

# Chapter 8

## Rate of Return Analysis

# **Systematic Economic Analysis Technique**

- 1. Identify the investment alternatives**
- 2. Define the planning horizon**
- 3. Specify the discount rate**
- 4. Estimate the cash flows**
- 5. Compare the alternatives**
- 6. Perform supplementary analyses**
- 7. Select the preferred investment**

# **Internal Rate of Return Analysis**

## **Single Alternative**

# Internal Rate of Return

- determines the interest rate ( $i^*$ ) that yields a future worth equal to zero over the planning horizon<sup>1</sup>
- the rate of interest earned on the unrecovered balance of the investment
- a very popular DCF method

$$0 = \sum_{t=0}^n A_t (1 + i^*)^{n-t}$$

<sup>1</sup> can also determine the interest rate that equates the present worth or annual worth to zero

# Internal Rate of Return

- Determining the value of  $i^*$  that satisfies the  $n$ -degree polynomial given below can be very challenging, since there can exist  $n$  distinct roots for an  $n$ -degree polynomial.
- *Descartes rule of signs* indicates an  $n$ -degree polynomial will have a single positive real root if there is a single sign change in the sequence of cash flows,  $A_1, A_2, \dots, A_{n-1}, A_n$ ; if there are 2 sign changes, there will be either 2 or 0 positive real roots; if there are 3 sign changes, there will be either 3 or 1 positive real roots; if there are 4 sign changes, there will be 4, 2, or 0 positive real roots; ...
- To determine if exactly one real and positive-valued root exists, use *Norstrom's criterion* – *if the cumulative cash flow begins with a negative value and changes only once to a positive-valued series, then there exists a unique positive real root.*

$$0 = \sum_{t=0}^n A_t (1 + i^*)^{n-t}$$

# Example 8.1

## SMP Investment

### Internal Rate of Return Analysis

<u>EOY</u>	<u>CF</u>
0	-\$500,000
1 - 10	\$92,500
10	\$50,000

$$\begin{aligned} \text{FW}(i^* \%) &= -\$500,000(F|P i^* \%, 10) + \$50,000 \\ &+ \$92,500(F|A i^* \%, 10) = \$0 \end{aligned}$$

# Example 8.1

## SMP Investment

### Internal Rate of Return Analysis

<u>EOY</u>	<u>CF</u>
0	-\$500,000
1 - 10	\$92,500
10	\$50,000

<u>i</u>	<u>FW</u>
12%	\$120,333.90
15%	-\$94,684.90

$$FW(i^*) = -\$500,000(F|P i^*, 10) + \$50,000 +$$

$$\$92,500(F|A i^*, 10) = \$0$$

$$i^* \approx 13.67893\%$$

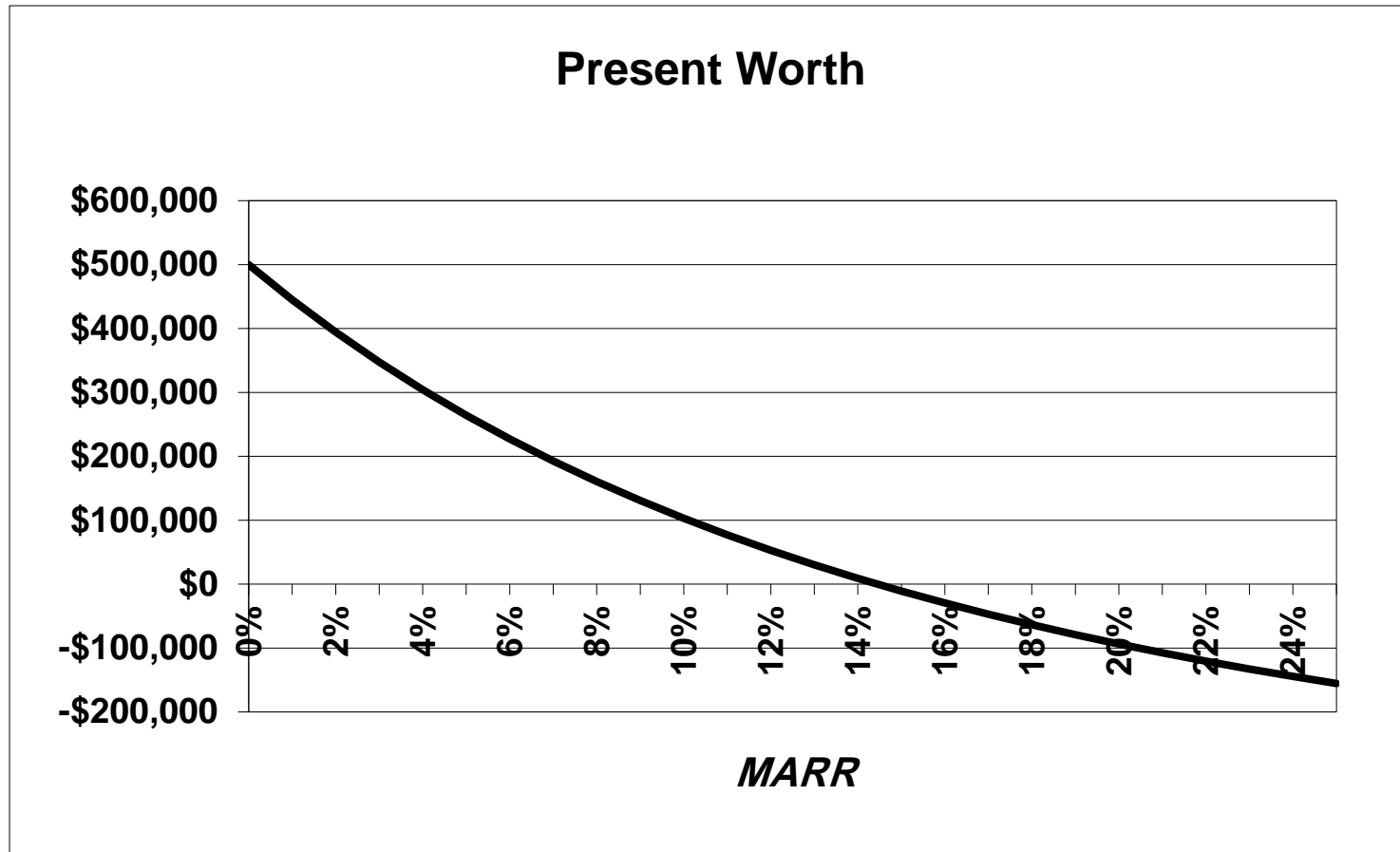
# Example 8.1

## Excel Solution

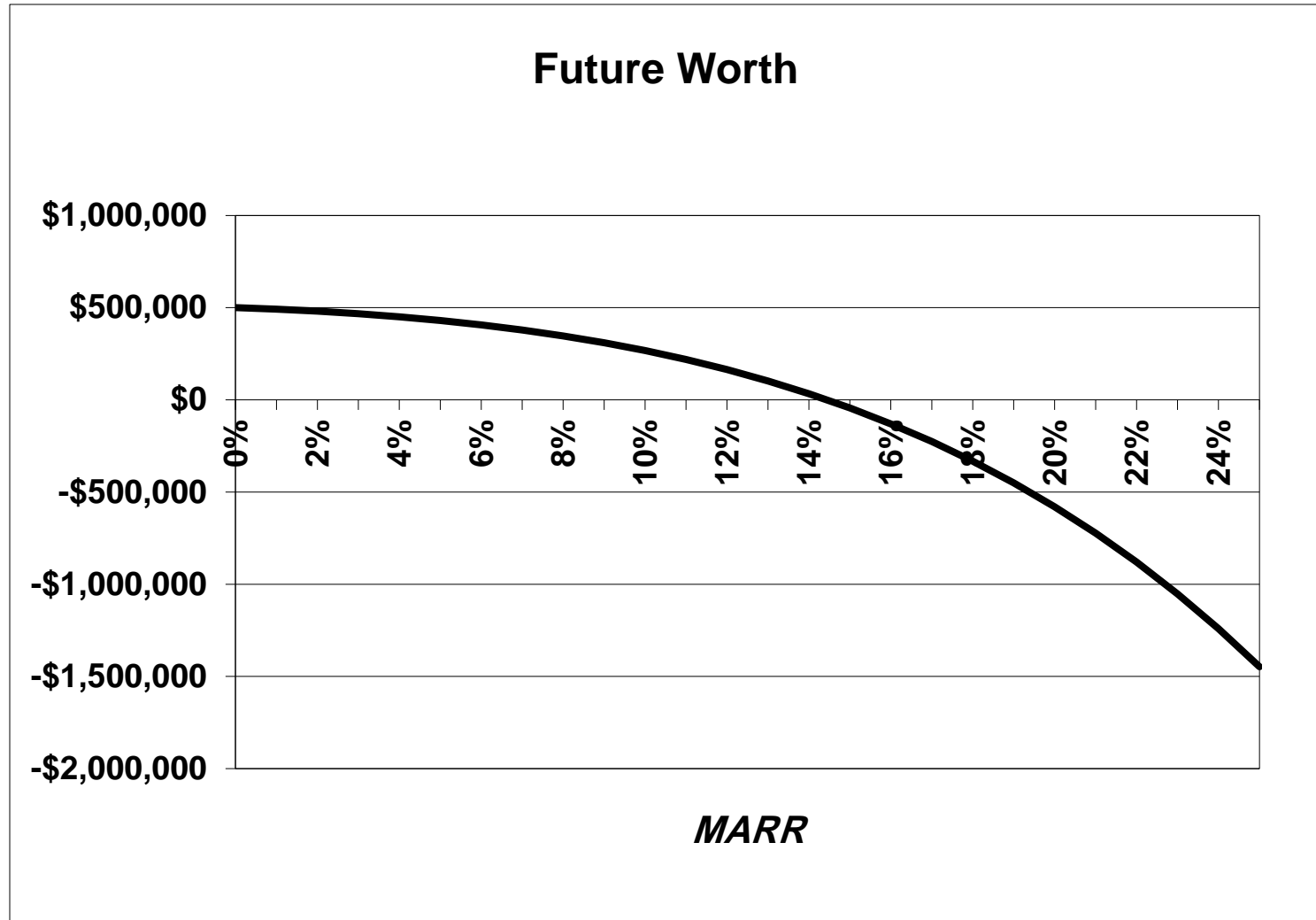
	A	B	
1	<u>EOY</u>	<u>CF</u>	
2	0	-\$500,000	
3	1	\$92,500	
4	2	\$92,500	
5	3	\$92,500	
6	4	\$92,500	
7	5	\$92,500	
8	6	\$92,500	
9	7	\$92,500	
10	8	\$92,500	
11	9	\$92,500	
12	10	\$142,500	
13	IRR =	[=IRR(B2:B12)]	
	IRR =	13.8003%	



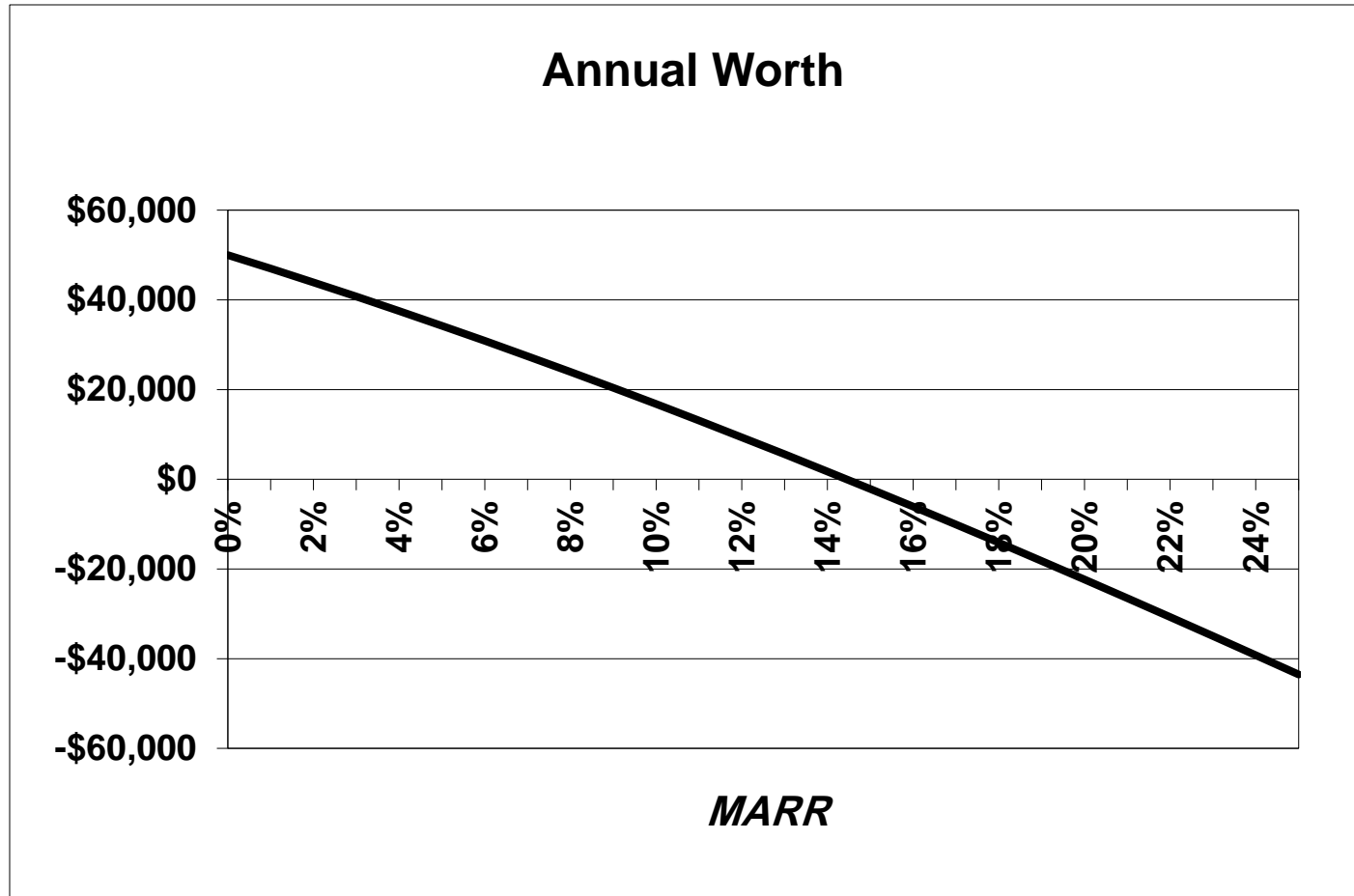
# SMP Investment



# SMP Investment



# SMP Investment



## Example 8.2

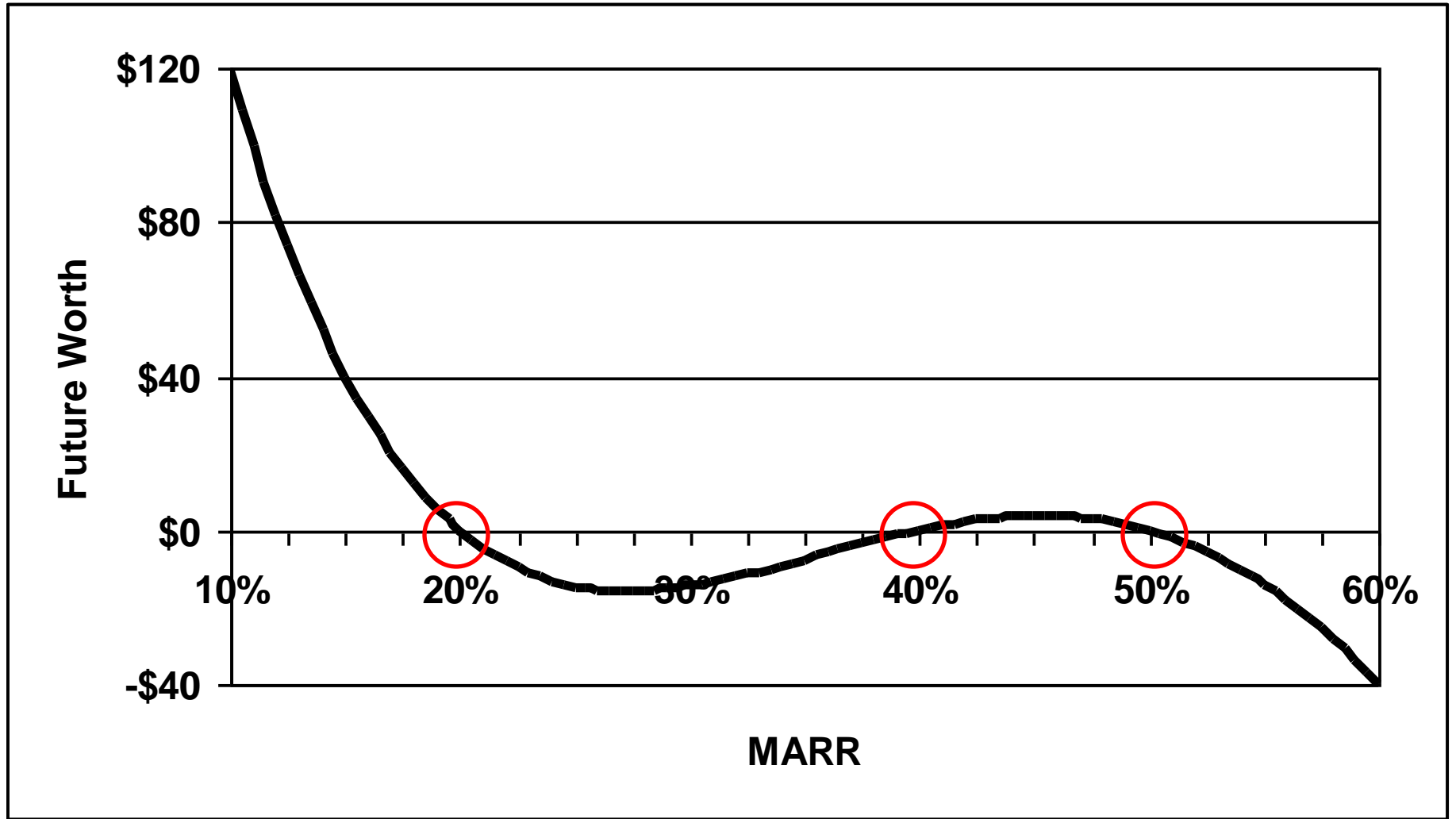
Consider the cash flow profile given below. The FW equals zero using a 20%, 40%, or 50% interest rate.

EOY	CF
0	-\$4,000
1	\$16,400
2	-\$22,320
3	\$10,080

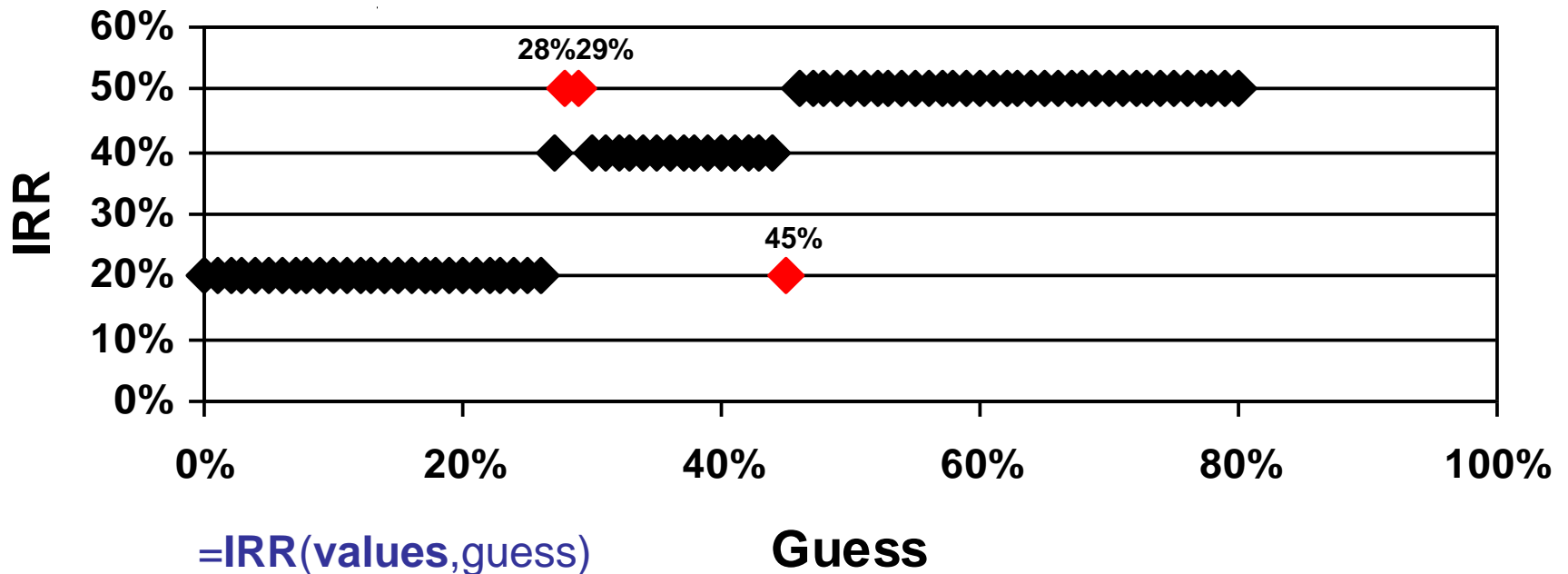
$$FW_1(20\%) = -\$4000(1.2)^3 + \$16,400(1.2)^2 - \$22,320(1.2) + \$10,080 = 0$$

$$FW_2(40\%) = -\$4000(1.4)^3 + \$16,400(1.4)^2 - \$22,320(1.4) + \$10,080 = 0$$

$$FW_3(50\%) = -\$4000(1.5)^3 + \$16,400(1.5)^2 - \$22,320(1.5) + \$10,080 = 0$$



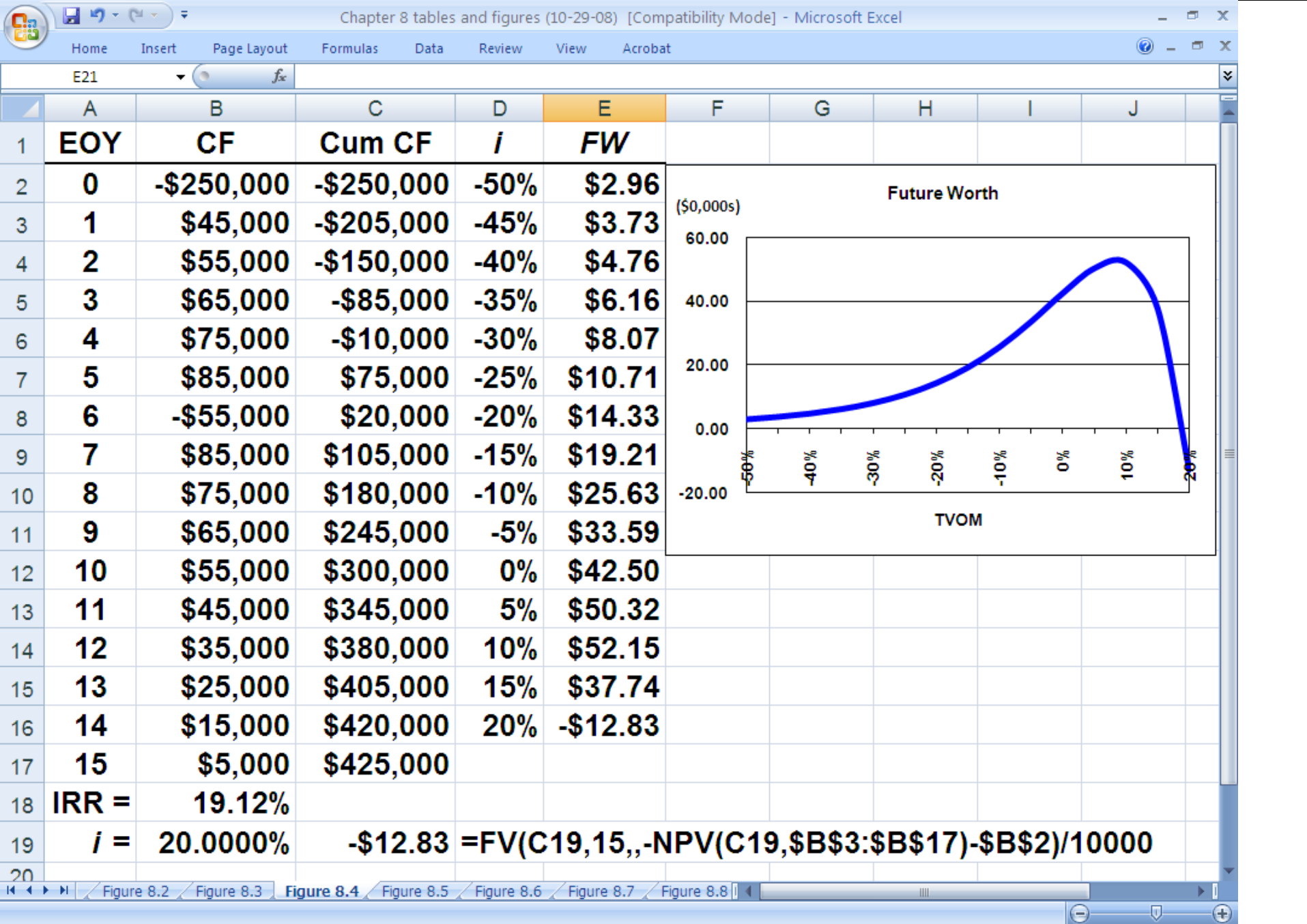
## Value Obtained from Excel's IRR Worksheet Function



# Example 8.3

**Julian Stewart invested \$250,000 in a limited partnership to drill for natural gas. The investment yielded annual returns of \$45,000 the 1<sup>st</sup> yr, followed by \$10,000 increases until the 6<sup>th</sup> yr, at which time an additional \$150,000 had to be invested for deeper drilling. Following the 2<sup>nd</sup> drilling, the annual returns decreased by \$10,000 per year, from \$85,000 to \$5,000. Using Excel, the IRR = 19.12%.**

**Plot future worth as a function of MARR and determine the MARR that maximizes FW.**





	A	B	C	D	E	F	G	H	I	J
1	<b>EOY</b>	<b>CF</b>	<b>Cum CF</b>	<b><i>i</i></b>	<b>FW</b>					
2	<b>0</b>	<b>-\$250,000</b>	<b>-\$250,000</b>	<b>-50%</b>	<b>\$2.96</b>					
3						(50,000c)				
4										
5										
6										
7										
8										
9										
10										
11										
12										
13	<b>13</b>	<b>\$25,000</b>	<b>\$405,000</b>	<b>15%</b>	<b>\$37.74</b>					
14	<b>14</b>	<b>\$15,000</b>	<b>\$420,000</b>	<b>20%</b>	<b>-\$12.83</b>					
15	<b>15</b>	<b>\$5,000</b>	<b>\$425,000</b>							
16	<b>IRR =</b>	<b>19.12%</b>								
17	<b><i>i</i> =</b>	<b>20.0000%</b>	<b>-\$12.83</b>	<b>=FV(C19,15,,-NPV(C19,\$B\$3:\$B\$17)-\$B\$2)/10000</b>						

**Solver Parameters**

Set Target Cell:

Equal To:  Max  Min  Value of:

By Changing Cells:

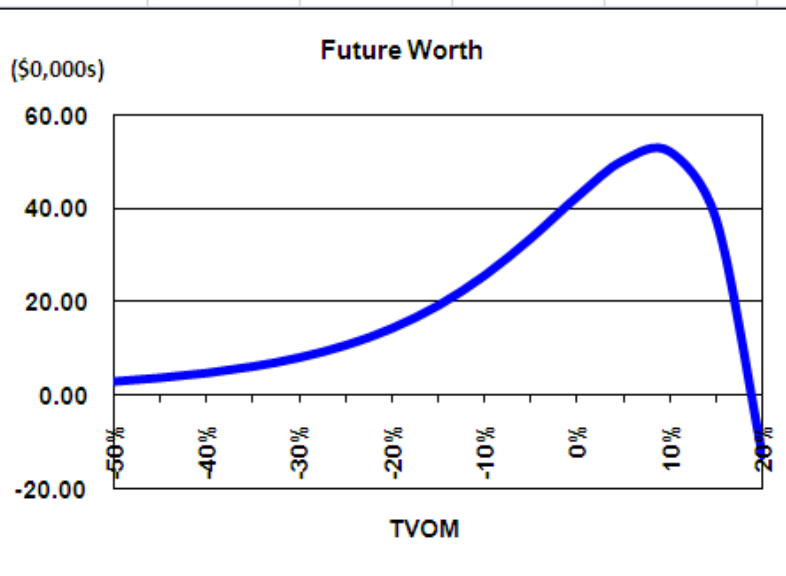
Subject to the Constraints:

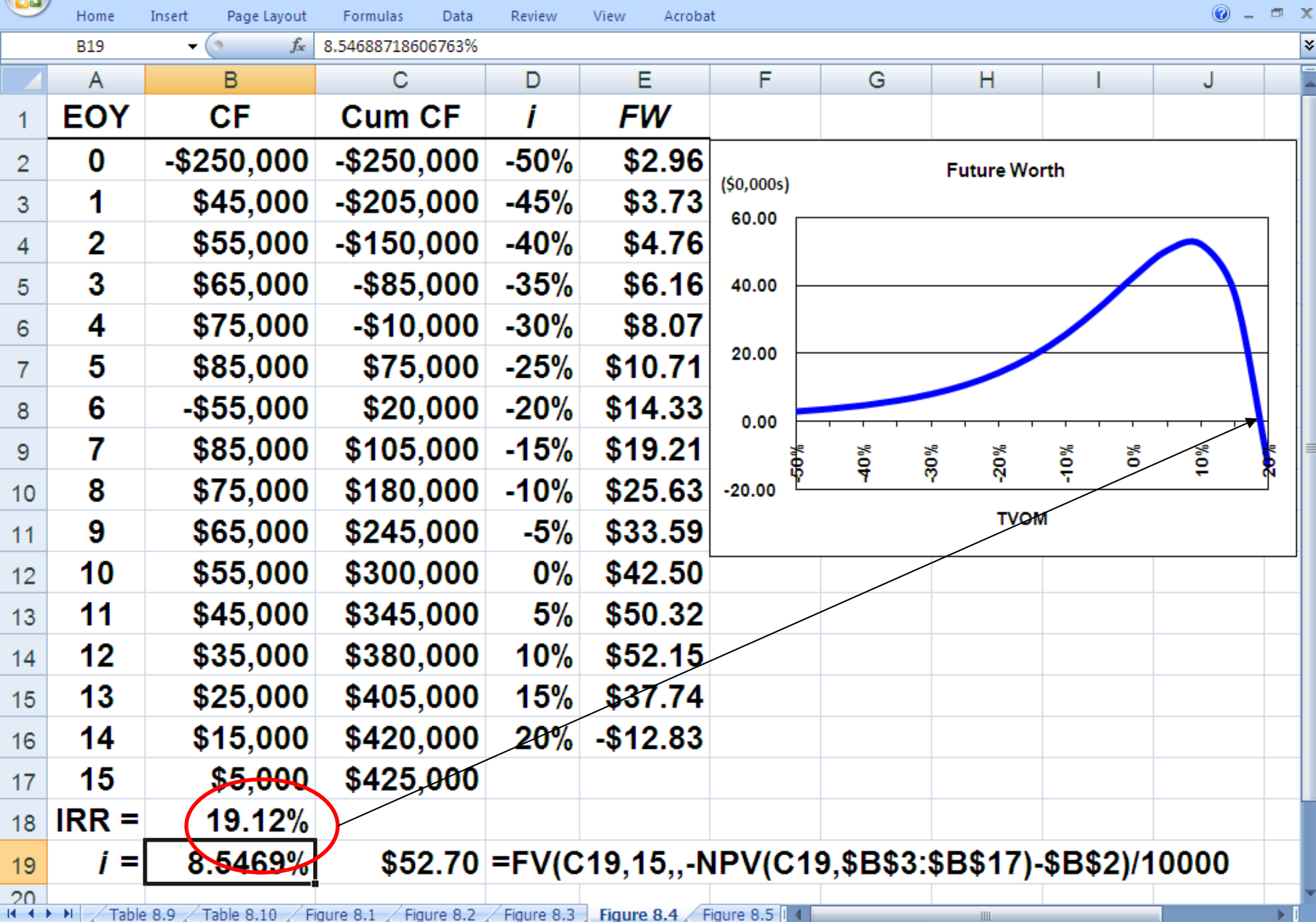
Future Worth

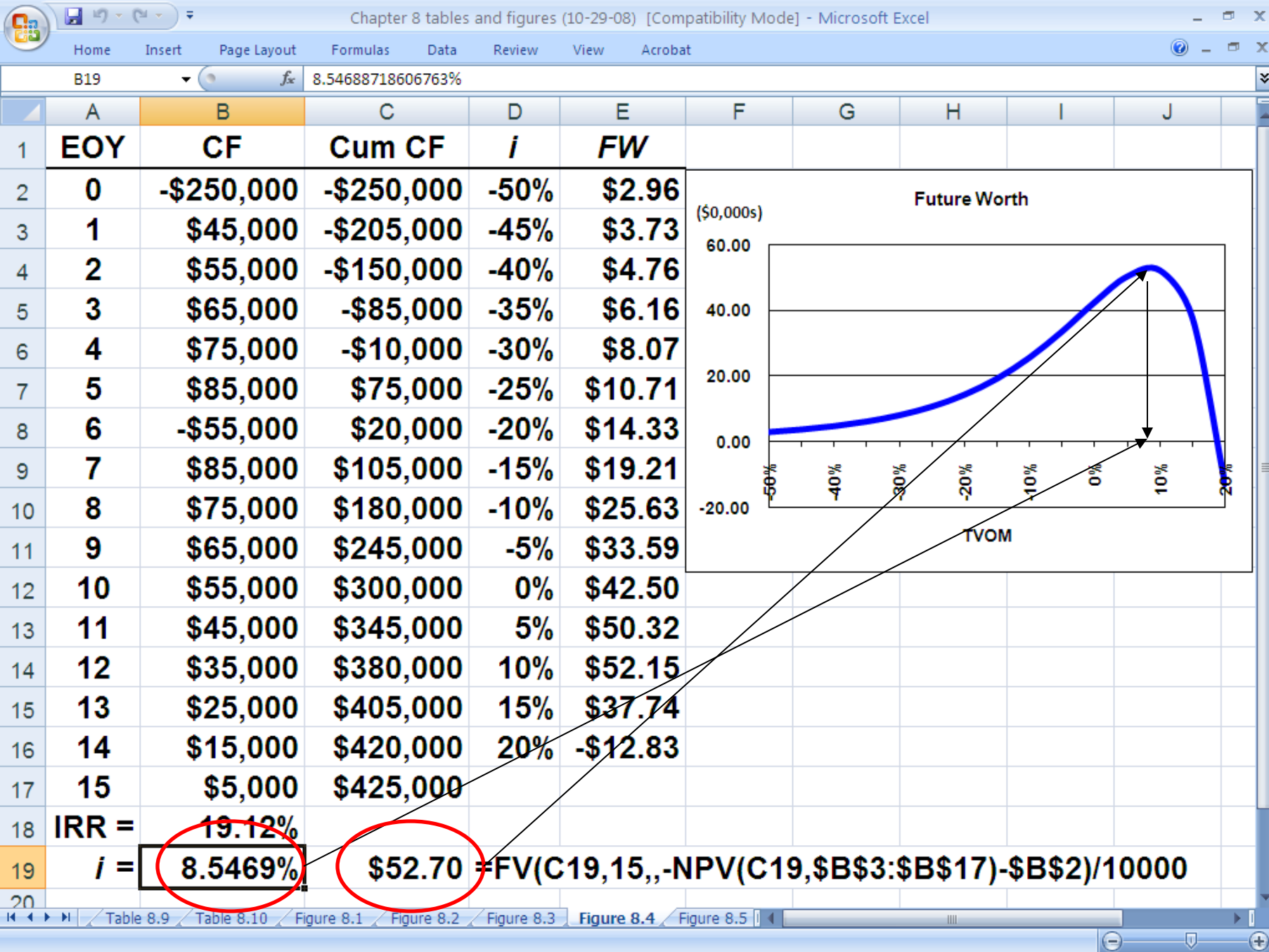
TVOM

B19 fx 8.54688718606763%

	A	B	C	D	E
1	<b>EOY</b>	<b>CF</b>	<b>Cum CF</b>	<b><i>i</i></b>	<b>FW</b>
2	0	-\$250,000	-\$250,000	-50%	\$2.96
3	1	\$45,000	-\$205,000	-45%	\$3.73
4	2	\$55,000	-\$150,000	-40%	\$4.76
5	3	\$65,000	-\$85,000	-35%	\$6.16
6	4	\$75,000	-\$10,000	-30%	\$8.07
7	5	\$85,000	\$75,000	-25%	\$10.71
8	6	-\$55,000	\$20,000	-20%	\$14.33
9	7	\$85,000	\$105,000	-15%	\$19.21
10	8	\$75,000	\$180,000	-10%	\$25.63
11	9	\$65,000	\$245,000	-5%	\$33.59
12	10	\$55,000	\$300,000	0%	\$42.50
13	11	\$45,000	\$345,000	5%	\$50.32
14	12	\$35,000	\$380,000	10%	\$52.15
15	13	\$25,000	\$405,000	15%	\$37.74
16	14	\$15,000	\$420,000	20%	-\$12.83
17	15	\$5,000	\$425,000		
18	<b>IRR =</b>	<b>19.12%</b>			
19	<b><i>i</i> =</b>	<b>8.5469%</b>	<b>\$52.70</b>	<b>=FV(C19,15,-,NPV(C19,\$B\$3:\$B\$17)-\$B\$2)/10000</b>	







	A	B	C	D	E	F	G	H	I	J
	EOY	CF	Cum CF	<i>i</i>	<i>FW</i>					
1	0	-\$250,000	-\$250,000	-50%	\$2.96					
2	1	\$45,000	-\$205,000	-45%	\$3.73					
3	2	\$55,000	-\$150,000	-40%	\$4.76					
4	3	\$65,000	-\$85,000	-35%	\$6.16					
5	4	\$75,000	-\$10,000	-30%	\$8.07					
6	5	\$85,000	\$75,000	-25%	\$10.71					
7	6	-\$55,000	\$20,000	-20%	\$14.33					
8	7	\$85,000	\$105,000	-15%	\$19.21					
9	8	\$75,000	\$180,000	-10%	\$25.63					
10	9	\$65,000	\$245,000	-5%	\$33.59					
11	10	\$55,000	\$300,000	0%	\$42.50					
12	11	\$45,000	\$345,000	5%	\$50.22					

Notice, the cumulative cash flow series changes from negative to positive and remains positive. Therefore, a unique positive real root exists. (Interestingly, if the interim investment is \$200,000, Norstrom's criterion is not met, but a unique positive real root exists. Recall, Norstrom's criterion is a sufficient condition, not a necessary condition.)

$$i = 8.5469\% \quad \$52.70 = FV(C19, 15, -, NPV(C19, \$B\$3:\$B\$17) - \$B\$2) / 10000$$

# **Internal Rate of Return Analysis**

## **Multiple Alternatives**

## Example 8.4

**You have available \$70,000 to invest and have been presented with 5 equal-lived, mutually exclusive investment alternatives with cash flows as depicted below. Currently, you are earning 18% on your investment of the \$70,000. Hence, you will not choose to invest in either of the alternatives if it does not provide a return on investment greater than 18%.**

**Using the internal rate of return method, which (if either) would you choose? What is its rate of return?**

## Data for Example 8.4

Investment	1	2	3	4	5
Initial Investment	\$15,000.00	\$25,000.00	\$40,000.00	\$50,000.00	\$70,000.00
Annual Return	\$3,750.00	\$5,000.00	\$9,250.00	\$11,250.00	\$14,250.00
Salvage Value	\$15,000.00	\$25,000.00	\$40,000.00	\$50,000.00	\$70,000.00
Internal Rate of Return	25.00%	20.00%	23.13%	22.50%	20.36%

**With an 18% MARR, which investment would you choose?**



**When the salvage value equals the initial investment and annual returns are a uniform annual series, the internal rate of return equals the quotient of the annual return and the initial investment**

# Solution to Example 8.4

Investment	1	2 - 1	3 - 1	4 - 3	5 - 4
$\Delta$ Investment					
$\Delta$ Annual Return					
$\Delta$ Salvage Value					
$\Delta$ IRR	25.00%	12.50%	22.00%	20.00%	15.00%
$\geq$ MARR?	Yes	No	Yes	Yes	No
Defender	1	1	3	4	4

# Portfolio Solution to Example 8.4

**“Do Nothing”**

$$\mathbf{\$70,000(0.18) = \$12,600/\text{year}}$$

**Invest in 1**

$$\mathbf{\$3,750 + \$55,000(0.18) = \$13,650}$$

**Prefer 1 to “Do Nothing”**

**Invest in 2**

$$\mathbf{\$5,000 + \$45,000(0.18) = \$13,100}$$

**Prefer 1 to 2**

# Portfolio Solution to Example 8.4

Invest in 3

$$\$9,250 + \$30,000(0.18) = \$14,650$$

Prefer 3 to 1

Invest in 4

$$\$11,250 + \$20,000(0.18) = \$14,850$$

Prefer 4 to 3

Invest in 5

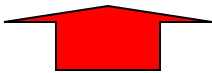
$$\$14,250$$

Prefer 4 to 5

**Choose 4**

# Present Worths with 10-Year Planning Horizon

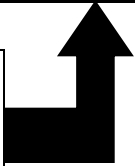
Investment	1	2	3	4	5
Initial Investment	\$15,000.00	\$25,000.00	\$40,000.00	\$50,000.00	\$70,000.00
Annual Return	\$3,750.00	\$5,000.00	\$9,250.00	\$11,250.00	\$14,250.00
Salvage Value	\$15,000.00	\$25,000.00	\$40,000.00	\$50,000.00	\$70,000.00
Present Worth	\$4,718.79	\$2,247.04	\$9,212.88	\$10,111.69	\$7,415.24



# Present Worths with 10-Year Planning Horizon

Investment	1	2	3	4	5
Initial Investment	\$15,000.00	\$25,000.00	\$40,000.00	\$50,000.00	\$70,000.00
Annual Return	\$3,750.00	\$5,000.00	\$9,250.00	\$11,250.00	\$14,250.00
Salvage Value	\$15,000.00	\$25,000.00	\$40,000.00	\$50,000.00	\$70,000.00
Present Worth	\$4,718.79	\$2,247.04	\$9,212.88	\$10,111.69	\$7,415.24

$$\begin{aligned}
 PW &= \$11,250(P/A\ 18\%,10) + \$50,000(P/F\ 18\%,10) - \$50,000 \\
 &= PV(18\%,10,-11250,-50000)-50000
 \end{aligned}$$



## **Principle #6**

**Continue to invest as long as each additional increment of investment yields a return that is greater than the investor's TVOM**

**“The object of management is not necessarily the highest rate of return on capital, but ... to assure profit with each increment of volume that will at least equal the economic cost of additional capital required.”**

**Donald Brown  
Chief Financial Officer  
General Motors  
1924**



# Example 8.5

Recall the theme park example involving two designs for the new ride, The Scream Machine: A costs \$300,000, has \$55,000/yr revenue, and has a negligible salvage value at the end of the 10-year planning horizon; B costs \$450,000, has \$80,000/yr revenue, and has a negligible salvage value. Based on an IRR analysis and a 10% MARR, which is preferred?

$$PW_A(12\%) = -\$300,000 + \$55,000(P/A\ 12\%,10) = \$10,762.10$$

$$PW_A(15\%) = -\$300,000 + \$55,000(P/A\ 15\%,10) = -\$23,967.65$$

interpolating,

$$IRR_A = 12\% + 3\%(\$10,762.10)/(\$10,762.10 + \$23,967.65) = 12.93\%$$

$$IRR_A = \text{RATE}(10, -55000, 300000)$$

$$= 12.87\% > \text{MARR} = 10\% \text{ (Alt. A is acceptable)}$$

$$PW_{B-A}(12\%) = -\$150,000 + \$25,000(P/A\ 12\%,10) = -\$8744.50$$

$$PW_{B-A}(10\%) = -\$150,000 + \$25,000(P/A\ 10\%,10) = \$37,951.35$$

interpolating,

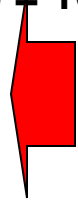
$$IRR_{B-A} = 10\% + 2\%(\$8744.50)/(\$8744.50 + \$37,951.35) = 10.375\%$$

$$IRR_{B-A} = \text{RATE}(10, -25000, 150000)$$

$$= 10.56\% > \text{MARR} = 10\% \text{ (Alt. B is preferred)}$$

$$IRR_B = \text{RATE}(10, -80000, 450000)$$

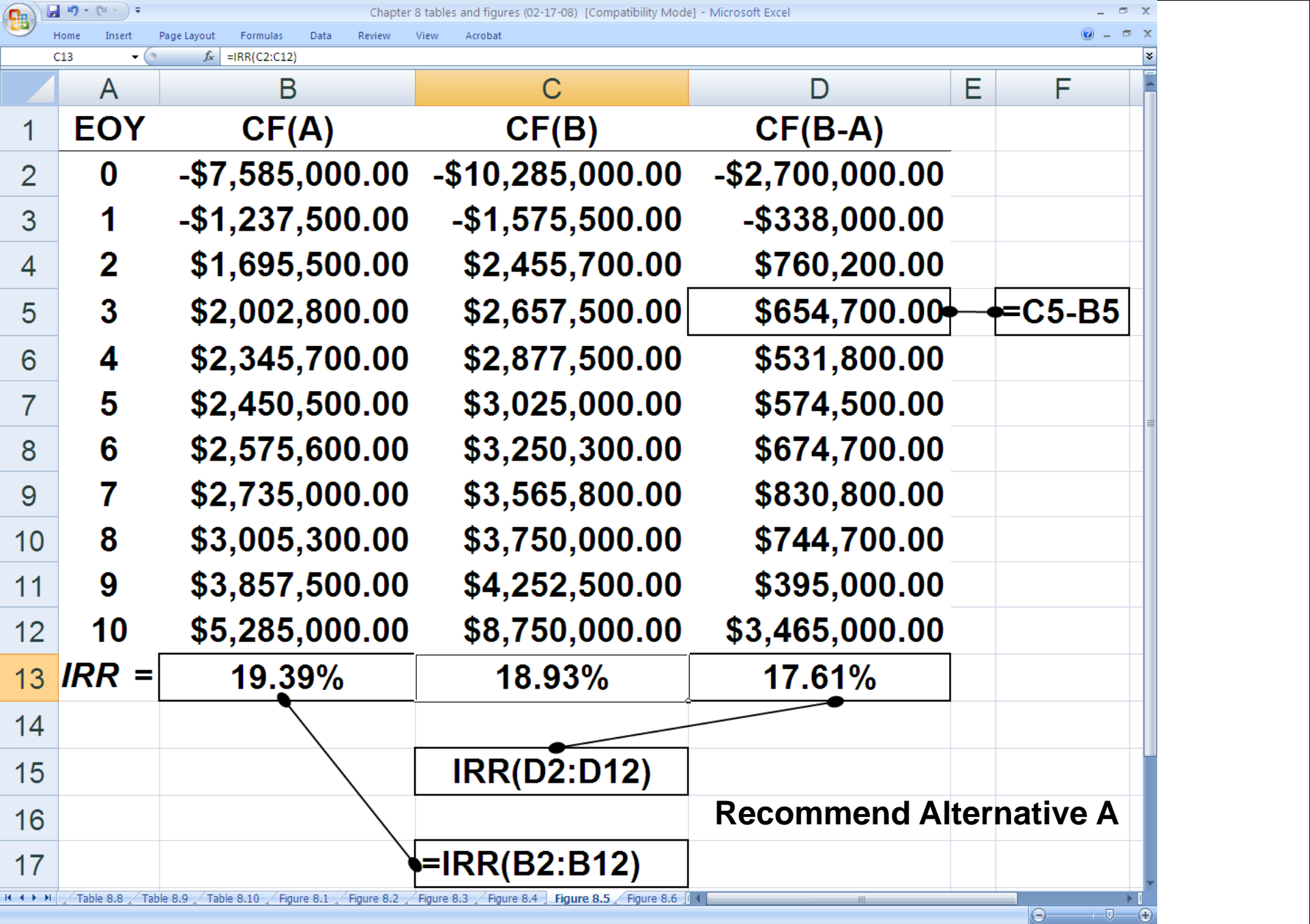
$$= 12.11\%$$



# Example 8.6

A batch chemical processing company is adding centrifuges. Two alternatives are in consideration. The estimated cash flow profiles are shown below. Using a MARR of 18.5%, which should be chosen?

EOY	CF(A)	CF(B)
0	-\$7,585,000.00	-\$10,285,000.00
1	-\$1,237,500.00	-\$1,575,500.00
2	\$1,695,500.00	\$2,455,700.00
3	\$2,002,800.00	\$2,657,500.00
4	\$2,345,700.00	\$2,877,500.00
5	\$2,450,500.00	\$3,025,000.00
6	\$2,575,600.00	\$3,250,300.00
7	\$2,735,000.00	\$3,565,800.00
8	\$3,005,300.00	\$3,750,000.00
9	\$3,857,500.00	\$4,252,500.00
10	\$5,285,000.00	\$8,750,000.00



# Example 8.7

Three mutually exclusive investment alternatives are being considered; the cash flow profiles are shown below. Based on a 15% MARR, which should be chosen?

EOY	CF(1)	CF(2)	CF(3)
0	-\$100,000	-\$125,000	-\$150,000
1	\$20,000	-\$25,000	-\$35,000
2	\$20,000	\$75,000	\$75,000
3	\$20,000	\$70,000	\$75,000
4	\$20,000	\$60,000	\$75,000
5	\$120,000	\$55,000	\$95,000

# Example 8.7 (Continued)

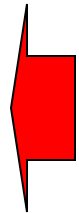
EOY	CF(1)	CF(2)	CF(3)	CF(2-1)	CF(3-2)
0	-\$100,000	-\$125,000	-\$150,000	-\$25,000	-\$25,000
1	\$20,000	-\$25,000	-\$35,000	-\$45,000	-\$10,000
2	\$20,000	\$75,000	\$75,000	\$55,000	\$0
3	\$20,000	\$70,000	\$75,000	\$50,000	\$5,000
4	\$20,000	\$60,000	\$75,000	\$40,000	\$15,000
5	\$120,000	\$55,000	\$95,000	-\$65,000	\$40,000
<b>IRR =</b>	20.00%	19.39%	18.01%	16.41%	13.41%

Recommend Alternative 2

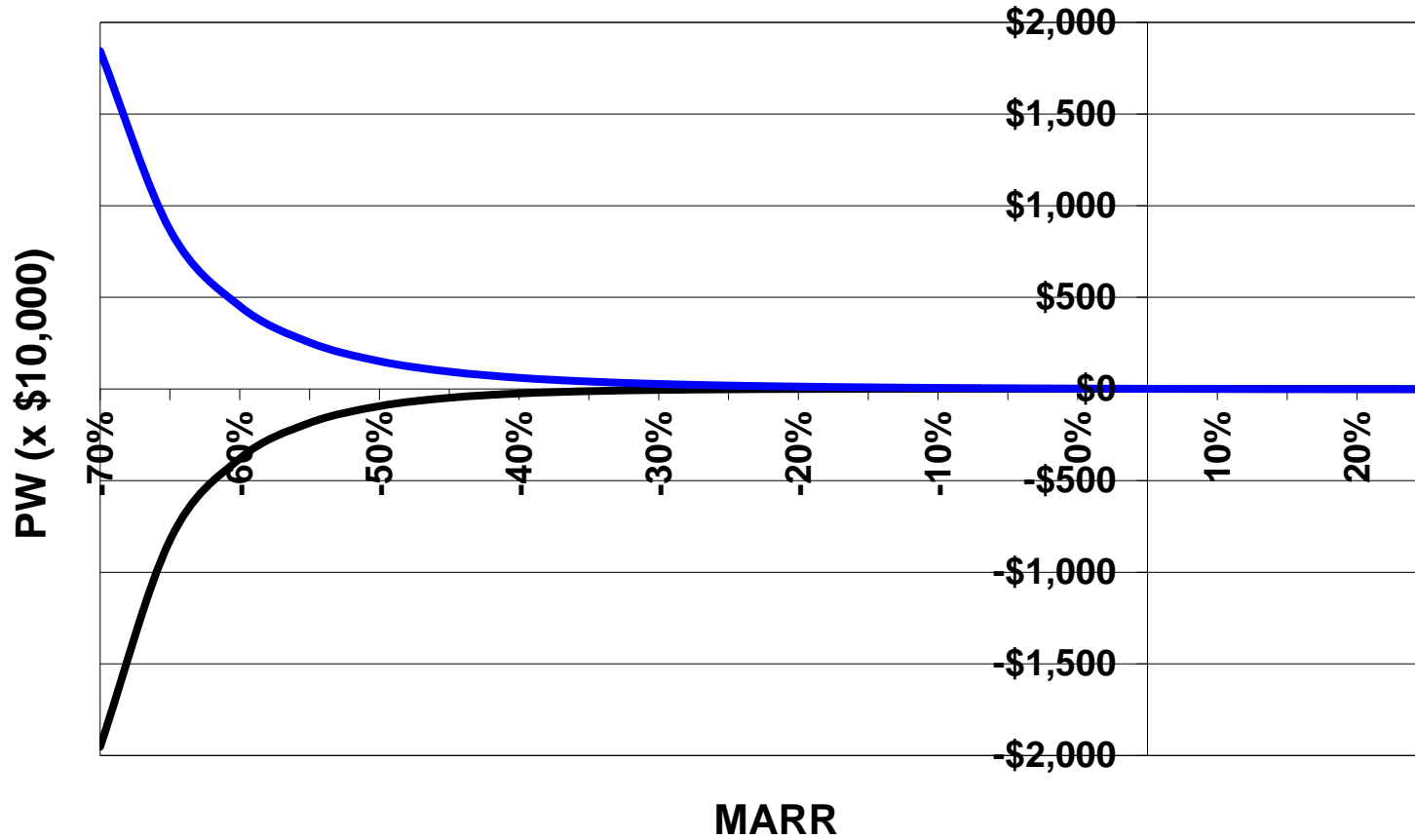
$$PW_1(15\%) = PV(0.15, 5, -20000, -100000) - 100000 = \$16,760.78$$

$$PW_2(15\%) = NPV(0.15, -25, 75, 70, 60, 55) * 1000 - 125000 = \$17,647.70$$

$$PW_3(15\%) = NPV(0.15, -35, 75, 75, 75, 95) * 1000 - 150000 = \$15,702.99$$

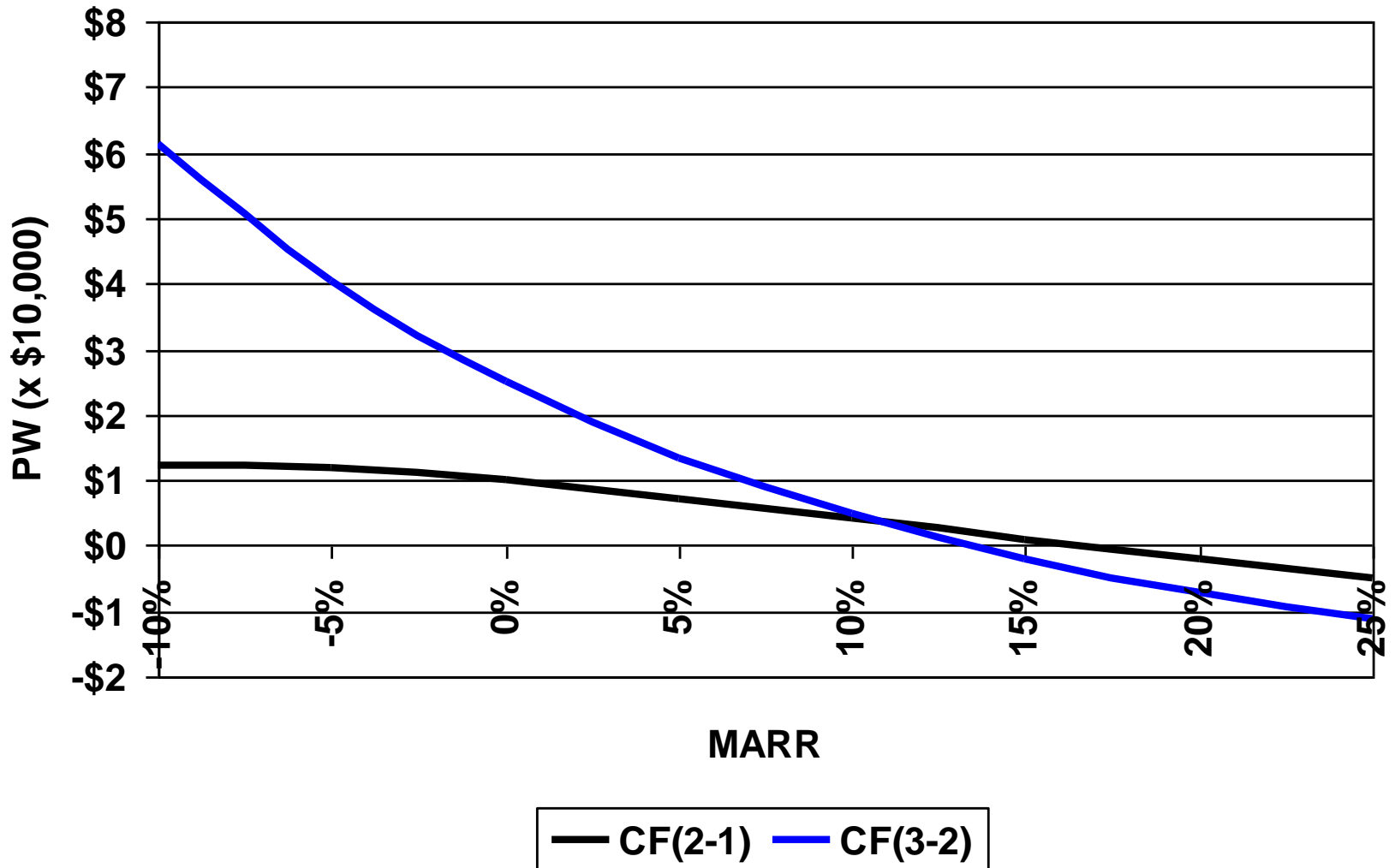


# Incremental IRR Comparison of Alternatives



— CF(2-1)    — CF(3-2)

## Incremental IRR Comparison of Alternatives



# **External Rate of Return Analysis**

## **Single Alternative**



# External Rate of Return Method

- equates the future worth of positive cash flows using the MARR to the future worth of negative cash flows using the ERR,  $i'$
- not a popular DCF method

$$\sum_{t=0}^n R_t (1 + r)^{n-t} = \sum_{t=0}^n C_t (1 + i')^{n-t}$$

$R_t$  is positive-valued cash flow and  $C_t$  is the absolute value of a negative-valued cash flow;  $r$  is the MARR

(useful way to avoid the “multiple root” problem of the IRR)

# Relationships among *MARR*, *IRR*, and *ERR*

- If  $IRR < MARR$ , then  $IRR < ERR < MARR$
- If  $IRR > MARR$ , then  $IRR > ERR > MARR$
- If  $IRR = MARR$ , then  $IRR = ERR = MARR$

# Example 8.8

## SMP Investment

### External Rate of Return Solution

<u>EOY</u>	<u>CF</u>			
0	-\$500,000			
1 - 10	\$92,500			
10	\$50,000			
	<b><math>\\$500,000(F P\ i'\%, 10) = \\$92,500(F A\ 10\%, 10)</math></b>			
			<b>+ \$50,000</b>	

# Example 8.8

## SMP Investment

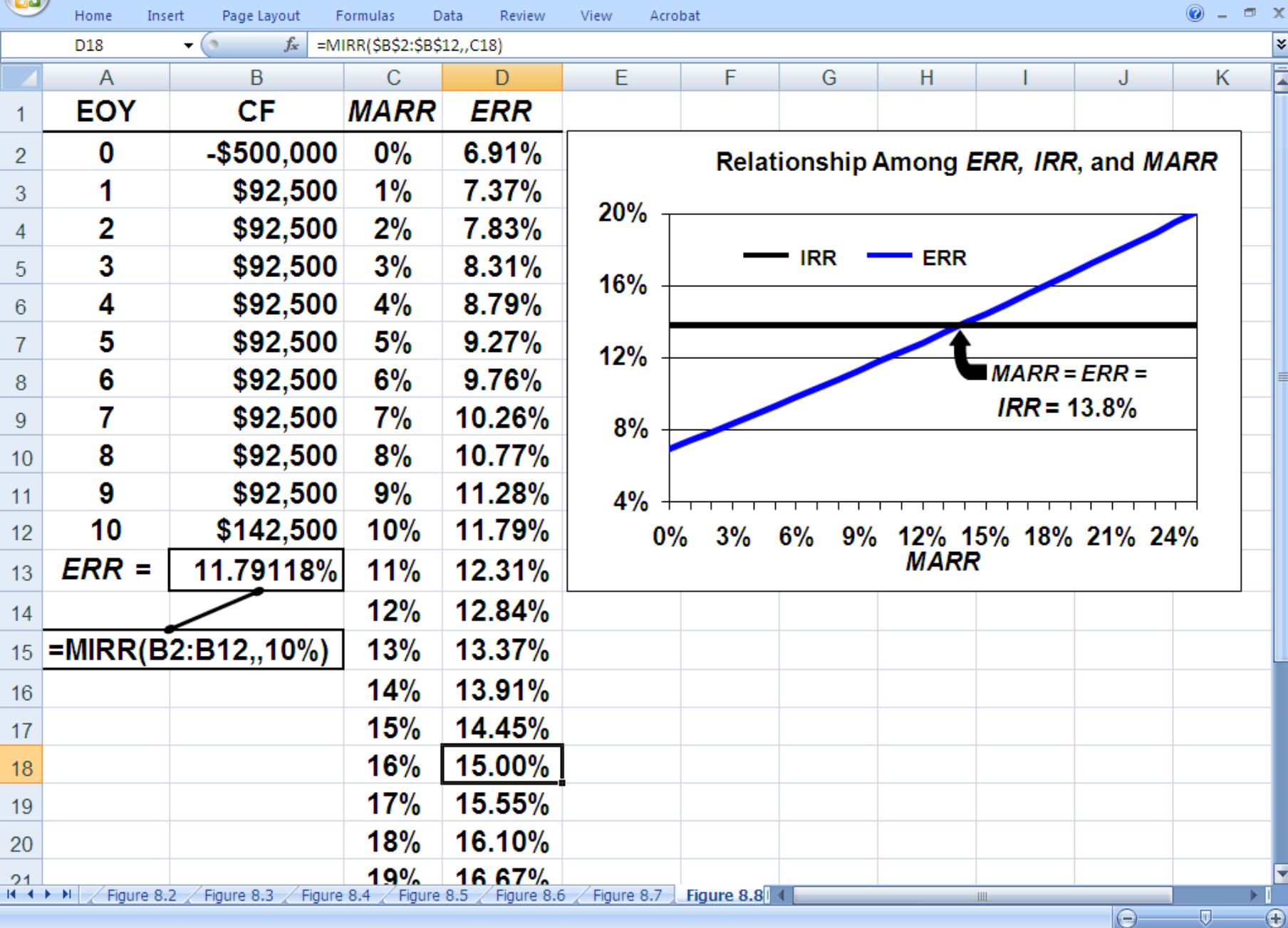
### External Rate of Return Solution

<u>EOY</u>	<u>CF</u>			
0	-\$500,000			
1 - 10	\$92,500			
10	\$50,000			
	<b><math>\\$500,000(F P\ i'\%, 10) = \\$92,500(F A\ 10\%, 10)</math></b>			
	<b>+ \$50,000</b>			
	<b><math>(F P\ i'\%, 10) = 3.048423</math></b>			
	<b><math>i' = 11.79117\%</math></b>			
	<b><math>i' = 11.79118\%</math> (using Excel)</b>			

# Example 8.8

## Excel Solution

	A	B		
1	<u>EOY</u>	<u>CF</u>		
2	0	-\$500,000		
3	1	\$92,500		
4	2	\$92,500		
5	3	\$92,500		
6	4	\$92,500		
7	5	\$92,500		
8	6	\$92,500		
9	7	\$92,500		
10	8	\$92,500		
11	9	\$92,500		
12	10	\$142,500		
13	ERR =	[=MIRR(B2:B12,,10%)]		
14	ERR =	11.79118%		




# Example 8.9

Recall the cash flow profile, given below, that produced 3 *IRR* values: 20%, 40%, or 50%. If *MARR* = 12%, what is the *ERR*? For various values of *MARR*, what are the corresponding values of *ERR*?

EOY	CF
0	-\$4,000
1	\$16,400
2	-\$22,320
3	\$10,080

$$\begin{aligned} \$4000(F/P i',3) + \$22,320(F/P i',1) = & \$16,400(F/P MARR,2) \\ & + \$10,080 \end{aligned}$$

	A	B	C	D	E
	<b>EOY</b>	<b>CF</b>			
1					
2	<b>0</b>	<b>-\$4,000</b>			
3	<b>1</b>	<b>\$16,400</b>			
4	<b>2</b>	<b>-\$22,320</b>			
5	<b>3</b>	<b>\$10,080</b>			
6	<b>ERR =</b>	<b>10.0000%</b>			
7	<b>EQ 8.2</b>	<b>\$776.16</b>			
8					
9	<b>=FV(B6,3,,4000)+FV(12%,2,,-16400)+FV(B6,1,,22320)+10080</b>				




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B7  $=FV(B6,3,,4000)+FV(12\%,2,, -16400)+FV(B6,1,,22320)+10080$


A B C D E

1  
2  
3  
4  
5  
6  
7  
8  
9

**Solver Parameters**

Set Target Cell:  


Equal To:  Max  Min  Value of:

By Changing Cells:  

Subject to the Constraints:

$=FV(B6,3,,4000)+FV(12\%,2,, -16400)+FV(B6,1,,22320)+10080$

Figure 8.4 Figure 8.5 Figure 8.6 Figure 8.7 Figure 8.8 Figure 8.9 Figure 8.10

	A	B	C	D	E
1	<b>EOY</b>	<b>CF</b>			
2	<b>0</b>	<b>-\$4,000</b>			
3	<b>1</b>	<b>\$16,400</b>			
4	<b>2</b>	<b>-\$22,320</b>			
5	<b>3</b>	<b>\$10,080</b>			
6	<b>ERR =</b>	<b>12.0911%</b>			
7	<b>EQ 8.2</b>	<b>\$0.00</b>			
8					
9	<b>=FV(B6,3,,4000)+FV(12%,2,, -16400)+FV(B6,1,,22320)+10080</b>				

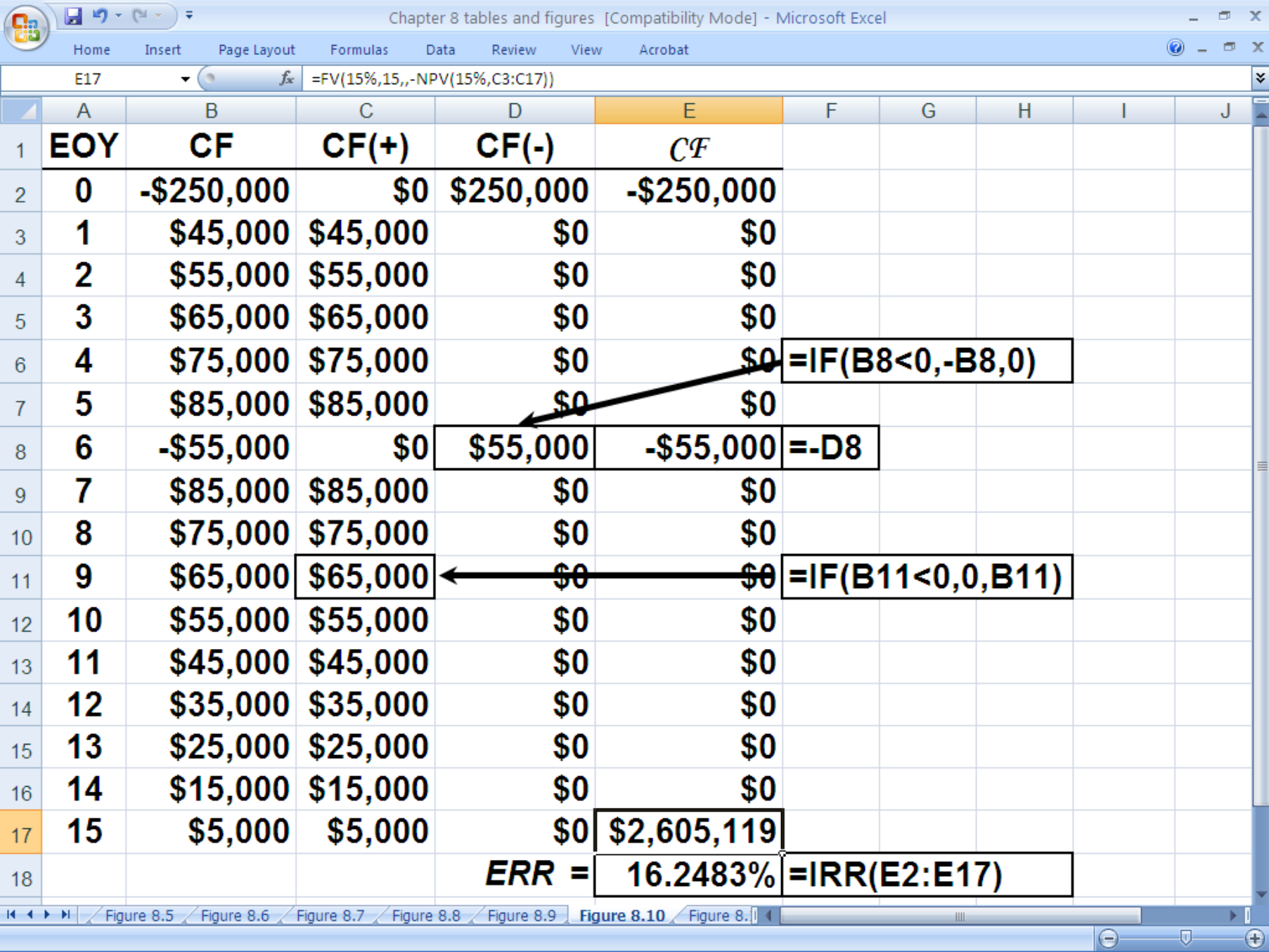
## ***ERR* Values for Various *MARR* Values in Example 8.9**

<b><i>MARR</i></b>	<b><i>ERR</i></b>	<b><i>MARR</i></b>	<b><i>ERR</i></b>
<b>0%</b>	<b>0.4654%</b>	<b>30%</b>	<b>29.9812%</b>
<b>2%</b>	<b>2.3768%</b>	<b>32%</b>	<b>31.9840%</b>
<b>4%</b>	<b>4.2999%</b>	<b>34%</b>	<b>33.9877%</b>
<b>6%</b>	<b>6.2338%</b>	<b>36%</b>	<b>35.9919%</b>
<b>8%</b>	<b>8.1775%</b>	<b>38%</b>	<b>37.9962%</b>
<b>10%</b>	<b>10.1302%</b>	<b>40%</b>	<b>40.0000%</b>
<b>12%</b>	<b>12.0911%</b>	<b>42%</b>	<b>42.0030%</b>
<b>14%</b>	<b>14.0592%</b>	<b>44%</b>	<b>44.0049%</b>
<b>16%</b>	<b>16.0339%</b>	<b>46%</b>	<b>46.0052%</b>
<b>18%</b>	<b>18.0144%</b>	<b>48%</b>	<b>48.0037%</b>
<b>20%</b>	<b>20.0000%</b>	<b>50%</b>	<b>50.0000%</b>
<b>22%</b>	<b>21.9900%</b>	<b>52%</b>	<b>51.9939%</b>
<b>24%</b>	<b>23.9837%</b>	<b>54%</b>	<b>53.9850%</b>
<b>26%</b>	<b>25.9805%</b>	<b>56%</b>	<b>55.9732%</b>
<b>28%</b>	<b>27.9799%</b>	<b>58%</b>	<b>57.9581%</b>

# Example 8.10

Recall Julian Stewart's \$250,000 investment in a limited partnership to drill for natural gas. The investment yielded annual returns of \$45,000 the 1<sup>st</sup> yr, followed by \$10,000 increases until the 6<sup>th</sup> yr, at which time an additional \$150,000 had to be invested for deeper drilling. Following the 2<sup>nd</sup> drilling, the annual returns decreased by \$10,000 per year, from \$85,000 to \$5,000. Since there were multiple negative values in the cash flow profile for the investment (EOY = 0 and EOY = 6), Excel's MIRR worksheet function cannot be used to solve for ERR.

When faced with multiple negative-valued cash flows, we construct a new  $CF$  profile that contains the negative-valued cash flows, zeroes, and the future worth of the positive-valued cash flows, with the FW based on the MARR.



# **External Rate of Return Analysis**

## **Multiple Alternatives**

# Example 8.11

Recall the 5 equal-lived, mutually exclusive investment alternatives which guaranteed your original investment back at any time you wished to end the investment. With \$70,000 to invest and an 18% *MARR*, you chose investment 4 using an *IRR* analysis.

Using the external rate of return method, which would you choose? What is its external rate of return? (We use a 10-year planning horizon.)

# Solution to Example 8.11

Investment	1	2	3	4	5
Initial Investment	\$15,000.00	\$25,000.00	\$40,000.00	\$50,000.00	\$70,000.00
Annual Return	\$3,750.00	\$5,000.00	\$9,250.00	\$11,250.00	\$14,250.00
Salvage Value	\$15,000.00	\$25,000.00	\$40,000.00	\$50,000.00	\$70,000.00
ERR	21.27%	19.02%	20.47%	20.19%	19.19%



Investment	1	2 - 1	3 - 1	4 - 3	5 - 4
$\Delta$ Investment	\$15,000.00	\$10,000.00	\$25,000.00	\$10,000.00	\$20,000.00
$\Delta$ Annual Return	\$3,750.00	\$1,250.00	\$5,500.00	\$2,000.00	\$3,000.00
$\Delta$ Salvage Value	\$15,000.00	\$10,000.00	\$25,000.00	\$10,000.00	\$20,000.00
$\Delta$ ERR	21.27%	14.70%	19.97%	19.02%	16.30%
$\geq$ MARR?	Yes	No	Yes	Yes	No
Defender	1	1	3	4	4

**Choose 4**



# Solution to Example 8.11

Investment	1	2	3	4	5
Initial Investment	\$15,000.00	\$25,000.00	\$40,000.00	\$50,000.00	\$70,000.00
Annual Return	\$3,750.00	\$5,000.00	\$9,250.00	\$11,250.00	\$14,250.00
Salvage Value	\$15,000.00	\$25,000.00	\$40,000.00	\$50,000.00	\$70,000.00
ERR	21.27%	19.02%	20.47%	20.19%	19.19%

`=RATE(10,,-50000,FV(18%,10,-11250)+50000)`



Investment	1	2 - 1	3 - 1	4 - 3	5 - 4
$\Delta$ Investment	\$15,000.00	\$10,000.00	\$25,000.00	\$10,000.00	\$20,000.00
$\Delta$ Annual Return	\$3,750.00	\$1,250.00	\$5,500.00	\$2,000.00	\$3,000.00
$\Delta$ Salvage Value	\$15,000.00	\$10,000.00	\$25,000.00	\$10,000.00	\$20,000.00
$\Delta$ ERR	21.27%	14.70%	19.97%	19.02%	16.30%
$\geq$ MARR?	Yes	No	Yes	Yes	No
Defender	1	1	3	4	4

**Choose 4**

# Example 8.12

Recall the example involving two designs (A & B) for a new ride at a theme park in Florida: A costs \$300,000, has \$55,000/yr revenue, and has a negligible salvage value at the end of the 10-year planning horizon; B costs \$450,000, has \$80,000/yr revenue, and has a negligible salvage value. Based on an *ERR* analysis and a 10% *MARR*, which is preferred?

$$\begin{aligned}ERR_A(10\%) &= \text{RATE}(10,,-300000,\text{FV}(10\%,10,-55000)) \\ &= 11.31814\% > MARR = 10\% \text{ (A is acceptable)}\end{aligned}$$

$$\begin{aligned}ERR_{B-A}(10\%) &= \text{RATE}(10,,-150000,\text{FV}(10\%,10,-25000)) \\ &= 10.26219\% > MARR = 10\% \text{ (B is preferred)}\end{aligned}$$


$$\begin{aligned}ERR_B(10\%) &= \text{RATE}(10,,-450000,\text{FV}(10\%,10,-80000)) \\ &= 10.97611\%\end{aligned}$$

# Example 8.13

Recall the batch chemical processing company that is considering two centrifuges for possible acquisition. The estimated cash flows are given below. With an 18.5% *MARR*, which should be chosen using an *ERR* analysis?

EOY	CF(A)	CF(B)
0	-\$7,585,000.00	-\$10,285,000.00
1	-\$1,237,500.00	-\$1,575,500.00
2	\$1,695,500.00	\$2,455,700.00
3	\$2,002,800.00	\$2,657,500.00
4	\$2,345,700.00	\$2,877,500.00
5	\$2,450,500.00	\$3,025,000.00
6	\$2,575,600.00	\$3,250,300.00
7	\$2,735,000.00	\$3,565,800.00
8	\$3,005,300.00	\$3,750,000.00
9	\$3,857,500.00	\$4,252,500.00
10	\$5,285,000.00	\$8,750,000.00



# Example 8.14

Recall, the three mutually exclusive investment alternatives having the cash flow profiles shown below. Based on a 15% *MARR* and *ERR* analysis, which should be chosen?

EOY	CF(1)	CF(2)	CF(3)
0	-\$100,000	-\$125,000	-\$150,000
1	\$20,000	-\$25,000	-\$35,000
2	\$20,000	\$75,000	\$75,000
3	\$20,000	\$70,000	\$75,000
4	\$20,000	\$60,000	\$75,000
5	\$120,000	\$55,000	\$95,000

	A	B	C	D	E	F	G	H	I	J
	EOY	CF(1)	CF(2)	CF(2-1)	CF(2-1)	CF(3)	CF(3-2)	CF(3-2)		
1	0	-\$100,000	-\$125,000	-\$25,000	-\$25,000	-\$150,000	-\$25,000	-\$25,000		
2	1	\$20,000	-\$25,000	-\$45,000	-\$45,000	-\$35,000	-\$10,000	-\$10,000		
3	2	\$20,000	\$75,000	\$55,000	\$0	\$75,000	\$0	\$0		
4	3	\$20,000	\$70,000	\$50,000	\$0	\$75,000	\$5,000	\$0		
5	4	\$20,000	\$60,000	\$40,000	\$0	\$75,000	\$15,000	\$0		
6	5	\$120,000	\$55,000	-\$65,000	\$130,773	\$95,000	\$40,000	\$63,863		
7	$ERR_1 =$	18.6198%		$ERR_{2-1} =$	15.3604%		$ERR_{3-2} =$	13.5670%	$=IRR(H2:H7)$	
8		$=MIRR(B2:B7,,15\%)$			$=IRR(E2:E7)$		$=FV(15\%,4,, -NPV(15\%,D4:D6))+D7$			

**Choose Investment 2**

# **Analyzing Alternatives with No Positive Cash Flows**

# Example 8.15

A company must purchase a new incinerator to meet air quality standards. Three alternatives have been identified, with cash flow profiles given below. Based on a 7-year planning horizon and a 12% *MARR*, which should be purchased? Perform *PW*, *AW*, *IRR*, and *ERR* analyses.

Alt	Initial Investment	Annual Operating Cost	Annual Maintenance Cost
A	\$250,000	\$105,000	\$42,000
B	\$385,000	\$78,000	\$28,000
C	\$475,000	\$65,000	\$18,000



# Solution to Example 8.15

$$\begin{aligned}PW_A(12\%) &= \$250,000 + \$147,000(P|A \ 12\%,7) \\ &= \$250,000 + \$147,000(4.56376) \\ &= \$920,872.72 \\ &= PV(12\%,7,-147000)+250000 \\ &= \$920,872.21\end{aligned}$$

$$\begin{aligned}PW_B(12\%) &= \$385,000 + \$106,000(P|A \ 12\%,7) \\ &= \$385,000 + \$106,000(4.56376) \\ &= \$868,758.56 \\ &= PV(12\%,7,-106000)+385000 \\ &= \$868,758.19\end{aligned}$$

$$\begin{aligned}PW_C(12\%) &= \$475,000 + \$83,000(P|A \ 12\%,7) \\ &= \$475,000 + \$83,000(4.56376) \\ &= \$853,792.08 \\ &= PV(12\%,7,-83000)+475000 \\ &= \$853,791.79\end{aligned}$$

**Choose C**



# Solution to Example 8.15 (Continued)

$$\begin{aligned} \text{EUAC}_A(12\%) &= \$250,000(\text{A|P } 12\%, 7) + \$147,000 \\ &= \$250,000(0.21912) + \$147,000 \\ &= \$201,780.00 \\ &= \text{PMT}(12\%, 7, -250000) + 147000 \\ &= \$201,779.43 \end{aligned}$$

$$\begin{aligned} \text{EUAC}_B(12\%) &= \$385,000(\text{A|P } 12\%, 7) + \$106,000 \\ &= \$385,000(0.21912) + \$106,000 \\ &= \$190,361.20 \\ &= \text{PMT}(12\%, 7, -385000) + 106000 \\ &= \$190,360.33 \end{aligned}$$

$$\begin{aligned} \text{EUAC}_C(12\%) &= \$475,000 + \$83,000(\text{P|A } 12\%, 7) \\ &= \$475,000 + \$83,000(4.56376) \\ &= \$187,082.00 \\ &= \text{PMT}(12\%, 7, -475000) + 83000 \\ &= \$187,080.92 \end{aligned}$$

**Choose C**



# Solution to Example 8.15 (Continued)

## IRR analysis

Incremental solution: B-A (\$135,000 incremental investment produces \$41,000 incremental reduction in annual costs)

$$\text{IRR}_{B-A}(12\%) = \text{RATE}(7, 41000, -135000) \\ = 23.4\% > 12\% \text{ (B} \gg \text{A)}$$

Incremental solution: C-B (\$90,000 incremental investment produces \$23,000 incremental reduction in annual costs)

$$\text{IRR}_{C-B}(12\%) = \text{RATE}(7, 23000, -90000) \\ = 17.082\% > 12\% \text{ (C} \gg \text{B)}$$



**Choose C**

# Solution to Example 8.15 (Continued)

## ERR analysis

Incremental solution: B-A (\$135,000 incremental investment yields \$41,000 reduction in annual costs)

$$\$135,000(1+ERR_{B-A})^7 = \$41,000(F|A\ 12\%,7)$$

$$(1+ERR_{B-A})^7 = \$41,000(10.08901)/\$135,000$$

$$ERR_{B-A} = 17.347\% > 12\% (B \gg A)$$

$$= \text{RATE}(7, -135000, FV(12\%, 7, -41000))$$

$$= 17.347\% > 12\%$$

Incremental solution: C-B (\$90,000 incremental investment yields \$23,000 reduction in annual costs)

$$\$90,000(1+ERR_{C-B})^7 = \$23,000(F|A\ 12\%,7)$$

$$(1+ERR_{C-B})^7 = \$23,000(10.08901)/\$90,000$$

$$ERR_{C-B} = 14.489\% > 12\% (C \gg B)$$

$$= \text{RATE}(7, -90000, FV(12\%, 7, -23000))$$

$$= 14.489\% > 12\%$$

Choose C



## Pit Stop #8—Halfway Home! Miles to Go!

1. True or False: For personal investment decision making, rates of return are used more frequently than present worth.
2. True or False: Unless non-monetary considerations dictate otherwise, you should choose the mutually exclusive investment alternative having the greatest rate of return over the planning horizon.
3. True or False: If  $ERR > MARR$ , then  $IRR > ERR > MARR$ .
4. True or False: If  $PW > 0$ , then  $IRR > MARR$ .
5. True or False: If  $ERR > MARR$ , then  $MIRR > MARR$ .
6. True or False: If  $IRR(A) > IRR(B)$ , then  $ERR(A) > ERR(B)$ .
7. True or False: If  $PW(A) > PW(B)$ , then  $FW(A) > FW(B)$ ,  $AW(A) > AW(B)$ ,  $CW(A) > CW(B)$ , and  $IRR(A) > IRR(B)$ .
8. True or False: Multiple roots can exist when using  $IRR$  and  $MIRR$  methods.
9. True or False: Excel's  $IRR$  worksheet function signals if multiple roots exist for a cash flow series.
10. True or False: Of all the equivalent  $DCF$  methods, the one that is the most difficult to use is the external rate of return method because of its requirement of a reinvestment rate for recovered capital.

## Pit Stop #8—Halfway Home! Miles to Go!

1. True or False: For personal investment decision making, rates of return are used more frequently than present worth. **TRUE**
2. True or False: Unless non-monetary considerations dictate otherwise, you should choose the mutually exclusive investment alternative having the greatest rate of return over the planning horizon. **FALSE**
3. True or False: If  $ERR > MARR$ , then  $IRR > ERR > MARR$ . **TRUE**
4. True or False: If  $PW > 0$ , then  $IRR > MARR$ . **TRUE**
5. True or False: If  $ERR > MARR$ , then  $MIRR > MARR$ . **FALSE**
6. True or False: If  $IRR(A) > IRR(B)$ , then  $ERR(A) > ERR(B)$ . **FALSE**
7. True or False: If  $PW(A) > PW(B)$ , then  $FW(A) > FW(B)$ ,  $AW(A) > AW(B)$ ,  $CW(A) > CW(B)$ , and  $IRR(A) > IRR(B)$ . **FALSE**
8. True or False: Multiple roots can exist when using  $IRR$  and  $MIRR$  methods. **FALSE**
9. True or False: Excel's  $IRR$  worksheet function signals if multiple roots exist for a cash flow series. **FALSE**
10. True or False: Of all the equivalent  $DCF$  methods, the one that is the most difficult to use is the external rate of return method because of its requirement of a reinvestment rate for recovered capital. **FALSE**