Discrete Mathematics
Chapter 04
Boolean Algebra

إعداد وتقديم

دكتور أحمد السيد

كلية العلوم– قسم الرياضيات –جامعة الملك سعود

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٠٥٥٩٥٩٦٧٢٠


بسم الله الرحمن الرحيم

تعرف على المدرب:

١.أستاذ جامعي بقسم الرياضيات كلية العلوم بجامعة الملك سعود.
٢.مستشار عميد كلية العلوم.
٣.عمل مستشاراً لوكيل الجامعة لتطوير الأعمال
٤.عمل مستشاراً لعميد تطوير المهارات
٥.أحد مؤسسي عمادة تطوير المهارات
٦.مدرب ومحكم دولي معتمد.
٧.مدقق جودة ومقيم معتمد.
٨.عمل مستشار تدريب ومحلل إحصائي لدى العديد من الجهات.
٩.قدم العديد من البرامج التدريبية.
١٠.شارك في تنظيم مؤتمرات وملتقيات وندوات ودورات تدريبية وأسبوع المهنة.
١١. لديه نشر علمي في مجالات ومؤتمرات علمية محكمة.
٢١. مدرب معتمد من **web of Science ISI**
٣١. شارك في تأسيس مراكز تدريبية وبحثية وبيوت خبرة.
١٣.شارك في إعداد وتطوير عدداً من البرامج وجلب الاعتمادات الدولية لها.
١٤.عمل كمدير مشاريع لبعض المشروعات الكبرى.
١٥.يحب الخير للجميع، ويحب العمل بروح الفريق.
.

Dr. Ahmed Alsayed

خطوات التعلم

Who

Why

What

When

Where

Steps

Tools

5WSTC Model Dr. Ahmed Alsayed in Training This Course

Continuum& Creative

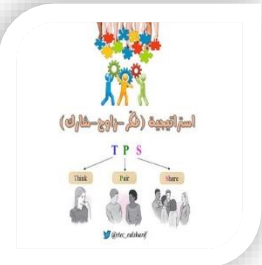
استراتيجيات التدريب والتدريس

المناقشة

العرض التقدمي

العصف الذهني

التطبيق العملي



فكر – زاوج –شارك

المحاضرة

الاكتشاف

البحث والاستقصاء

لنتفق معاً لتحقيق أهدافنا

لنتفق

أساس نجاح التدريب أنتم لذا نأمل التركيز والإضافة.

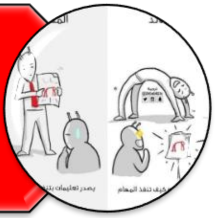
الإبداع يحتاج إلى صفاء الذهن لذا نأمل تجنب المشتتات (جوال، حديث جانبي،....).

السؤال والمناقشة والتفاعل سر المعرفة لذا نسأل ونناقش مع كامل الاحترام بيننا.

الاختلاف في الرأي لا يفسد للود قضية لذا إذا اختلفنا فهذا سر من أسرار النجاح.

الفئات المستهدفة:

الطلاب



المحاضرين والمعيدين وطلبة الدراسات العليا

الجدول الزمني

الوقت	Activity النشاط
٦:٠٠–٦:١٠	تسجيل
٦:١٠–٦:٢٠	Welcome and introduction ترحيب وتعارف
٦:٢٠–٨:٠٠	Session1 جلسة ١
٨:٠٠–٨:١٥	Pray استراحة وصلاة
٨:١٥–٩:٢٠	Session2 جلسة ٢
٩:٢٠–١٠:٠٠	Session3 جلسة (٣)

Dr.

تعارف

Dr. Ahmed Alsayed

1- Course Syllabus (Credit Hours: 4 (3+2))

No	List of Topics
1	Introduction to Number Systems: • Binary System (Binary to Decimal Conversion - Decimal to Binary Conversion – Arithmetic: addition, subtraction, multiplication) • Octal Number System (Conversions and Arithmetic) • Hexadecimal Number System (Conversions and Arithmetic)
2	Logic: • Proposition calculus and connectives • Truth tables • Propositional Equivalence.
3	Sets: • Set operations
4	Boolean Algebra: • Boolean Functions • Representation Boolean Functions • Logic Gates • Minimization of Circuit
5	Basic Concepts of Graph Theory: • Graph Terminology and Special Types of Graphs • Connectivity

King Saud University
College of Applied Studies & Community Service
Department of Computer Science & Engineering

	خطة تدريس المقرر (مقترح) **Course plan**
رمز ورقم المقرر: 153 ريض **Math. 153**	مـقرر: الرياضيات المحددة **Discrete Mathematics**

2- Learning Resources

Required Textbooks:

Discrete Mathematics and its application, seven edition, Kenneth H. Rosen

Essential References Materials

Any Book in Discrete Mathematics.

Electronic Materials

1. http://www.wikihow.com/Convert-from-Binary-to-Decimal
2. http://www.cci-compeng.com/Unit_1_Representing_Data/1309_Fractions.htm
3. http://syedatnsu.tripod.com/chap1.pdf
4. Digital logic and computer design, Morris mano:
5. http://www.4shared.com/office/NKpFEyey/Digital_Logic_And_Computer_Des.htm
6. http://www.exploringbinary.com/binary-addition/
7. The Pearson Guide to MCA Entrance Examinations by Edgar Thorpe
8. http://www.robotroom.com

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5. http://www.4shared.com/office/NKpFEyey/Digital_Logic_And_Computer_Des.htm
6. http://www.exploringbinary.com/binary-addition/
7. The Pearson Guide to MCA Entrance Examinations by Edgar Thorpe
8. http://www.robotroom.co

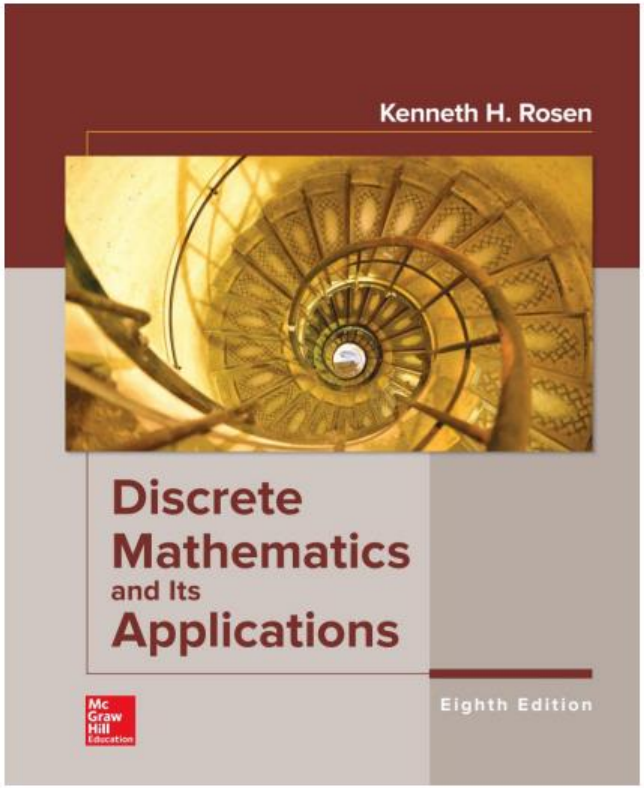
Topic	المحاضرة	م
Logic and Prof Chapter2		
https://www.youtube.com/watch?v=eFDzhn1Inc4&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz	١	
https://www.youtube.com/watch?v=dOZ6Bam4Bks&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=2	٢	
https://www.youtube.com/watch?v=-BxvBFJaN6E&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=3	٣	
https://www.youtube.com/watch?v=xCDQHgDQEUk&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=4	٤	
https://www.youtube.com/watch?v=MipjqNYp3T4&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=5	٥	
https://www.youtube.com/watch?v=nYtOiEtcYIs&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=6	٦	
https://www.youtube.com/watch?v=mk0krQZNzoE&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=7	٧	
https://www.youtube.com/watch?v=bNNpZa3fwq0&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=8	٨	
https://www.youtube.com/watch?v=BAfbs5N4uC8&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=9	٩	
Sets Chapter 2		
https://www.youtube.com/watch?v=1FEEjRCWo6E&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=10	١	
https://www.youtube.com/watch?v=RdbOHQddn3Y&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=11	٢	

Topic	المحاضرة	م
Logic and Prof Chapter2		
https://www.youtube.com/watch?v=eFDzhn1Inc4&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz	١	
https://www.youtube.com/watch?v=dOZ6Bam4Bks&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=2	٢	
https://www.youtube.com/watch?v=-BxvBFJaN6E&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=3	٣	
https://www.youtube.com/watch?v=xCDQHgDQEUk&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=4	٤	
https://www.youtube.com/watch?v=MipjqNYp3T4&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=5	٥	
https://www.youtube.com/watch?v=nYtOiEtcYIs&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=6	٦	
https://www.youtube.com/watch?v=mk0krQZNzoE&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=7	٧	
https://www.youtube.com/watch?v=bNNpZa3fwq0&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=8	٨	
https://www.youtube.com/watch?v=BAfbs5N4uC8&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=9	٩	

Topic	المحاضرة	
Sets Chapter 3		
https://www.youtube.com/watch?v=1FEEjRCWo6E&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=10	١	
https://www.youtube.com/watch?v=RdbOHQddn3Y&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=11	٢	
https://www.youtube.com/watch?v=iSuD96uQ2zU&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=12	٣	
https://www.youtube.com/watch?v=0E8ek2FSxWE&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=13	٤	
https://www.youtube.com/watch?v=S7jh1BH_UU8&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=14	٥	
https://www.youtube.com/watch?v=Wc7RW5EVaBw&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=15	٦	
https://www.youtube.com/watch?v=MFRVt2zwf0Y&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=16	٧	

- Course code: 153 Math
- Course name: Discrete Mathematics
- Level: 1
- Second Semester 2st Year / B.Sc.
- Course Credit: 3 credits + 2 h Taterao's
- Instructor: Dr. Ahmed Alsayed

Lectures Reference



Textbook

2019

Course Objectives

- Learn how to think mathematically.
- Grasp the basic logical and reasoning mechanisms
- of mathematical thought.
- Acquire logic and proof as the basics for abstract
- thinking.
- Improve problem-solving skills.
- Grasp the basic elements of induction, recursion, combination and discrete structures.

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What is discrete mathematics? Discrete mathematics is the part of mathematics devoted to the study of discrete objects. (Here discrete means consisting of distinct or unconnected elements.) The kinds of problems solved using discrete mathematics include:

- How many ways are there to choose a valid password on a computer system?
- What is the probability of winning a lottery?
- Is there a link between two computers in a network?
- How can I identify spam e-mail messages?
- How can I encrypt a message so that no unintended recipient can read it?
- What is the shortest path between two cities using a transportation system?
- How can a list of integers be sorted so that the integers are in increasing order?
- How many steps are required to do such a sorting?
- How can it be proved that a sorting algorithm correctly sorts a list?
- How can a circuit that adds two integers be designed?
- How many valid Internet addresses are there?

You will learn the discrete structures and techniques needed to solve problems such as these.

More generally, discrete mathematics is used whenever objects are counted, when relationships between finite (or countable) sets are studied, and when processes involving a finite number of steps are analyzed. A key reason for the growth in the importance of discrete mathematicsis that information is stored and manipulated by computing machines in a discrete fashion.

WHY STUDY DISCRETE MATHEMATICS? There are several important reasons for studying discrete mathematics. First, through this course you can develop your mathematical maturity: that is, your ability to understand and create mathematical arguments. You will not get very far in your studies in the mathematical sciences without these skills. Second, discrete mathematics is the gateway to more advanced courses in all parts of the mathematical sciences. Discrete mathematics provides the mathematical foundations for many computer science courses, including data structures, algorithms, database theory, automata theory, formal languages, compiler theory, computer security, and operating systems. Students find these courses much more difficult when they have not had the appropriate mathematical foundations from discrete mathematics. One student sent me an e-mail message saying that she used the contents of this book in every computer science course she took! Math courses based on the material studied in discrete mathematics include logic, set theory, number theory, linear algebra, abstract algebra, combinatorics, graph theory, and probability theory (the discrete part of the subject).

Introduce Myself

Dr. Ahmed Alsayed

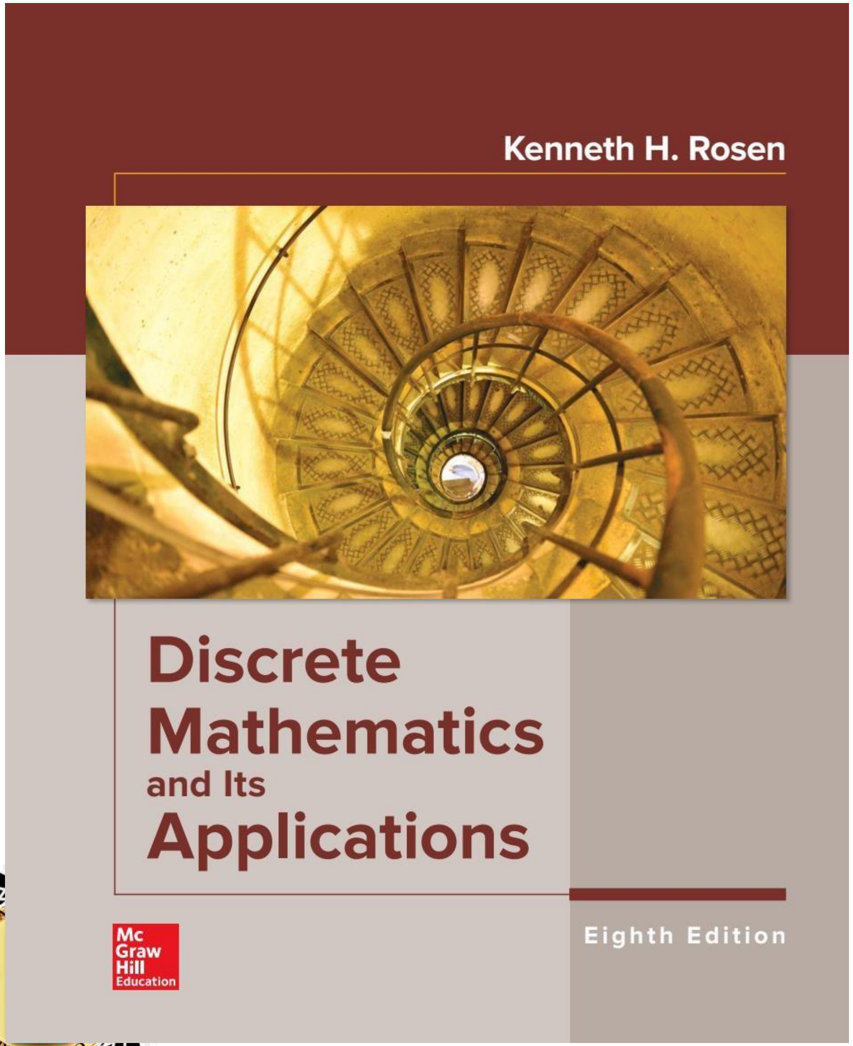
College of Sciences

Email: dalsayed@ksu.edu.sa

Basic Course Information

- Course code: **153Math**
- Course name: **Discrete Mathematics**
- Level: **1st Year / B.Sc.**
- Course Credit: **3 credits**
- Instructor:
  - **Dr. Ahmed Alsayed**

Lectures Reference



Textbook

2019

Course Objectives

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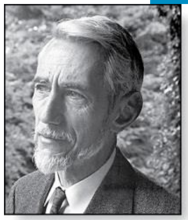
Boolean Algebra

Chapter 12

Dr. Ahmed Alsayed- 0559596720
King Saud University- College of Science

Chapter Summary



Claude Shannon
(1916 - 2001)

- Boolean Functions
- Representing Boolean Functions
- Logic Gates
- Minimization of Circuits (*not currently included in overheads*)

Boolean Functions

Section 12.1

Section Summary

Introduction to Boolean Algebra
Boolean Expressions and Boolean Functions
Identities of Boolean Algebra
Duality
The Abstract Definition of a Boolean Algebra

Introduction to Boolean Algebra

Boolean algebra has rules for working with elements from the set $\{0, 1\}$ together with the operators + (Boolean sum), $\cdot$ (Boolean product), and $\bar{\ }$ (complement).

These operators are defined by:

Boolean sum: $1 + 1 = 1,\ 1 + 0 = 1,\ 0 + 1 = 1,\ 0 + 0 = 0$

Boolean product: $1 \cdot 1 = 1,\ 1 \cdot 0 = 0,\ 0 \cdot 1 = 0,\ 0 \cdot 0 = 0$

complement: $\bar{0} = 1,\ \bar{1} = 0$

Example: Find the value of $1 \cdot 0 + \overline{(0 + 1)}$

Solution : $1 \cdot 0 + \overline{(0 + 1)} = 0 + \bar{1}$

$= 0 + 0$

$= 0$

Boolean Expressions and Boolean Functions

Definition: Let $B = \{0, 1\}$. Then $B^n = \{(x_1, x_2, \ldots, x_n) \mid x_i \in B \text{ for } 1 \leq i \leq n\}$ is the set of all possible n-tuples of 0s and 1s. The variable x is called a *Boolean variable* if it assumes values only from B, that is, if its only possible values are 0 and 1. A function from B^n to B is called a *Boolean function of degree n*.

Example: The function $F(x, y) = x$ from the set of ordered pairs of Boolean variables to the set {0, 1} is a Boolean function of degree 2.

TABLE 1

x	y	$F(x, y)$
1	1	1
1	0	0
0	1	0
0	0	1

Boolean Expressions and Boolean Functions (*continued*)

Example: Find the values of the Boolean function represented by $F(x, y, z) = xy + \bar{z}$.

Solution: We use a table with a row for each combination of values of *x*, *y*, and *z* to compute the values of $F(x,y,z)$.

TABLE 2

x	y	z	xy	$\bar{z}$	$F(x, y, z) = xy + \bar{z}$
1	1	1	1	0	1
1	1	0	1	1	1
1	0	1	0	0	0
1	0	0	0	1	1
0	1	1	0	0	0
0	1	0	0	1	1
0	0	1	0	0	0
0	0	0	0	1	1

Boolean Expressions and Boolean Functions (*continued*)

Definition: Boolean functions F and G of n variables are equal if and only if $F(b_1, b_2, \ldots, b_n) = G(b_1, b_2, \ldots, b_n)$ whenever $b_1, b_2, \ldots, b_n$ belong to B. Two different Boolean expressions that represent the same function are *equivalent*.

Definition: The complement of the Boolean function F is the function $\bar{F}$, where $\bar{F}(x_1, x_2, \ldots, x_n) = \overline{F(x_1, x_2, \ldots, x_n)}$.

Definition: Let F and G be Boolean functions of degree n. The Boolean sum $F + G$ and the Boolean product FG are defined by

$(F + G)(x_1, x_2, \ldots, x_n) = F(x_1, x_2, \ldots, x_n) + G(x_1, x_2, \ldots, x_n)$

$(FG)(x_1, x_2, \ldots, x_n) = F(x_1, x_2, \ldots, x_n)G(x_1, x_2, \ldots, x_n)$

Boolean Functions

Example: How many different Boolean functions of degree *n* are there?

Solution: By the product rule for counting, there are 2^n different *n*-tuples of 0s and 1s. Because a Boolean function is an assignment of 0 or 1 to each of these different n-tuples, by the product rule there are 2^{2^n} different Boolean functions of degree *n*.

TABLE 4 The Number of Boolean Functions of Degree *n*.

Degree	*Number*
1	4
2	16
3	256
4	65,536
5	4,294,967,296
6	18,446,744,073,709,551,616

The example tells us that there are 16 different Boolean functions of degree two. We display these in Table 3.

TABLE 3 The 16 Boolean Functions of Degree Two.

x	y	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}	F_{16}
1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
1	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0
0	1	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0
0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0

Identities of Boolean Algebra

TABLE 5 Boolean Identities.

Identity	*Name*
$\overline{\overline{x}} = x$	Law of the double complement
$x + x = x$ $x \cdot x = x$	Idempotent laws
$x + 0 = x$ $x \cdot 1 = x$	Identity laws
$x + 1 = 1$ $x \cdot 0 = 0$	Domination laws
$x + y = y + x$ $xy = yx$	Commutative laws
$x + (y + z) = (x + y) + z$ $x(yz) = (xy)z$	Associative laws
$x + yz = (x + y)(x + z)$ $x(y + z) = xy + xz$	Distributive laws
$\overline{(xy)} = \overline{x} + \overline{y}$ $\overline{(x + y)} = \overline{x}\ \overline{y}$	De Morgan's laws
$x + xy = x$ $x(x + y) = x$	Absorption laws
$x + \overline{x} = 1$	Unit property
$x\overline{x} = 0$	Zero property

Each identity can be proved using a table.

All identities in Table 5, except for the first and the last two come in pairs. Each element of the pair is the *dual* of the other (obtained by switching Boolean sums and Boolean products and 0's and 1's.

The Boolean identities correspond to the identities of propositional logic (Section 1.3) and the set identities (Section 2.2).

Identities of Boolean Algebra

Example: Show that the distributive law
$x(y + x) = xy + xz$ is valid.
Solution: We show that both sides of this identity always take the same value by constructing this table.

TABLE 6 Verifying One of the Distributive Laws.

x	y	z	$y + z$	xy	xz	$x(y + z)$	$xy + xz$
1	1	1	1	1	1	1	1
1	1	0	1	1	0	1	1
1	0	1	1	0	1	1	1
1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0

Formal Definition of a Boolean Algebra

Definition: A *Boolean algebra* is a set B with two binary operations $\vee$ and $\wedge$, elements 0 and 1, and a unary operation $^{-}$ such that for all x, y, and z in B:

$$x \vee 0 = x$$
$$x \wedge 1 = x$$

identity laws

$$x \vee \bar{x} = 1$$
$$x \wedge \bar{x} = 0$$

complement laws

$$(x \vee y) \vee z = x \vee (y \vee z)$$
$$(x \wedge y) \wedge z = x \wedge (y \wedge z)$$

associative laws

$$x \vee y = y \vee x$$
$$x \wedge y = y \wedge x$$

commutative laws

$$x \vee (y \wedge z) = (x \vee y) \wedge (y \vee z)$$
$$x \wedge (y \vee z) = (x \wedge y) \vee (y \wedge z)$$

distributive laws

The set of propositional variables with the operators $\wedge$ and $\vee$, elements **T** and **F**, and the negation operator $\neg$ is a Boolean algebra.

The set of subsets of a universal set with the operators $\cup$ and $\cap$, the empty set ($\emptyset$), universal set (U), and the set complementation operator ($^{-}$) is a Boolean algebra.

Representing Boolean Functions

Section 12.2

Section Summary

Sum-of-Products Expansions
Functional Completeness

Sum-of-Products Expansion

Example: Find Boolean expressions that represent the functions (i) $F(x, y, z)$ and (ii) $G(x, y, z)$ in Table 1.

Solution:

(i) To represent F we need the one term $x\bar{y}z$ because this expression has the value 1 when $x = z = 1$ and $y = 0$.

(ii) To represent the function G, we use the sum $xy\bar{z} + \bar{x}y\bar{z}$ because this expression has the value 1 when $x = y = 1$ and $z = 0$, or $x = z = 0$ and $y = 1$.

TABLE 1

x	y	z	F	G
1	1	1	0	0
1	1	0	0	1
1	0	1	1	0
1	0	0	0	0
0	1	1	0	0
0	1	0	0	1
0	0	1	0	0
0	0	0	0	0

The general principle is that each combination of values of the variables for which the function has the value 1 requires a term in the Boolean sum that is the Boolean product of the variables or their complements.

Sum-of-Products Expansion (*cont.*)

Definition: A *literal* is a Boolean variable or its complement. A *minterm* of the Boolean variables $x_1, x_2, \ldots, x_n$ is a Boolean product $y_1 y_2 \cdots y_n$, where $y_i = x_i$ or $y_i = \overline{x_i}$. Hence, a minterm is a product of n literals, with one literal for each variable.

The minterm $y_1, y_2, \ldots, y_n$ has value has value 1 if and only if each x_i is 1.This occurs if and only if $x_i = 1$ when $y_i = x_i$ and $x_i = 0$ when $y_i = \overline{x_i}$.

Definition: The sum of minterms that represents the function is called the *sum-of-products expansion* or the *disjunctive normal form* of the Boolean function.

Sum-of-Products Expansion (*cont*)

Example: Find the sum-of-products expansion for the function $F(x,y,z) = (x + y)\,\bar{z}$.

Solution: We use two methods, first using a table and second using Boolean identities.

(*i*) Form the sum of the minterms corresponding to each row of the table that has the value 1.

TABLE 2

x	y	z	$x+y$	$\bar{z}$	$(x+y)\bar{z}$
1	1	1	1	0	0
1	1	0	1	1	1
1	0	1	1	0	0
1	0	0	1	1	1
0	1	1	1	0	0
0	1	0	1	1	1
0	0	1	0	0	0
0	0	0	0	1	0

Including a tem for each row of the table $F(x,y,z) = 1$ gives us $F(x, y, z) = xy\bar{z} + x\bar{y}\bar{z} + \bar{x}y\bar{z}$.

Sum-of-Products Expansion (*cont*)

(*ii*) We now use Boolean identities to find the disjunctive normal form of F(*x*,*y*,*z*):

$$F(x,y,z) = (x+y)\,\bar{z}$$

$$= x\bar{z} + y\bar{z}$$ *distributive law*

$$= x1\bar{z} + 1y\bar{z}$$ *identity law*

$$= x(y+\bar{y})\bar{z} + (x+\bar{x})y\bar{z}$$ *unit property*

$$= xy\bar{z} + x\bar{y}\bar{z} + xy\bar{z} + \bar{x}y\bar{z}$$ *distributive law*

$$= xy\bar{z} + x\bar{y}\bar{z} + \bar{x}y\bar{z}$$ *idempotent law*

Functional Completeness

Definition: Because every Boolean function can be represented using the Boolean operators $\cdot$, $+$, and $^{-}$, we say that the set $\{\cdot, +, ^{-}\}$ is *functionally complete*.

The set $\{\cdot, ^{-}\}$ is functionally complete since $x + y = \overline{\bar{x}\bar{y}}$.

The set $\{+, ^{-}\}$ is functionally complete since $xy = \overline{\bar{x} + \bar{y}}$.

The *nand* operator, denoted by $|$, is defined by $1|1 = 0$, and $1|0 = 0|1 = 0|0 = 1$. The set consisting of just the one operator nand $\{|\}$ is functionally complete. Note that $\bar{x} = x \mid x$ and $xy = (x|y)|(x|y)$.

The *nor* operator, denoted by $\downarrow$, is defined by $0 \downarrow 0 = 1$, and $1 \downarrow 0 = 0 \downarrow 1 = 1 \downarrow 1 = 0$. The set consisting of just the one operator nor $\{\downarrow\}$ is functionally complete. (*see Exercises* 15 *and* 16)

Logic Gates

Section 12.3

Section Summary

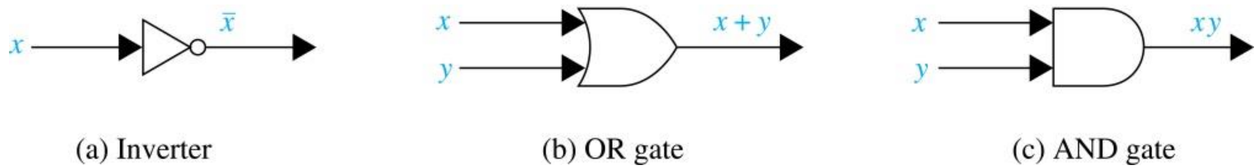
Logic Gates

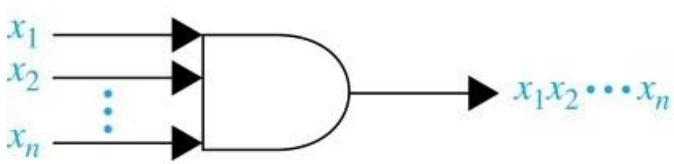
Combinations of Gates

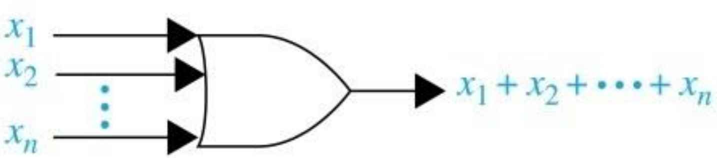
Examples of Circuits

Logic Gates

We construct circuits using *gates*, which take as input the values of two or more Boolean variables and produce one or more bits as output, and *inverters*, which take the value of a Boolean variable as input and produce the complement of this value as output.



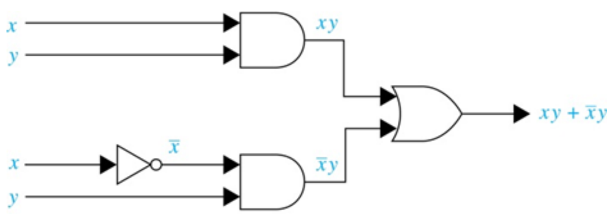


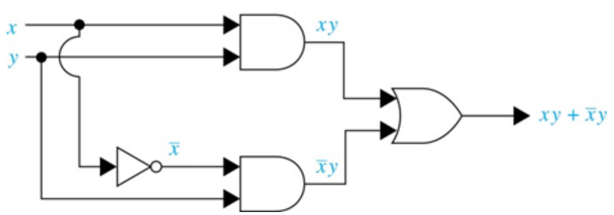


Combinations of Gates

Combinatorial circuits can be constructed using a combination of inverters, OR gates, and AND gates. Gates may share input and the output of one or more gates may be input to another.

We show two ways of constructing a circuit that produces the output $xy + \bar{x}y$.





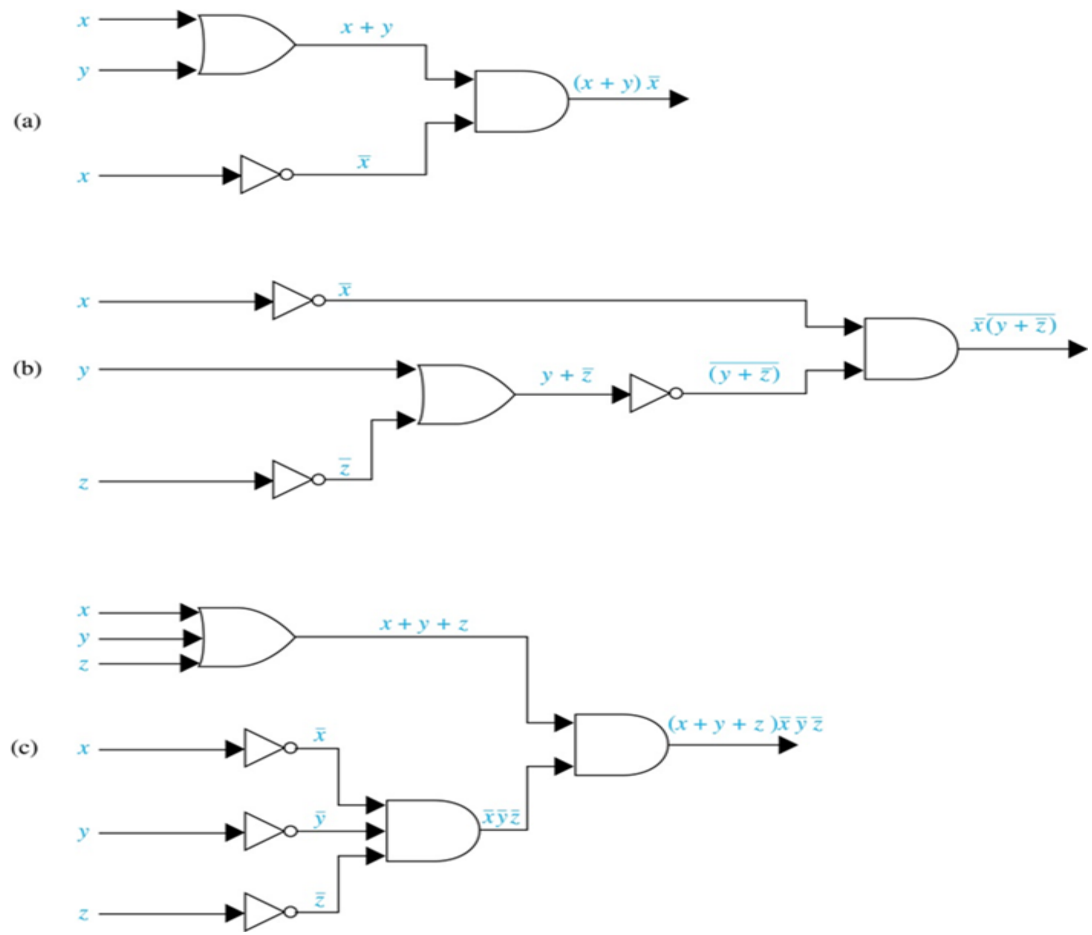
Combinations of Gates

Example: Construct circuits that produce these outputs

(a) $(x + y)\bar{x}$

(b) $\bar{x}\,\overline{(y + \bar{z})}$

(c) $(x + y + z)(\bar{x}\,\bar{y}\,\bar{z})$



Adders

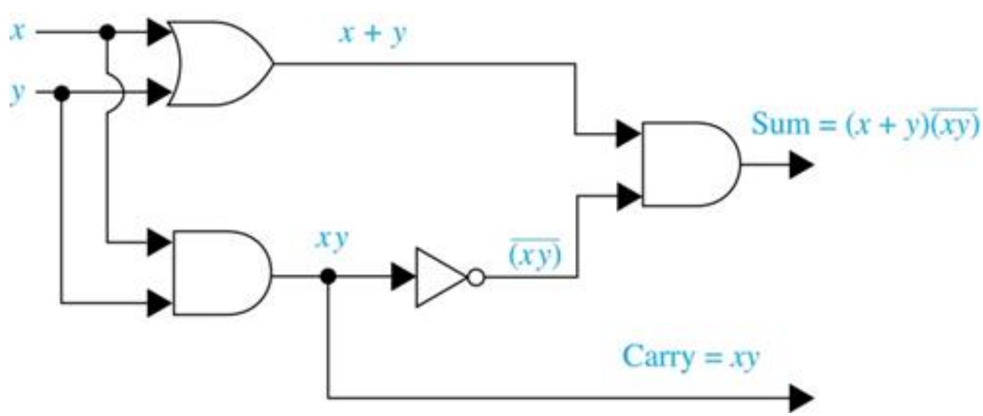
Logic circuits can be used to add two positive integers from their binary expansions.

The first step is to build a *half adder* that adds two bits, but which does not accept a carry from a previous addition.

Since the circuit has more than one output, it is a *multiple output circuit*.

TABLE 3 Input and Output for the Half Adder.

Input		*Output*	
x	*y*	*s*	*c*
1	1	0	1
1	0	1	0
0	1	1	0
0	0	0	0

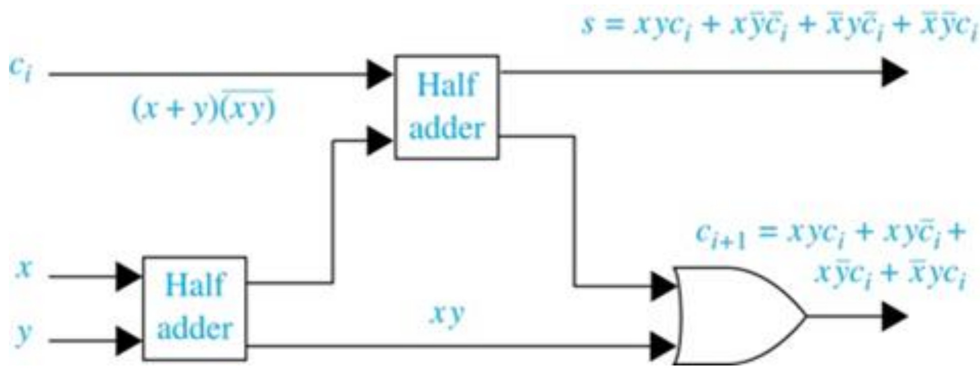


Adders (continued)

A *full adder* is used to compute the sum bit and the carry bit when two bits and a carry are added.

TABLE 4 Input and Output for the Full Adder.

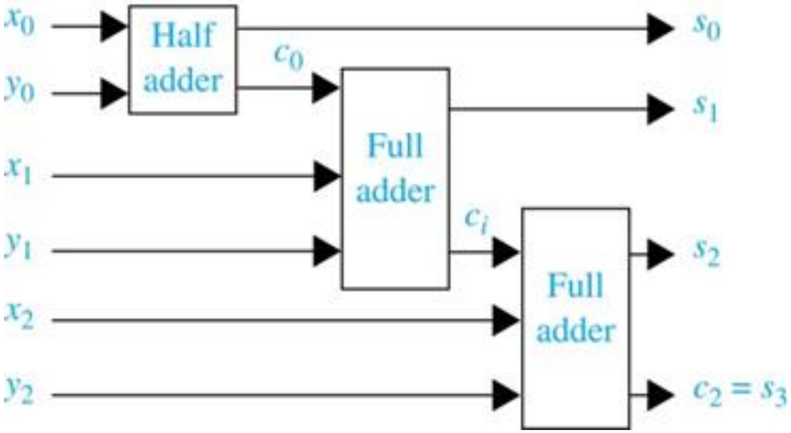
Input			Output	
x	y	c_i	s	c_{i+1}
1	1	1	1	1
1	1	0	0	1
1	0	1	0	1
1	0	0	1	0
0	1	1	0	1
0	1	0	1	0
0	0	1	1	0
0	0	0	0	0



Adders (continued)

A half adder and multiple full adders can be used to produce the sum of *n* bit integers.

Example: Here is a circuit to compute the sum of two three-bit integers.



شكرًا لحسن استماعكم

Thank you