigma^2 = \lambda$ $S.D = \sigma = \sqrt{\lambda}$

$$p(\mu - 2\sigma < X < \mu + 2\sigma)$$

$$P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!} \; ; \; x = 0, 1, 2, \ldots$$

$$\Rightarrow p(9 - 2\sqrt{9} < X < 9 + 2\sqrt{9})$$

$$\Rightarrow p(3 < X < 15)$$

$$\Rightarrow \sum_{x=4}^{14} \frac{9^x e^{-9}}{x!} = 0.9373$$