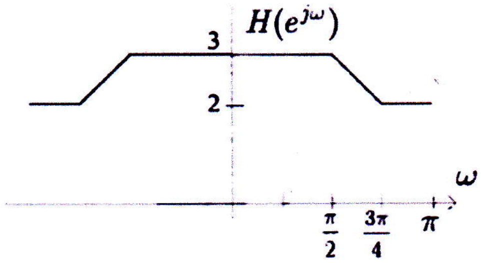
Q[2]: Mid1-321

(a) h[n] is a FIR system of length 3. The DTFT of this system is shown.

i. Find the 3-point DFT H[k] of this system.

ii. Find the 4-point DFT H[k] of this system.

iii. Using the DFT method, find the output y[n] of this system if the input is x[n]={4,7}.



(b) The 3-point DFT of g[n]={1,2,2} is given by G[k]={5,-1,-1}. Find f[n], the 3-point IDFT of F[k]={-2 ,10 ,-2}

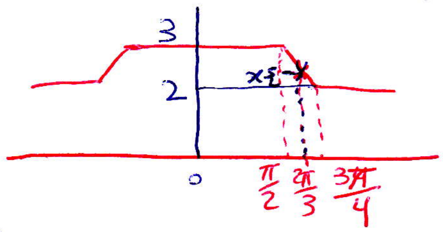
(a) i) $\omega = \frac{2\pi k}{N}$ (here, $N=3$)

at $k=0 \rightarrow \omega = 0$

$\rightarrow H[0] = 3$ (from the graph)

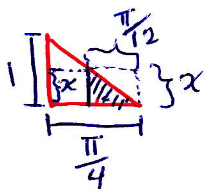
similarly, at $k=1 \rightarrow \omega = \frac{2\pi}{3}$

hence,



$H[1] = 2 + x$

to find $x \rightarrow$



$$\frac{1}{\frac{\pi}{4}} = \frac{x}{\frac{\pi}{12}}$$

$$\Rightarrow x = \frac{1}{3}$$

hence, $\rightarrow H[1] = 2 + \frac{1}{3} = \frac{7}{3}$

$\rightarrow H[2] = \frac{4\pi}{3}$

① Complete the graph until $\frac{4\pi}{3}$ and find $H[2]$

② Since the graph is repeated every 2π $\rightarrow 2\pi - \frac{4\pi}{3} = \frac{2\pi}{3}$

$\frac{2\pi}{3}$ is the same point of $H[1]$

hence, $H[2] = H[1] = \frac{7}{3}$

$\rightarrow H[k] = \{3, \frac{7}{3}, \frac{7}{3}\}$

ii) Here, $\omega = \frac{2\pi k}{4} = \frac{\pi k}{2}$ ($N=4$)

$H[0] = 3$

$H[1] = 3$

$H[2] = 2$

$H[3] = 3$

iii) Zero pad $x[n]$

$\Rightarrow x[n] = \{4, 7, 0, 0\}$

$X[k] = D_4 x$

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 4 \\ 7 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 11 \\ 4-j7 \\ -3 \\ 4+j7 \end{bmatrix}$$

$$Y[k] = \begin{bmatrix} X[0]\,H[0] \\ X[1]\,H[1] \\ X[2]\,H[2] \\ X[3]\,H[3] \end{bmatrix}$$

$$Y[k] = \begin{bmatrix} 11 \times 3 \\ (4-j7) \times 3 \\ -3(2) \\ (4+j7)(3) \end{bmatrix} = \begin{bmatrix} 33 \\ 12-j21 \\ -6 \\ 12+j21 \end{bmatrix}$$

$\rightarrow$ P.T.O

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Now, find $y[n]$

$$y[n] = D_N^{-1} Y[k] \qquad \left(D_N^{-1} = \frac{1}{N} D_N^{*} \right)$$

$$y[n] = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 33 \\ 12-j21 \\ -6 \\ 12+j21 \end{bmatrix}$$

$$y[n] = \{12.75,\ 20.25,\ 0.75,\ -0.75\}$$

(b) Using the properties

(linearity) $\alpha g[n] \longleftrightarrow \alpha G[k]$

(frequency-shifting) $W_N^{-k_0 n} g[n] \longleftrightarrow G[\langle k-k_0 \rangle_N]$

Since, $F[k] = 2G[\langle k-1 \rangle_3]$

Hence, $f[n] = 2g[n] W_3^{-(-1)n} = 2g[n] W_3^{n} \quad \left(W_3^{n} = e^{\frac{2\pi j n}{3}} \right)$

$\rightarrow f[0] = 2(1) = 2$

$f[1] = 2(2) e^{\frac{j2\pi(1)}{3}} = 4e^{\frac{j2\pi}{3}}$

$f[2] = 2(2) e^{\frac{j2\pi(2)}{3}} = 4e^{\frac{j4\pi}{3}}$

$\Rightarrow f[n] = \{2,\ 4e^{\frac{j2\pi}{3}},\ 4e^{\frac{j4\pi}{3}}\}$