

# Statistical Methods 105

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August 2020

# Chapter 1

## Discrete Random Variable

- 1 Discrete Probability Distributions
- 2 Some Discrete Probability Distributions
  - Discrete Uniform Random Variable
  - Binomial Distribution
  - Hypergeometric Distribution
  - Poisson Distribution

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# 1) Discrete Probability Distributions

1

## Definition (Probability function)

The set of ordered pairs  $(x, f(x))$  is a probability function, probability mass function, or probability distribution of the discrete random variable  $X$  if, for each possible outcome  $x$ ,

①  $f(x) \geq 0$ ,

②  $\sum_{x \in X} f(x) = 1$ ,

③  $P(X = x) = f(x)$ .

2

## Definition (cumulative distribution function)

The cumulative distribution function  $F(x)$  of a discrete random variable  $X$  with probability distribution  $f(x)$  is

$$F(x) = P(X \leq x) = \sum_{t \leq x} f(t), \text{ for } -\infty < x < +\infty.$$

③

## Definition (Mean of a Random Variable)

Let  $X$  be a random variable with probability distribution  $f(x)$ . The mean, or expected value, of  $X$  is

$$\mu_X = \mu = E(x) = \sum_x x f(x).$$

توقع  
X

③

## Theorem

Let  $X$  be a random variable with probability distribution  $f(x)$ . The expected value of the random variable  $g(X)$  is

$$\mu_{g(X)} = E[g(X)] = \sum_x g(x) f(x).$$

توقع  
X  
والتي  
g(X)

### Example

Suppose that the number of cars  $X$  that pass through a car wash between 4:00 P.M. and 5:00 P.M. on any sunny Friday has the following probability distribution:

|        |                |                |               |               |               |               |
|--------|----------------|----------------|---------------|---------------|---------------|---------------|
| $x$    | 4              | 5              | 6             | 7             | 8             | 9             |
| $f(x)$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |

1. Find  $E(X)$ .
2. Let  $g(X) = 2X - 1$  represent the amount of money, in dollars, paid to the attendant by the manager. Find the attendant's expected earnings for this particular time period.

$$E(g(x)) = \sum_{\text{all } x} g(x) \cdot f(x)$$

## Solution

Simple calculations yield:

|            |                |                |                |                |                |                |
|------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $x$        | 4              | 5              | 6              | 7              | 8              | 9              |
| $f(x)$     | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{4}$  | $\frac{1}{4}$  | $\frac{1}{6}$  | $\frac{1}{6}$  |
| $xf(x)$    | $\frac{1}{3}$  | $\frac{5}{12}$ | $\frac{3}{2}$  | $\frac{7}{4}$  | $\frac{4}{3}$  | $\frac{3}{2}$  |
| $g(x)$     | 7              | 9              | 11             | 13             | 15             | 17             |
| $g(x)f(x)$ | $\frac{7}{12}$ | $\frac{9}{12}$ | $\frac{11}{4}$ | $\frac{13}{4}$ | $\frac{15}{6}$ | $\frac{17}{6}$ |

1.  $E(X) = \frac{1}{3} + \frac{5}{12} + \frac{3}{2} + \frac{7}{4} + \frac{4}{3} + \frac{3}{2} = 6.83 \rightarrow$

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بالدالة الاحصائية

2. The attendant's expected earnings for this particular time period is equal to:

$$E[g(X)] = \frac{7}{12} + \frac{9}{12} + \frac{11}{4} + \frac{13}{4} + \frac{15}{6} + \frac{17}{6} = 14.67. \$$$

↓  
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## Properties of the mean:

$$= E(aX) + E(b)$$

$$E(X) = \square$$

$$E(g(x)) = \sum g(x) \cdot f(x)$$

### ③ Theorem

Let  $X$  a random variable. If  $a$  and  $b$  are constants, then

$$E(aX + b) = aE(X) + b.$$

التوقع لـ  $aX + b$

$$= E(aX) + E(b)$$

$$= aE(X) + b$$

### ③ Theorem

The expected value of the sum or difference of two or more functions of a random variable  $X$  is the sum or difference of the expected values of the functions. That is,

$$E[g(X) \pm h(X)] = E[g(X)] \pm E[h(X)].$$

التوقع لـ حاصل الجمع أو الفرق  
حول في المتغير العشوائي  $X$

### Example

Let  $X$  be a random variable with probability distribution as follows:

|        |               |               |   |               |
|--------|---------------|---------------|---|---------------|
| $x$    | 0             | 1             | 2 | 3             |
| $f(x)$ | $\frac{1}{3}$ | $\frac{1}{2}$ | 0 | $\frac{1}{6}$ |

$E(Y)$

Find the expected value of  $Y = (X - 1)^2$ .

Solution *ممكن ان يكون بطرق مختلفة*

\* Simple calculations yield:

|            |               |               |   |               |
|------------|---------------|---------------|---|---------------|
| $x$        | 0             | 1             | 2 | 3             |
| $f(x)$     | $\frac{1}{3}$ | $\frac{1}{2}$ | 0 | $\frac{1}{6}$ |
| $g(x)$     | 1             | 0             | 1 | 4             |
| $f(x)g(x)$ | $\frac{1}{3}$ | 0             | 0 | $\frac{2}{3}$ |

$$\begin{aligned} &= E(X^2 - 2X + 1) \\ &= E(X^2) \\ &\quad + E(-2X) \\ &\quad + E(1) \end{aligned}$$

Therefore, the expected value of  $Y$  is equal to:

*ممكن ان نحصل على ذلك بالاحصاء*

$$E(Y) = E[g(X)] = 1.$$

*دعنا نثبت*

\* Another solution: by using the properties of the mean theorems,  
 $E(Y) = E((X - 1)^2) = E(X^2) - 2E(X) + 1.$  (Hint!)

# Variance of Random Variable

$$E(g(x)) = \sum_{\text{all } x} g(x) \cdot f(x)$$

## Theorems (Variance of Random Variable)

(4)

Let  $X$  be a random variable with probability distribution  $f(x)$  and mean  $\mu$ . The **variance of  $X$**  is

$$Var(X) = \sigma^2 = E[(X - \mu)^2] = \sum_x \underbrace{(x - \mu)^2}_{g(x)} \underbrace{f(x)}_{\frac{1}{n}}$$

or it can be written as:

$$\sigma_x^2 = \sigma^2 = E(X^2) - \underbrace{E(X)^2}_{[E(X)]^2}$$

(4)

$$Var(g(X)) = \sigma_{g(X)}^2 = E[(g(x) - \mu_{g(X)})^2] = \sum_x (g(x) - \mu_{g(X)})^2 f(x).$$

تباين دالة في  $X$

The positive square root of the variance,  $\sigma$ , is called the **standard deviation of  $X$** .

$$\updownarrow$$
$$\sigma_x$$

## Properties of the Variance:

$$\begin{aligned} &= \text{Var}(aX) + \text{Var}(b) \\ &= a^2 \text{Var}(X) + 0 \end{aligned}$$

$$\geq 0$$

②

### Theorem

Let  $X$  a random variable. If  $a$  and  $b$  are constants, then  $\text{Var}(aX + b) = a^2 \text{Var}(X)$ .

باین برای ثابت  $a$  و  $b$

$$\begin{aligned} &= \text{V}(aX) + \text{V}(b) \\ &= a^2 \text{V}(X) \end{aligned}$$

### Corollary

Setting  $a = 1$ , then  $\text{Var}(X + b) = \text{Var}(X)$ .

### Corollary

Setting  $b = 0$ , then  $\text{Var}(aX) = a^2 \text{Var}(X)$ .

### Example

Calculate the variance of  $g(X) = 2X + 3$ , where  $X$  is a random variable with probability distribution:

|        |               |               |               |               |
|--------|---------------|---------------|---------------|---------------|
| $x$    | 0             | 1             | 2             | 3             |
| $f(x)$ | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{1}{2}$ | $\frac{1}{8}$ |

### Solution

\* Simple calculations yield

$$\sigma_{g(X)}^2 = \sum_{\forall x} [g(x) - \mu_{g(X)}]^2 f(x)$$

له طريقتين في الحل

$$g(x) = 2x + 3$$

|          |               |               |               |               |
|----------|---------------|---------------|---------------|---------------|
| x        | 0             | 1             | 2             | 3             |
| f(x)     | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{1}{2}$ | $\frac{1}{8}$ |
| g(x)     | 3             | 5             | 7             | 9             |
| g(x)f(x) | $\frac{3}{4}$ | $\frac{5}{8}$ | $\frac{7}{2}$ | $\frac{9}{8}$ |

$E(g(x)) = \sum g(x) \cdot f(x)$

$$E(g(x))$$

Therefore, The expected value of  $g(X)$  is equal to

$$E[g(X)] = \frac{3}{4} + \frac{5}{8} + \frac{7}{2} + \frac{9}{8} = 6$$

يمكننا حل ذلك  
بالطريقة الثانية

So, the variance of  $g(X) = 2X + 3$  is equal to

$$\sigma_{g(X)}^2 = (3-6)^2 * \frac{1}{4} + (5-6)^2 * \frac{1}{8} + (7-6)^2 * \frac{1}{2} + (9-6)^2 * \frac{1}{8} = 4$$

and the standard deviation of  $g(X)$  is equal to:  $\sigma = \sqrt{4} = 2$ .

\* Another solution: By using the properties of the variance

$$\sum_{i=1}^n (x_i - \mu_x)^2 f(x_i)$$

$$\text{Var}(2X + 3) = 2^2 \text{Var}(X) = 4 \text{Var}(X) = 4[E(X^2) - E(X)^2]$$

|        |               |               |               |               |
|--------|---------------|---------------|---------------|---------------|
| $x$    | 0             | 1             | 2             | 3             |
| $f(x)$ | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{1}{2}$ | $\frac{1}{8}$ |
| $x^2$  | 0             | 1             | 4             | 9             |

Therefore,  $E[X^2] = \frac{1}{8} + 2 + \frac{9}{8} = \frac{26}{8}$ , and  $E[X] = \frac{1}{8} + 1 + \frac{3}{8} = \frac{12}{8}$ .

Then, the variance of  $g(X) = 2X + 3$  is equal to

$$\sigma^2 = 4 \left[ \frac{26}{8} - \left( \frac{12}{8} \right)^2 \right] = 4.$$

- 1 Discrete Probability Distributions
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  - Discrete Uniform Random Variable
  - Binomial Distribution
  - Hypergeometric Distribution
  - Poisson Distribution

## 2.1) Discrete Uniform Random Variable

### Definition (Discrete Uniform Random Variable)

① A random variable  $X$  is called discrete uniform if it has a finite number of specified outcomes, say  $x_1, x_2, \dots, x_k$  and each outcome is equally likely. Then, the discrete uniform mass function is given by:

$$f(x) = P(X = x) = \begin{cases} \frac{1}{k}, & x = x_1, x_2, \dots, x_k \\ 0, & \text{otherwise.} \end{cases}$$

**Note:**  $k$  is called the parameter of the distribution.

### Theorem

② The expected value (mean) and variance of the discrete uniform distribution are:

$$\mu = E(X) = \sum_{i=1}^k \frac{x_i}{k}, \text{ and } \sigma^2 = \frac{1}{k} \sum_{i=1}^k [x_i - E(X)]^2.$$

### Example

Suppose that you select a ball from a box contains 6 balls labeled  $1, 2, \dots, 6$ . Let  $X$  = the number that is observed when selecting a ball. Find  $E(X)$  and  $Var(X)$ .

### Solution

The probability distribution of  $X$  is:

$$P(X = x) = \begin{cases} \frac{1}{6}, & x = x_1, x_2, \dots, x_6 \\ 0, & \text{otherwise.} \end{cases}$$

The expected value:

$$\mu = E(X) = \sum_{i=1}^k \frac{x_i}{k} = \frac{1 + 2 + 3 + 4 + 5 + 6}{6} = 3.5.$$

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بالطريقة البسيطة



The variance:

$$\sigma^2 = \frac{1}{k} \sum_{i=1}^k [x_i - E(X)]^2 = \frac{(1 - 3.5)^2 + \dots + (6 - 3.5)^2}{6} = 2.92.$$

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## 2.2) Binomial Distribution

### Definition (Bernoulli Process)

The process is said to be a Bernoulli process if:

- The outcomes of process is either success or failure.
- The probability of success is  $P(X = 1) = p$  and the probability of failure is  $P(X = 0) = 1 - p = q$ .

Strictly speaking, trials of random experiment are called binomial trials if satisfy the following conditions:

- 1 The experiment consists of finite number of repeated trials.
- 2 Each trial has exactly two outcomes: success or failure.
- 3 The repeated trials are independent.
- 4 The probability of success remains the same in each trial.

# Binomial distribution

- نشره في ١ - ١.  $n$  من المحاولات المستقلة (أي  $n$  من عملية برنولي)  
٢ - كل محاولة من هذه المحاولات لها وجهان: الاحتمال  $p$  والاحتمال  $q$

## Definition (Binomial Distribution)

The binomial distribution is defined based on the Bernoulli process. It is made up of  $n$  independent Bernoulli processes. Suppose that  $X_1, X_2, \dots, X_n$  are independent Bernoulli random variables, then  $Y = \sum X_i$  will conform binomial distribution. The probability mass function of the binomial random variable  $X$  is given by:

$$(1) \quad f(x) = P(X = x) = \binom{n}{x} p^x q^{n-x}, \quad x = 0, 1, 2, \dots, n.$$

\*We denote the binomial distribution with the parameters  $n$  and  $p$ , by  $Bin(n, p)$  and  $\binom{n}{x} = \frac{n!}{x!(n-x)!} = n \text{ C } x$  من الآلة الحاسبة

- ٣ - الاحتمال  $p$  والاحتمال  $q$  ثابت لكل محاولة  
٤ -  $p + q = 1$  دائماً لأنهما مكملان لبعضهما  
٥ - محاولات متتالية متكررة

## The Cumulative Distribution Function (CDF)

2 The cumulative distribution function (CDF) of binomial distribution  $\text{Bin}(n, p)$  is:

$$F_X(x) = P(X \leq x) = \sum_{i=0}^x \binom{n}{i} p^i q^{n-i}.$$

ممكن الاستدلال  
الآلة الطالسة  
غيرها يدخل علامة (طالسة)

### 3 Theorem

The mean and variance of the binomial distribution  $\text{Bin}(n, p)$  are

$$\mu = np \text{ and } \sigma^2 = npq.$$

### Example

If the mean and the variance of a binomial distribution are 10 and 5 respectively, then:

- 1 Determine the probability mass function.
- 2 Calculate the probability  $P(X = 0)$ .
- 3 Calculate the probability  $P(X \geq 2)$ .

$$\begin{aligned} E(X) &= np \Rightarrow 10 = np \\ \text{Var}(X) &= npq \Rightarrow 5 = npq \\ \therefore 5 &= 10q \Rightarrow q = \frac{1}{2} \Rightarrow p = \frac{1}{2} \\ \therefore 10 &= n \frac{1}{2} \Rightarrow n = 20 \end{aligned}$$

### Solution

- 1 By solving  $E(X) = np = 10$  and  $\text{Var}(X) = np(1 - p) = 5$ , we get  $p = 0.5$  and  $n = 20$ . The probability mass function is:

$$P(X = x) = \binom{20}{x} (0.5)^x (0.5)^{20-x}, \quad x = 0, 1, \dots, 20$$

$$\begin{aligned} \text{2 } P(X = 0) &= \binom{20}{0} (0.5)^0 (0.5)^{20} = 0.5^{20} \\ &= \sum_{x=0}^{20} \binom{20}{x} \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{20-x} \end{aligned}$$

$$\begin{aligned} \text{3 } P(X \geq 2) &= 1 - P(X < 2) = 1 - [P(X = 0) + P(X = 1)] \\ &= 1 - 0.00002 = 0.99998. \end{aligned}$$

### Example

Suppose that the probability that a person dies when he or she contracts a certain disease is 0.4. A sample of 10 persons who contracted this disease is randomly chosen. Find the following:

- 1 The probability that exactly 4 persons will die.
- 2 The probability that less than 3 persons will die.
- 3 The probability that more than 8 persons will die.
- 4 The expected number of persons who will die.
- 5 The variance of the number of persons who will die.

### Solution

The probability mass function is:

$n = 10$   
 $p = 0.4$  ,  $q = 0.6$  }  $\Rightarrow \text{Bin}(10, 0.4)$

احتمال الموت  $\leftarrow$

$$P(X = x) = \binom{10}{x} (0.4)^x (0.6)^{10-x}, x = 0, 1, \dots, 10$$

1.  $P(X = 4) = \binom{10}{4} (0.4)^4 (0.6)^{10-4} = 0.251$

2.

$$\begin{aligned} P(X < 3) &= P(X = 0) + P(X = 1) + P(X = 2) \\ &= \sum_{x=0}^2 \binom{10}{x} (0.4)^x (0.6)^{10-x} = 0.167. \end{aligned}$$

3.

ممكن ان يكون ذلك بالاحتمال بالستة ام  $\Sigma$

$$\begin{aligned} P(X > 8) &= P(X = 9) + P(X = 10) \\ &= \sum_{x=9}^{10} \binom{10}{x} (0.4)^x (0.6)^{10-x} = 0.0017. \end{aligned}$$

4.  $E(X) = np = (10)(0.4) = 4$

5.  $Var(X) = np(1 - p) = (10)(0.4)(0.6) = 2.4$

## 2.3) Hypergeometric Distribution

### Definition

The probability distribution of the hypergeometric random variable  $X$  describes the probability of  $K$  successes (random draws for which the object drawn has a specified feature) in  $n$  draws, without replacement, from a finite population of size  $N$  that contains exactly  $K$  objects with that feature, where each draw is either a success or a failure.

There are two methods of selection:

1. Selection **with replacement**: If we select the elements of the sample at random and with replacement, then  $X \sim \text{Bin}(n, p)$ ; where  $p = \frac{K}{N}$
2. Selection **without replacement**: When the selection is made without replacement, the random variable  $X$  has a hypergeometric distribution with parameters  $N, n$ , and  $K$ . and we write  $X \sim h(x; N, n, K)$ .

The probability mass function for hypergeometric random variable  $X$  is:

$$\textcircled{1} f(x) = P(X = x) = \begin{cases} \frac{\binom{K}{x} \binom{N-K}{n-x}}{\binom{N}{n}} & \max\{0, n-(N-K)\} \leq x \leq \min\{K, n\} \\ 0, & \text{otherwise.} \end{cases}$$

~~$x = 0, 1, 2, \dots, \min(K, n)$~~

## $\textcircled{2}$ Theorem

The mean and variance of the hypergeometric distribution  $h(x; N, n, K)$  are

$$\mu = n \frac{K}{N} \text{ and } \sigma^2 = n \frac{K}{N} \left(1 - \frac{K}{N}\right) \frac{N-n}{N-1}.$$

هذا الذي اهتم به على الحد  
الكل  $\approx$  احتمال النجاح

الاحتمال الفشل  
وهو مكمال الاحتمال النجاح

معامل  
التصحیح  
ونذلك لان  
العملية بدون ارجاع

### Example

Suppose there are 50 officers, 10 female officers and 40 male officers. Suppose 20 of them will be selected for promotion. Let  $X$  represent the number of female promotions. Find:

- 1 The probability that exactly one female is found in the sample.
- 2 The expected value (mean) and the variance of the number of females in the sample.

### Solution

$50 = N$  Female  $\frac{10}{50}$   $\left\{ \begin{array}{l} N=K \\ \text{male } \frac{40}{50} \end{array} \right.$   $\rightarrow n = 20$  (sample size)

- Note that the binomial distribution doesn't apply here, as the officers are **without replacement** once they are drawn. **In other words**, the trials are **not independent** events.
- $X$  has a hypergeometric distribution with  $N = 50$ ,  $n = 20$ , and  $K = 10$ ; i.e.  $X \sim h(x; N, n, K) = h(x; 50, 20, 10)$ .

$$P(X = x) = \begin{cases} \frac{\binom{10}{x} \binom{50-10}{20-x}}{\binom{50}{20}} & x = 0, 1, 2, \dots, 10 \\ 0, & \text{otherwise.} \end{cases}$$

- ① The probability that exactly one female is found in the sample is:

$$f(1) = P(X = 1) = \frac{\binom{10}{1} \binom{40}{19}}{\binom{50}{20}} = 0.0279$$

- ② The expected value (mean) is  $E(X) = n \frac{K}{N} = 20 \times \frac{10}{50} = 4$ .

- ③ The variance is
- $$\sigma^2 = n \frac{K}{N} \left(1 - \frac{K}{N}\right) \frac{N-n}{N-1} = 20 \times \frac{10}{50} \times \left(1 - \frac{10}{50}\right) \times \frac{50-20}{50-1} = 1.9592$$

3

## (Binomial Approximation Theorem)

If  $n$  is small compared to  $K$ , then a binomial distribution  $Bin(n, p = \frac{K}{N})$  can be used to approximate the hypergeometric distribution  $h(x; N, n, K)$ .

تقريب توزيع hyper الى توزيع Bin عندما تكون  $K$  كبيرة مقارنة مع  $n$   
 Example

A manufacturer of automobile tires reports that among a shipment of 5000 sent to a local distributor, 1000 are slightly blemished. If one purchases 10 of these tires at random from the distributor, what is the probability that exactly 3 are blemished?

### Solution

Since  $K = 1000$  is large relative to the sample size  $n = 10$ , we shall approximate the desired probability by using the binomial distribution. The probability of obtaining a blemished tire is 0.2. Therefore, the probability of obtaining exactly 3 blemished tires is

$$h(3; 5000, 10, 1000) \approx Bin(10, p = \frac{1000}{5000}) = \binom{10}{3} (0.2)^3 (0.8)^7 = 0.2013.$$

$N = 5000$      $K = 1000$      $N - K = 4000$      $n = 10$   
 $2 = \frac{1000}{5000}$      $PLX = 3$

## 2.4) Poisson Distribution

### Definition

Let  $X$  the number of outcomes occurring during a given time interval.  $X$  is called a Poisson random variable, with parameter  $\lambda$ , when its probability mass function is given by

$$P(x, \lambda) = P(X = x) = e^{-\lambda} \frac{\lambda^x}{x!}, \quad x = 0, 1, 2, \dots$$

where  $e$  is an irrational number approximately equal to 2.71828 and  $\lambda$  is the average number of occurrences per interval unit.

### Theorem

$X \sim \text{Poisson}(\lambda)$

If a random variable  $X$  has a Poisson distribution. Then both the mean and the variance of  $X$  are  $\lambda$ .

$$\mu = \lambda \text{ and } \sigma^2 = \lambda$$

### Example

The mean number of accidents per month at a certain intersection is 3.  $\Rightarrow \lambda = 3$  for 1 month  $\Rightarrow X \sim \text{Poisson}(3) \Rightarrow P(X) = \frac{e^{-3} 3^x}{x!}, x: 0, 1, \dots$

- 1 What is the probability that in **any given month** 4 accidents will occur at this intersection?
- 2 What is the probability that more than 4 accidents will occur in **any given month** at the intersection?
- 3 What is the probability that 4 accidents will occur in **5 months**?

### Solution

- 1  $f(4) = P(X = 4) = e^{-3} \frac{3^4}{4!} = 0.168$ .  $\sum_{x=4}^{\infty} P(X)$    
  $\rightarrow$   $\frac{e^{-3} 3^4}{4!} \times \frac{3}{4} = \frac{e^{-3} 3^5}{4!} = 0.1847$
- 2  $P(X > 4) = 1 - P(X \leq 4) = 1 - [P(X = 0) + \dots + P(X = 4)] = 1 - [\sum_{x=0}^4 e^{-3} \frac{3^x}{x!}] = 0.1847$ .
- 3 Since the average number of accidents at a certain intersection per month is 3, thus the average number of accidents in 5 months is 15. Let  $X$  represent the number of accidents in 5 months,  $f(x) = P(X = x) = e^{-15} \frac{15^x}{x!}, x = 0, 1, 2, \dots$   
Then,  $f(4) = P(X = 4) = e^{-15} \frac{15^4}{4!} = 0.00065$ .



### Example

لا نعلم هل هو 1 أو 2، لأننا لا نعلم بين نوعه على حسب خفة أو ثقل

In a certain industrial facility, accidents occur infrequently. It is known that the probability of an accident on any given day is 0.005 and accidents are independent of each other.

- 1 What is the probability that in any given period of 400 days there will be an accident on one day?
- 2 What is the probability that there are at most three days with an accident?

### Solution

$X \sim \text{Bin}(400, 0.005) \Rightarrow X \sim \text{Poisson}(2 \text{ for 1 day}) \Rightarrow$

Let  $X$  be a binomial random variable with  $n = 400$  and  $p = 0.005$ .

Thus,  $np = 2$ . Use the Poisson approximation,

$$P(X) = \frac{e^{-2} 2^x}{x!},$$

$$P(X = 1) = e^{-2} 2^1 = 0.271.$$

$X = 0, 1, \dots$

$$P(X \leq 3) = \sum_{x=0}^3 e^{-2} \frac{2^x}{x!} = 0.857.$$